Abstract
This paper deals with a unique multi-objective interactive exploration approach. The purpose of the exploration is to identify superior concepts from a large number of conceptual solutions. The superiority is to be decided based on both performance and un-modeled decision-maker preferences. The sought concepts and their particular solutions are expected to serve as candidates for further search and selection of a conceptual or a particular solution. A novel interactive approach is proposed, which employs aggregation of concepts into families of concepts. Interactive suppressing of families of concepts is suggested to cope with runs involving a large number of concepts and a strict run-time limitation.

The proposed approach is demonstrated using a real-life problem of searching for design alternatives of a propulsion system. The paper is concluded with suggestions for future research.

1 Introduction
The main motivation for the proposed search approach is the need to develop efficient search methods to support design space exploration and concept selection (e.g., [Łukasiewycz et al., 2008], [Mattson and Messac, 2005]). Our approach to solving such exploration problems involves pre-defined meaningful sets of feasible solutions. Each of these sets is termed a concept (also known as conceptual solution) [Moshaiov et al., 2015]. In essence, each concept is represented as a set of potential solution alternatives that have some common features. This representation approach, which has been termed the Set-Based Concept (SBC) approach, allows exploring design spaces on both the conceptual and particular solution levels. In the SBC approach, the performance vectors of particular designs, from all concepts, are compared in a mutual objective space. Figure 1 illustrates the SBC approach. Three concepts of aircrafts are shown. The (generally different) design spaces of the concepts are marked by ellipses of different gray levels. Also shown are some examples of performance vectors in the objective space.

As in [Moshaiov et al., 2015] and [Snir et al., 2015], the current study employs SBC representations and the concept-based relaxed-Pareto approach, as a means for Concept-based Design Space Exploration (C-DSE). In such exploration, the front of each concept is searched for within a relaxation zone, which is defined with respect to the so-called s-Pareto front. The s-Pareto front is the (global) front of the non-dominated solutions of the entire set of solutions (the union of all the concepts’ sets). Figure 2 highlights the idea of such an exploration.

The figure concerns the optimal performances of ten concepts by showing their individual fronts in the objective space.

Figure 1 Illustration of the SBC approach

Figure 2 Concept-based relaxed-Pareto approach
space. Each of the ten fronts is marked by a different symbol. The s-Pareto front for the shown case is the front that is marked by v. The associated relaxation zone is the part of the gray area in the objective space, which is bounded by the bold black curves and the s-Pareto front.

In the aforementioned C-DSE, the Decision Makers (DMs) are interested in filtering-out concepts with fronts outside the relaxation zone. It should be noted that prior to the search the relaxation zone is unknown, nor are the fronts within that zone. This is because the relaxation is measured relative to the s-Pareto front, which is a part of the sought information. It is further noted that in contrast to the well-known island model for evolutionary search with sub-populations [Whitley and Starkweather, 1990], the basic assumption in the SBC-approach is that solutions from different concepts do not mate. Hence, when considering a parallel search for the described C-DSE, the main concern is how to distribute the computational resources among the searched concepts. In principle, while searching, it is desired to reduce resources from non-promising concepts.

The studies in [Moshiaiov et al., 2015] and [Snir et al., 2015] provide two alternative Evolutionary Algorithms (EAs) for C-DSE using the concept-based relaxed-Pareto approach. They differ in the technique taken for the allocation of the computational resources among the concepts. In [Moshiaiov et al., 2015], a predefined relaxation decay profile has been used to support the distribution of computational resources among the concepts, whereas in [Snir et al., 2015] a run-time control approach has been devised for that purpose.

The SBC approach provides DMs with multiple solutions. As shown in [Moshiaiov and Rizakov, 2013], this is true not only for multi-objective problems, but also for single-objective problems. In fact, as discussed in [Moshiaiov 2016], it can be used as a kind of an alternative technique to multi-modal optimization.

It is noted that the proposed EAs of [Moshiaiov et al., 2015] and [Snir et al., 2015] may be used to support solving any multi-objective problem, by a-priori partition of the searched space in any arbitrary way, which is not necessarily meaningful. However, these algorithms were developed for solving problems with meaningful concepts.

Similar to [Moshiaiov et al., 2015] and [Snir et al., 2015], this paper concerns the case of meaningful concepts, where the DMs have knowledge about the concepts. This knowledge is not necessarily modeled, yet such tacit knowledge could be used to interactively support the search.

Interactive EC is a well-known search approach in which humans interact with the computer to influence an evolutionary search by their knowledge [Takagi, 2001]. In evolutionary multi-objective optimization, the use of interactive techniques is becoming widespread, as evident from a recent review in [Purshouse et al., 2014]. Such techniques hybridize Pareto-optimization with multi-criteria decision-making techniques. In the hybrid approaches, the DMs commonly influence the search via the articulation of their preferences towards the objectives. In contrast, here interactivity involves the articulation of preferences towards concepts and types of concepts (families of concepts). It is suggested that families of concepts, to be defined by the DMs, will support coping with a large number of concepts. Such families are expected to be meaningful subsets of the set of all concepts, and may include many concepts.

The use of preferences towards concepts and sub-concepts is not new, as can be observed from [Avigad and Moshiaiov, 2009] and [Moshiaiov and Rizakov, 2013]. In contrast to such studies, the current work employs preferences towards families of concepts.

To understand the motivation for the use of families of concepts, the reader is referred to the example in Section 3. It concerns the design of a propulsion system. For this particular problem, we have identified over 10,000 meaningful concepts and many families of concepts that are of interest to the DMs. It is noted that a non-interactive exploration run with 100 of such concepts, using the EA of [Moshiaiov et al., 2015] may take about a week (on a standard workstation). Clearly, the curse-of-dimensionality serves as a strong motivation for interactive elimination of a large number of concepts during the search.

Families of concepts allow interactivity when a large number of concepts are to be examined. This is because the use of families helps coping with the curse-of-dimensionality of the information to be presented to the DMs. Second, it helps the DMs to understand the influence of specific common features that are associated with a defined family. Finally, suppressing a family is expected to interactively cope with run-time limitations in a non-tedious way.

The main contribution of this paper is the introduction of the interactive approach, which may handle a search with a very large number of concepts under strict run-time limitations. The purpose of the search is to filter-out concepts and serve as a pre-process for a more refined search with a much lower number of concepts.

The rest of this paper is organized as follows. Following [Moshiaiov et al., 2015] and [Snir et al., 2015], section 2 provides a description of the problem and the proposed interactive approach. Section 3 describes the demonstration and discusses its implications. Finally, section 4 provides a summary and conclusions for this study.

2 Methodology

2.1 Introducing the problem

The general problem, which is dealt with here, involves a "competition" among candidate concepts. As explained in the introduction, a concept is a subset of the entire set of feasible solutions. It is noted that the notion of competition, as used here, is not that of a classical competitive co-evolution. Namely, there is no fitness coupling; competition is about the distribution of computational resources among the concepts. The problem, which is described below, involves three sets: \( C_{\text{sup}} \subset C_{\text{red}} \subset C \). The set \( C \) is defined by the DMs prior to the search, whereas the set \( C_{\text{red}} \) results from decisions by the DMs during an interactive process.
Next, the set \( C_{\sup} \) is to be found by computations based on \( C_{\text{red}} \). This is further explained in the following.

Given a set \( C \) of \( n \) candidate concepts, the problem is to find, within a time-limit, a set \( C_{\sup} \subseteq C \) of \( n_{\sup} \) concepts, where \( n_{\sup} < n \). Solving the problem involves finding the approximated relaxed-Pareto front, by which the superior concepts are defined (see Eq. 4 below). The superior set of concepts \( C_{\sup} \) is to be found based on solving an interactive multi-objective problem. The interaction results in a reduced set of concepts \( C_{\text{red}} \subseteq C \), with \( n_{\text{red}} \) concepts. The reduced set is based on interactive filtering-out of candidate concepts as described in subsection 2.2. During the interactive process the filtering-out of concepts may be done in stages. The reduced set of concepts \( C_{\text{red}} \) is the one that results from all the filtering-out stages. The following defines \( C_{\sup} \), where \( C_{\sup} \subseteq C_{\text{red}} \) assuming \( C_{\text{red}} \) is given.

Let \( n_{m} \) be the dimension of the objective-space \( \mathbb{R}^{n_{m}} \). Let \( X_{m} \in \mathbb{R}^{n_{m}} \) be the design-space of the \( m \)-th concept, and \( f_{m} : X_{m} \to \mathbb{R}^{n_{m}} \) is the concept’s objective-function. Let \( n_{m} \) be the dimension of \( X_{m} \). Furthermore, let \( s \) be any particular solution and let \( m_{s} \) and \( x_{s} \) represent the concept index and the parameter vector of \( s \), respectively. Also, \( x_{sj} (j = 1, \ldots, n_{m}) \) represents the \( j \)-th element of \( x_{s} \), and \( y_{sj} = f_{x_{s}}[x_{s}] \) represents the performance vector of \( s \), with \( y_{ij} (i = 1, \ldots, n_{s}) \) representing the \( i \)-th element of \( y_{s} \).

Consider, without loss of generality, the problem of finding all the feasible Pareto-optimal solutions and fronts, for each minimization problem of the following \( n_{s} \) independent problems:

\[
\min f_{m}[x] \quad \text{for} \quad m = 1, \ldots, n_{s} \quad (1)
\]

Let \( P_{m}^{*} \) be the set of all feasible particular solutions of the \( m \)-th concept, then the Pareto-optimal set of the concept is defined as follows:

\[
G_{m} = \left\{ s \in P_{m}^{*} \mid \nexists s' \in P_{m}^{*} : y_{s} \succ y_{s}' \right\} \quad (2)
\]

Where \( a \succ b \) stands for \( a \) dominates \( b \).

Let \( P_{\text{red}}^{*} \) be the set of all feasible particular solutions from all concepts of \( C_{\text{red}} \). Then the s-Pareto set of the reduced set of concepts is defined as follows:

\[
G_{\text{red}} = \left\{ s \in P_{\text{red}}^{*} \mid \nexists s' \in P_{\text{red}}^{*} : y_{s} \succ y_{s}' \right\} \quad (3)
\]

Let \( F_{\text{red}} \) be the union of the Pareto-optimal sets from all concepts of \( C_{\text{red}} \). Then, the superior concept-based relaxed Pareto-optimal set, is defined as follows:

\[
R_{\sup} = \left\{ s \in F_{\text{red}} \mid \exists s' \in G_{\text{red}} : y_{s} \succ y_{s} + r \right\} \quad (4)
\]

Where vector \( r = (r_{1}, \ldots, r_{n}) \in \mathbb{R}^{n} \) is a vector of relaxation, which is defined by the DMs prior to the search.

It is noted that the difficulty is that the relaxation zone, which contains the sought information \( R_{\sup} \), is relative to the s-Pareto front, which in itself is unknown prior to the search. The run-time limitation and the large number of concepts make it hard to find the s-Pareto, unless a sufficient number of concepts is filtered out during the search. The superior set \( C_{\sup} \) is composed of all concepts having at least one solution in \( R_{\sup} \).

### 2.2 The interactive search

The interactive search problem for the superior concepts, which is dealt with here, assumes that there is a large number of candidate concepts and a substantial run-time limitation. Namely, it is hard to expect, under the given run-time limitation, that the fronts of all candidate concepts can be found. Moreover, it is assumed that a drastic number of concepts should be suppressed to make the problem tractable within the time limitation. This means that eliminating individual concepts during the search can be too laborious for the DMs. On the other hand, it is assumed that meaningful sets of concepts (families of concepts) can be defined by the DMs and used to effectively categorize concepts during the search process. Figure 3 outlines the interactive process of the current implementation.

![Figure 3 Scheme of the interactive search](image-url)
epoch duration. The epoch duration refers to the interval (number of iterations) between updates of the displayed search results. Following initialization, the proposed algorithm simultaneously searches for better solutions in each concept.

When a predefined number of iterations (the epoch size) is reached, the DMs are provided with up-to-date information that may support a categorization decision on a family of concepts. In the current implementation, the information that the DMs are provided with, consists of all current fronts of the concepts. Moreover, to make a decision on a family of concepts, the DMs should be able to distinguish between the fronts of concepts that belong to that family and those that are not. For this purpose, a coloring option is provided. For example, consider a request to color family A by red and family B by blue. Also assume that family A consists of the following set of concepts {#1, #2, #6, #7, #9, #11}, whereas family B consists of the set of concepts {#3, #4, #6, #7, #8, #10, #16}. In this case, the display will show in red the fronts of the following set of concepts {#1, #2, #9, #11}, whereas the fronts colored in blue will be those of the set {#3, #4, #8, #10, #16}. The concepts of the intersection between the two families will be colored in purple. These will include concepts {#6, #7}. The fronts of all other concepts will be colored in gray.

Next, the DMs categorize the concepts by categorizing the families. The possible categories, in the current implementation, are either default or suppressed. In general, a concept that belongs to a suppressed family will be categorized as suppressed concept. Otherwise, a concept is categorized as default. The implication of these categories are:

- **Default**: The current non-dominated solutions of such a concept might be selected for mutation.
- **Suppressed**: No current solutions of such a concept will be selected to create an offspring.

In making a decision on a family of concepts, the DMs consider the revealed information, namely the current fronts, including their absolute and relative positions in the objective space. Moreover, they may use their tacit knowledge on the problem, the concepts and the families. Such a decision can also be based on subjective preferences, which can not necessarily be explained.

It should be noted that a concept can also be categorized as **Converged**. Yet, this category is not declared by the DMs. A converged concept is a concept that all its allocated computational resources have been utilized. Other types of concept categories, such as in [Moshiaiov and Rizakov, 2013] might be used in future extensions of the proposed approach.

With the above categorization, the reduced set of concepts will be obtained. The reduced set ($C_{red}$) contains all concepts that are not categorized as suppressed within the interactive evolutionary process.

### 3 Demonstration

The core part of the (non-interactive) algorithm, was tested in [Snir et al. 2015]. It involved a unique benchmarking approach, using the simultaneous evolution of forty artificial concepts. The design-space and objective-function of each such concept is based on one of the standard test-functions, which are commonly used for the development of multi-objective evolutionary algorithms (e.g., [Deb et al. 2012]).

Here, the focus is shifted to demonstrating the proposed interactive version using a real-life problem. It concerns the design of a propulsion system for an aircraft. This real-life problem consists of selecting an electric motor, a transmission, a propeller design, and the number of propellers on the aircraft.

In the considered case the system may have either one or two propellers on the aircraft. The propeller may be constructed from various materials, the number of blades should also be decided, the type of airfoil profile should be decided and also the geometrical parameters along the blade. Clearly there are plenty of decisions to be made.

The problem is defined as a bi-objective one. One objective is to minimize the acoustic signature of the aircraft, and the other one is to maximize the flight range of the aircraft. Such objectives are conflicting. To evaluate particular designs from the different concepts that are considered here, we have employed an analysis software, which was provided by the Israel Aerospace Industries. Its description is given in [Gur and Rosen 2009].

As noted in [Moshiaiov 2016], expert designers of the considered system may end up defining thousands of meaningful concepts of interest to them. It is quite clear that in such a situation, there is a large body of relevant knowledge that is not necessarily presented in the analysis code that is used for the optimization. It is also clear, that a thorough search of the entire spaces of such a large number of concepts involves a prohibitive cost. Here we demonstrate how family-based interactivity can be employed to reduce the search effort, while still providing useful results.

In this study the exploration involved 180 concepts of the propulsion system. These were pre-defined as follows. First a set of motors and a set of transmissions were defined. Next, a range of the number of blades was defined, and a set of airfoil profiles was selected. Each concept was defined by selecting from the above a particular motor and a particular transmission, as well as the number of blades and their particular profile.

Figure 4 provides the resulting 180 fronts from a particular non-interactive run, with no run-time limitation. In Figure 4, the light gray curves are the parts of fronts that are outside the relaxation zone, whereas the thin black curves are the parts within that zone. The global front (s-Pareto) is marked by a bold black curve.
We compare the results of the current study to a reference set of fronts that we accumulated in the past for the considered concepts. We termed any concept with a reference front inside the relaxation zone as a superior concept. In the following we compare the obtained concepts, with fronts inside the relaxation zone, with the superior concepts. In Figure 5 the obtained percentage of superior concepts is shown as a function of the run-time.

The reference curve contains four data points, namely the percentage at different run-times for the non-interactive (automatic) search. Also shown in Figure 5 two data points that are described below.

Below, we report the results of an interactive session. Figure 6 shows the fronts as obtained after 10 hours. At this stage, observing the revealed fronts, the DMs noticed that many, of what appear to be inferior concepts, are concepts with a particular motor type. Hence, they defined the set of all concepts with that motor to be a family of concepts and removed it from the rest of the search process.

In Figure 6, the fronts of the removed concepts are marked in red. This run was continued until 25 hours. The percentage of the obtained superior concepts for this interactive run is shown in Figure 5. Clearly, the percentage is higher as compared with the one obtained by the automatic search. As a reference to what can be achieved by a more drastic reduction of the search, we provide an additional result in Figure 5, which was achieved by a method that we call sample removal (details will be provided in future work). Clearly, for the considered case, the latter method improved dramatically the percentage of revealed superior concepts with 25 hours. While tempting to go for the drastic approach by sample removal, one may realize that there is a risk of losing important information if this will be done in a too early stage.

4. Conclusions

This paper deals with a unique multi-objective exploration approach. It employs a set-based concept approach. The purpose of the exploration is to identify a list of superior concepts from a large number of concepts. The sought concepts and their particular solutions are expected to serve as candidates for further search and selection of a conceptual or a particular solution. In order to achieve this goal, with limited computational resources, a novel interactive approach is suggested. It involves the aggregation of concepts into families of concepts, as a means to cope with a large number of concepts. A novel interactive evolutionary algorithm is devised in which the decision-makers are able to make decisions on the families of concepts in order to bias the search. The interactive search is compared with a non-interactive search, under substantial run-time limitation. The comparison is performed on a real-life problem concerning the design of propulsion systems.

In general, machine learning is related to optimization, but it has a different scope [Bennett and Parrado-Hernandez 2006]. In the current implementation the search can be considered as machine learning, since that it is set such that
the optimization process results with information that is revealed on conceptual solutions and their relative performances, and not with optimization in the narrow sense of this term. In the future machine learning can be used to help filtering-out concepts and families in automatic way, based on rules that are pre-set by the DMs, or learnt from the way DMs prefered concepts and families in previous decision situations.

The purpose of the current study is to outline and demonstrate the proposed interactive method. At present, no comprehensive statistical analysis is done. This is left for future work, in which several improvements to the current interactive EA are planned. Among such improvements are the addition of processed information to be displayed to support the interactive decision-making process, as well as additional family categories. The reader is referred to [Moshaiov and Rizakov, 2013], for some ideas on possible improvements as applied to a similar type of problem. Another possible option to be studied is the hybridization of the Pareto-based search with interactive multi-criteria decision-making on the preferences towards the objectives.

References


