Multifrequency Complementary Phase-Coded Radar Signal

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1. Introduction

The multifrequency complementary phase-coded (MCPC) radar signal, first described in the IEEE RADAR'2000 conference [1], is further expanded on in this paper. MCPC employs M subcarriers *simultaneously.* The subcarriers are phase modulated by M different sequences that constitute a *complementary* set. Such a set can be constructed, for example, from the M cyclic shifts of a perfect phase-coded sequence of length M (e.g., P4). The subcarriers are separated by the inverse of the duration of a phase element t_b, yielding Orthogonal Frequency Division Multiplexing (OFDM), well known in communications. The signal exhibits a thumbtack ambiguity function with delay resolution of t_b/M . The power spectrum is relatively flat, with width of M/t_b . The signal can be constructed by power combining M fix-amplitude signals. The resulting signal, however, is of variable amplitude. A single MxM MCPC pulse, based on a specific complementary set of sequences, can be generated in $M!$ different permutations. In each permutation the M sequences are arranged in a different order along the M frequencies. With some permutations the peak-to-mean envelope power ratio (PMEPR) can be maintained below 2.

This paper emphasizes the use of MCPC in a coherent train of M pulses. The pulses are not completely identical. Each pulse is constructed using a different permutation of the same complementary set. The permutations are chosen so that a complementary set is found both in each pulse and in each frequency. Combining such a coding arrangement with a weight function along the frequency axis yields significant range sidelobe reduction. The behavior in the presence of Doppler shift is demonstrated in plots of the ambiguity function.

2. Single MCPC pulse

A schematic description of an MxM MCPC pulse is given in Fig. 1. It shows M (=5) sequences modulating M subcarriers. Each sequence is constructed from M bits each of duration t_b . The autocorrelation mainlobe width of such a signal is t_b/M . This is M times shorter than the mainlobe width of a single frequency digital radar signal (e.g. P4). We will also show that the MCPC pulse exhibits an efficient spectrum usage. As depicted in Fig. 1, the power spectrum is nearly rectangular with cutoff at $f \approx M/(2t_b)$.

The autocorrelation sidelobes will be lower if instead of repeating the same sequence on all frequencies, the sequences will be different, but will constitute a complementary set. A complex valued sequence X_i , whose k^{th} element is $s_i(k)$, forms a complementary set if the sum $Z(p)$ of the a-periodic autocorrelation function R_i of all sequences from the set is equal to zero for all nonzero time shifts p, i.e.,

$$Z(p) = \sum_{i=0}^{M-1} \sum_{k=0}^{M-1-p} s_i(k) s_i^*(k+p) = \begin{cases} \sum_{i=0}^{M-1} R_i(0)\,, & p = 0 \\ 0 \quad , & p \neq 0 \end{cases} \tag{1}$$

where * denotes complex conjugate, p is the (positive) time shift, and $R_i(0)$ is the energy of the sequence X_i . When the set has only two sequences (a complementary pair), the two sequences (of equal length M) must have aperiodic autocorrelation functions whose sidelobes are equal in magnitude but opposite in sign. The sum of the two autocorrelation functions has a peak of $2M$ and a sidelobe level of zero.

An MxM complementary set can be constructed using Popovic's [2] result which states that *all the different cyclic time shifted versions of any sequence having an ideal periodic autocorrelation function, form a complementary set*. Useful polyphase sequences which exhibit ideal periodic autocorrelation function are the P3 and P4 signals [3]. Useful two-phase sequences with ideal periodic autocorrelation were suggested by Golomb [4]. Binary phase sequences (using 0^0 and 180^0) with ideal periodic autocorrelation are rare. Barker code of length 4 is an example. However, binary NxM complementary sets can be constructed by methods other than cyclic shifts. The mathematical expression of the complex envelope of a phase coded NxM MCPC pulse is given by

$$u(t) = \begin{cases} \sum_{n=1}^{N} W_n \exp\left\{ j\left[2\pi\left(\tfrac{M+1}{2} - n\right)t/t_b + \theta_n\right]\right\} \sum_{m=1}^{M} u_{n,m}\left[t-(m-1)t_b\right], & 0 \le t \le Mt_b \\ 0 \quad , \quad \text{elsewhere} \end{cases} \tag{2}$$

where

$$u_{n,m}(t) = \begin{cases} \exp\left(j\phi_{n,m}\right) \; , & 0 \le t \le t_b \\ 0 \quad , & \text{elsewhere} \end{cases} \tag{3}$$

$\phi_{n,m}$ is the m^{th} phase element of the n^{th} sequence, θ_n is an arbitrary phase shift added by the transmitter hardware to each carrier (known to the receiver) and W_n is the amplitude weight assigned to the n^{th} subcarrier.

We first consider a phase element $\phi_{n,m}$ of the MxM matrix constructed from all the cyclic shifts of a P4 phase coded sequence, described by

$$\phi_m = \frac{\pi}{M}(m-1)^2 - \pi(m-1), \; m = 1,2,...,M \quad (4)$$

or a P3 sequence described by

$$\phi_m = \frac{\pi}{M}(m-1)^2, \; m = 1,2,...,M, \; M \text{ even} \quad (5)$$

0	22.5	90	202.5	0	202.5	90	22.5
22.5	90	202.5	0	202.5	90	22.5	0
90	202.5	0	202.5	90	22.5	0	22.5
202.5	0	202.5	90	22.5	0	22.5	90
0	202.5	90	22.5	0	22.5	90	202.5
202.5	90	22.5	0	22.5	90	202.5	0
90	22.5	0	22.5	90	202.5	0	202.5
22.5	0	22.5	90	202.5	0	202.5	90

Table 1. Phase matrix [deg.] of 8x8 MCPC based on P3

For P3 and M= 8, the phase (modulo 2π) matrix is given in Table 1. The set of complex elements with uniform magnitude, whose phase is described by Table 1, will remain a complementary set for any reordering of the rows. When one particular order (stated on the illustrations) is used as $\phi_{n,m}$ in (2) and (3), the resulting complex envelope will exhibit an autocorrelation function and power spectral density as depicted in Figs. 2 and 3. Note that it was also assumed that $\theta_n = 0$ and $W_n = 1$ for all n. The main results for such a multifrequency signal is an autocorrelation function (Fig. 2) whose main lobe extends as far as t_b / M with low sidelobes elsewhere. In a single-frequency P3 or P4 signal, the main lobe will extend as far as t_b. Note also a relatively flat spectrum extending as far as $M / 2t_b$. A P3 or P4 signal will exhibit a sinc-squared shaped power-spectrum with first null at $1/t_b$. On the other hand, while each one of the subcarriers of an MCPC signal has a fix real-amplitude, their sum exhibits a variable real-amplitude, as depicted in Fig. 4. (In Fig. 4, PMEPR = 2.93.)

Finally we observe in Fig. 7 the 1st and 2nd quadrants of the ambiguity function of this particular MCPC signal. Its shape resembles the ideal thumbtack. Recall that the ambiguity function of single-frequency P3, P4 or Linear FM signals exhibit a diagonal ridge.

3. Coherent train of complementary MCPC pulses

A train of M MCPC pulses can be complementary in time as well as in frequency. This happens when each pulse in the train exhibits a different order of sequences such that a set of complementary phase sequences is obtained in each frequency. There are also M! different ways to order the pulses. The autocorrelation sidelobes are further reduced as demonstrated by comparing Fig. 5 to Fig. 2. Both pertain to a 8x8 MCPC signal. The order of sequences in the 8 pulses is outlined within the drawing. We note from Fig. 5 that the sidelobe-reduction applies to all but the sidelobes within the first bit. This should be expected because a complementary set yields zero autocorrelation sidelobes only for $|\tau| > t_b$.

The delay axis in Fig. 5 is limited to the duration of a pulse (= Mt_b). The autocorrelation within that delay is not affected by the pulse repetition interval T_r as long as T_r is larger then twice the pulse width, namely $T_r > 2Mt_b$. The pulse interval does affect the ambiguity function for non-zero Doppler. The dramatic improvement in sidelobe reduction for $t_b < |\tau| < Mt_b$ by a train of complementary MCPC pulses, invites a method for further sidelobe-reduction in the delay range of $|\tau| < t_b$. Frequency weighting is a well-established method for reducing autocorrelation sidelobes in linear FM radar signals [5]. We found out that it was not very effective in a *single* MCPC pulse because it yielded meaningful sidelobe reduction only over the limited delay range $|\tau| < t_b$, but did not help over the larger remaining delay range of $t_b < |\tau| < Mt_b$. However, once we found out that a complementary train of MCPC pulses dramatically reduces sidelobes in that larger delay range $t_b < |\tau| < Mt_b$, it became obvious that combining complementary pulse train and frequency weighting can reduce autocorrelation sidelobes over the entire delay range $0 < |\tau| < Mt_b$.

In conventional constant-amplitude radar signals, weighting is usually implemented only at the receiver, in order not to loose the constant-amplitude property of the transmitted signal. This is effectively a deviation from matched filter processing and results in a small SNR loss. In our case, the signal is already of variable amplitude (but of fix amplitude at each subcarrier). Hence applying different amplitude to each subcarrier adds no difficulty. Despite the extensive knowledge regarding weighting windows, we limited our numerical trials to a simple family of weighting described by

$$W_n = \left[a_0 + a_1 \cos \frac{2\pi\left(n - \frac{1}{2}\right)}{M} \right]^{\alpha} \quad , \quad n = 1,....M \quad (6)$$

Note that setting $a_0 = 0.53836$, $a_1 = -0.46164$ and $\alpha = 0.5$ is equivalent to adding a Hamming window at the receiver side. We found out that values of α slightly different from 0.5 yielded smaller peak sidelobes. The weight W_n now multiplies the signal of the n'th subcarrier as noted in (2). To the M=8 MCPC complementary pulse train used in Fig. 5, we added weighting according to (6). The resulted magnitude of the autocorrelation function is plotted in Fig. 6.

The ambiguity function of a complementary train of M MCPC pulses, with or without weighting, depends on the pulse interval T_r. The partial ambiguity function plotted in Fig. 8 was obtained for an arbitrary case in which the pulse interval was 4 times the MCPC pulse duration, namely $T = 4Mt_b$ and the weighting was according to (6). Because of the periodicity in time, the response in Doppler exhibits peaks at multiples of $\nu = \frac{1}{T_r} = \frac{0.25}{Mt_b}$, the first of which is seen in Fig. 8. However, since the pulses in the train are different from each other there are no pronounced peaks at multiples of T_r . For comparison, Fig. 9 presents partial ambiguity function of a coherent train of 8 single-frequency P3 pulses, each with M=64 chips (each of duration t_c) and with intrapulse and interpulse weighting. Note a similar main lobe (when $t_c = t_b / 8$) , much lower Doppler sidelobes, but repeated peaks at multiples of T_r. The constant volume property of the ambiguity function is maintained by shifting volume from the additional peaks (in P3) to the pedestal strips (in MCPC).

Removing the spacing between the MCPC pulses creates a **CW** signal. Fig. 10 presents the *periodic* ambiguity function of a train of 16 contiguous 16x16 MCPC pulses with different permutations (selected randomly). The resulting signal exhibits pulse compression of $M^3 = 4096$. Note how the periodicity in Doppler is almost gone. The sidelobe pedestal is relatively uniform, below –20 dB.

4. MCPC based on 2-valued complementary set

The P3 and P4 phase sequences used so far to construct the MCPC complementary set are polyphase sequences. There are 2-valued phase sequences that also exhibit perfect periodic autocorrelation, and can serve to construct a complementary set. One such alternative is the sequences described by Golomb [4]. One example of such a sequence is based on Barker code of length 7 [+ + + - - + -], in which the two phase values are not 0 and 180^0 but 0 and 138.59^0 (= arccos(-3/4)). Golomb codes exist for lengths 3,7,11,15,19,23,31,35,43,47,59…. Implementing two-valued sequences is especially simple if the two are binary values (–1, +1). Binary complementary sets can be constructed from Hadamard matrices. Hadamard-based MCPC pulses exhibit especially low autocorrelation sidelobes, but relatively higher Doppler sidelobes and higher PMEPR.

References

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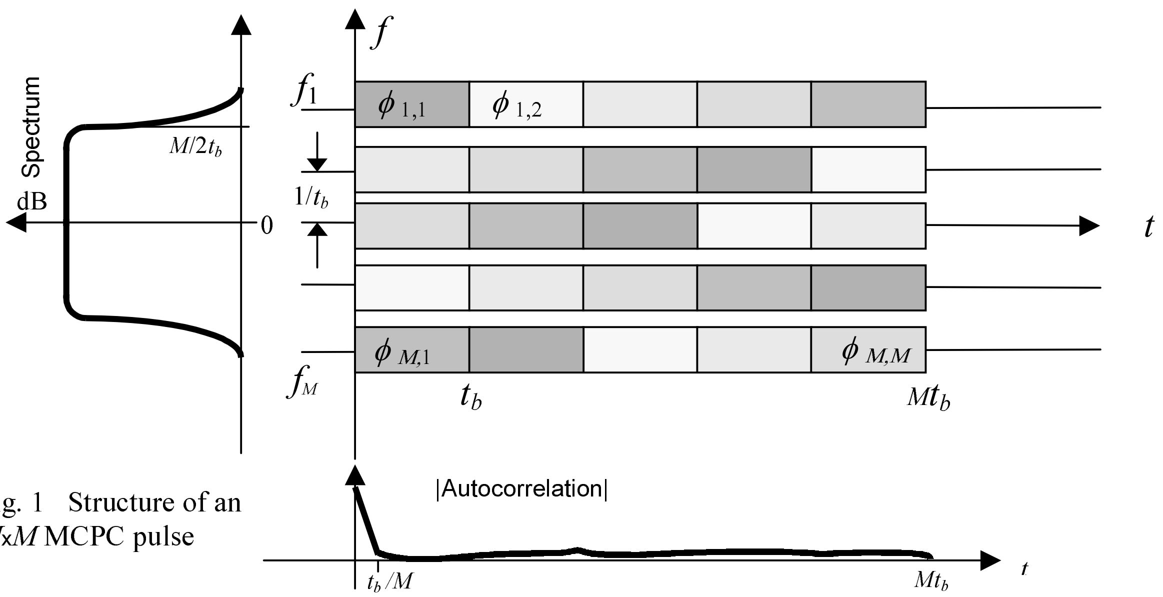

Fig. 1 Structure of an MxM MCPC pulse

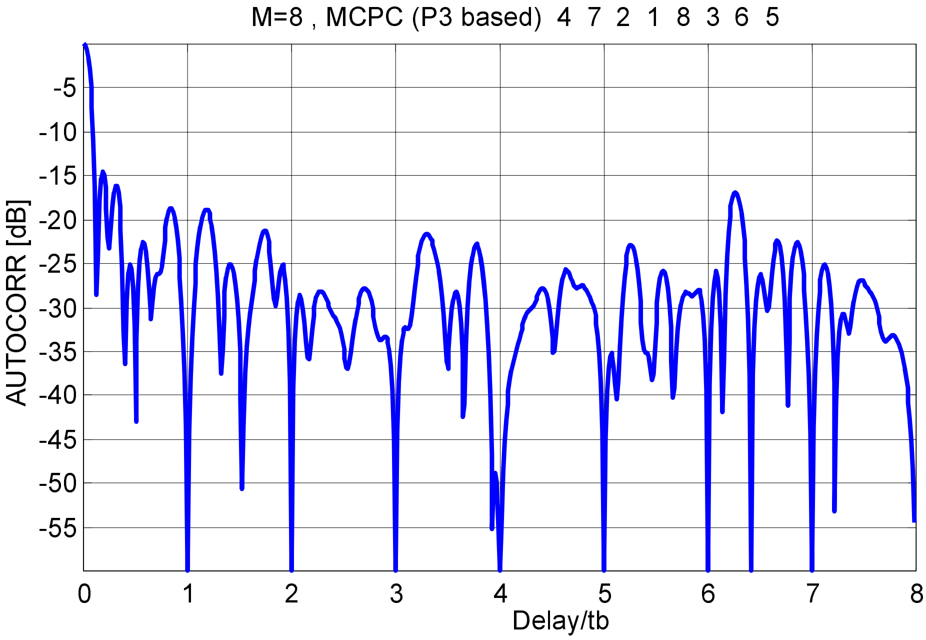

Fig. 2 Autocorrelation of an 8x8 MCPC pulse based on a P3 sequence

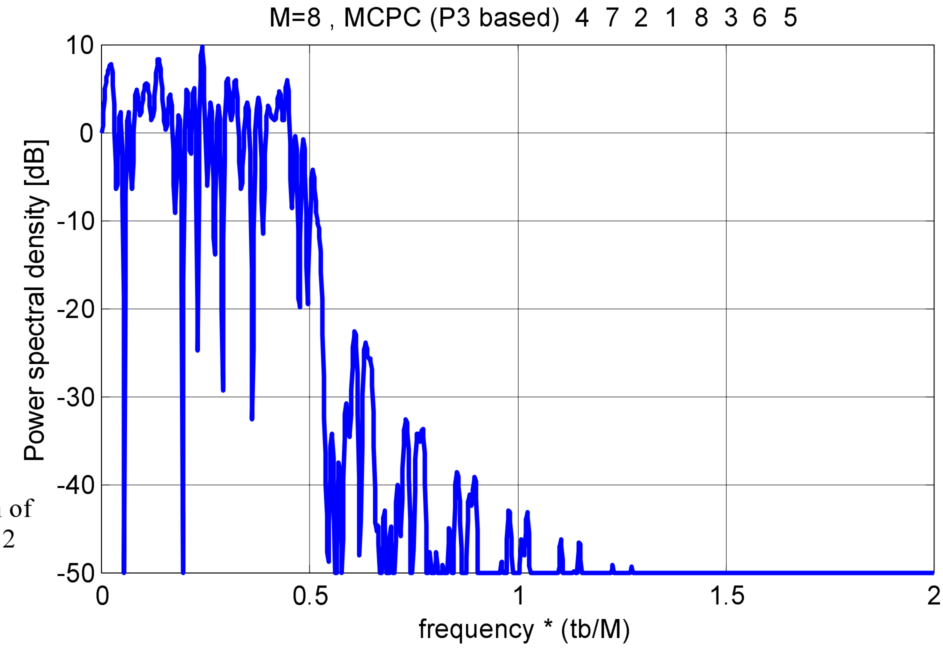

Fig. 3 Power spectrum of the signal used in Fig. 2

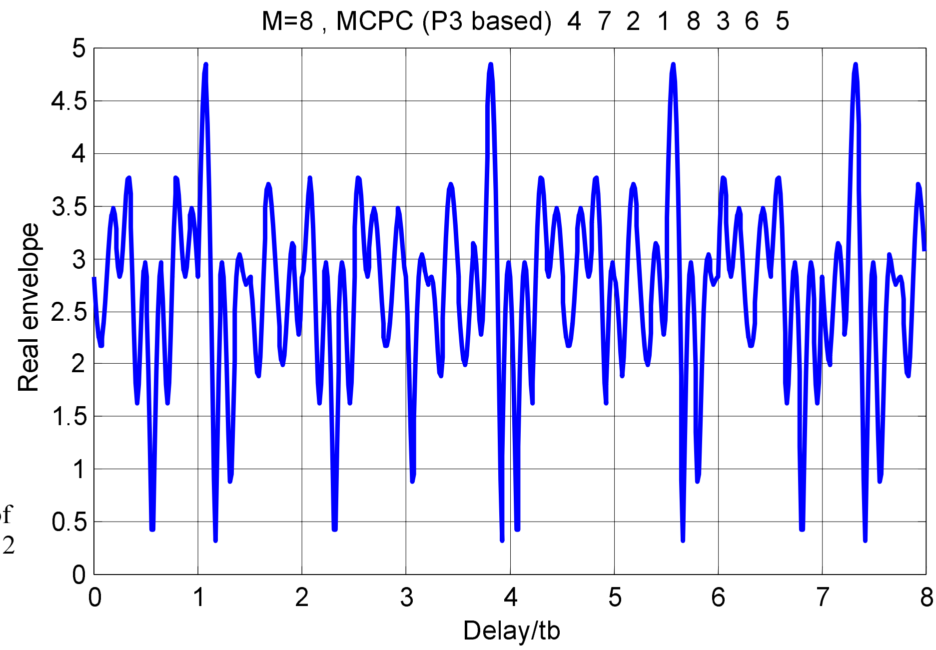

Fig. 4 Real envelope of the signal used in Fig. 2

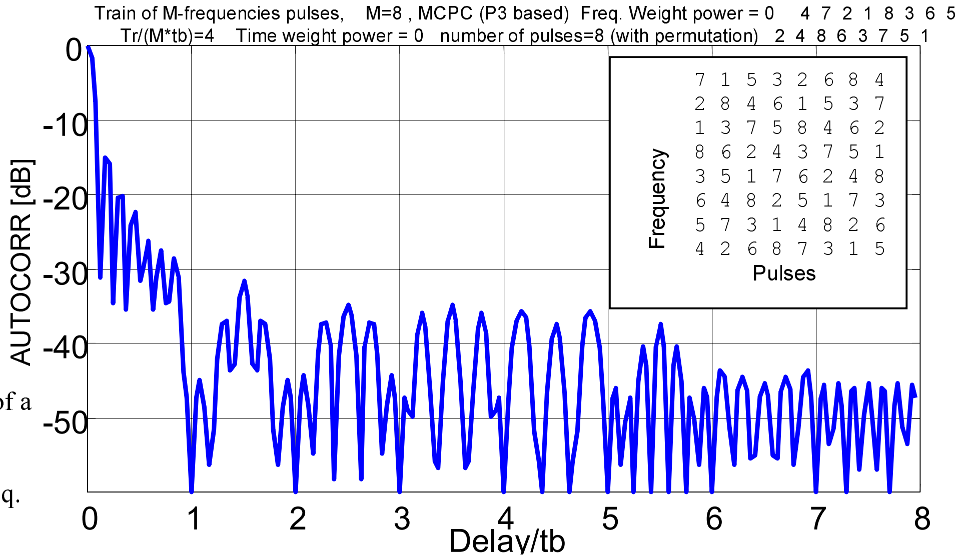

Fig. 5 Autocorrelation of a coherent train of 8 complementary MCPC pulses (P3 based, no freq. weighting)

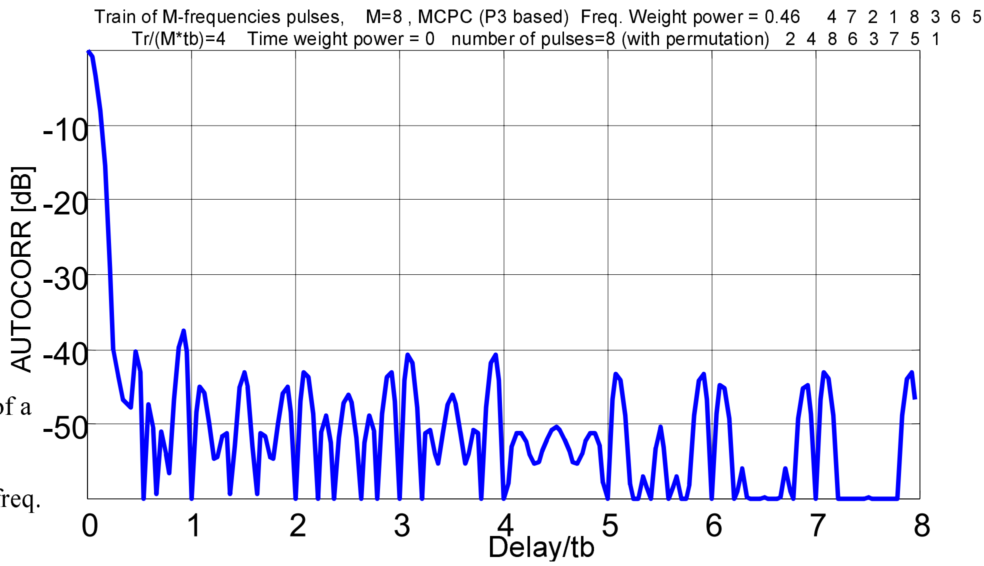

Fig. 6 Autocorrelation of a coherent train of 8 complementary MCPC pulses (P3 based, with freq. weighting)

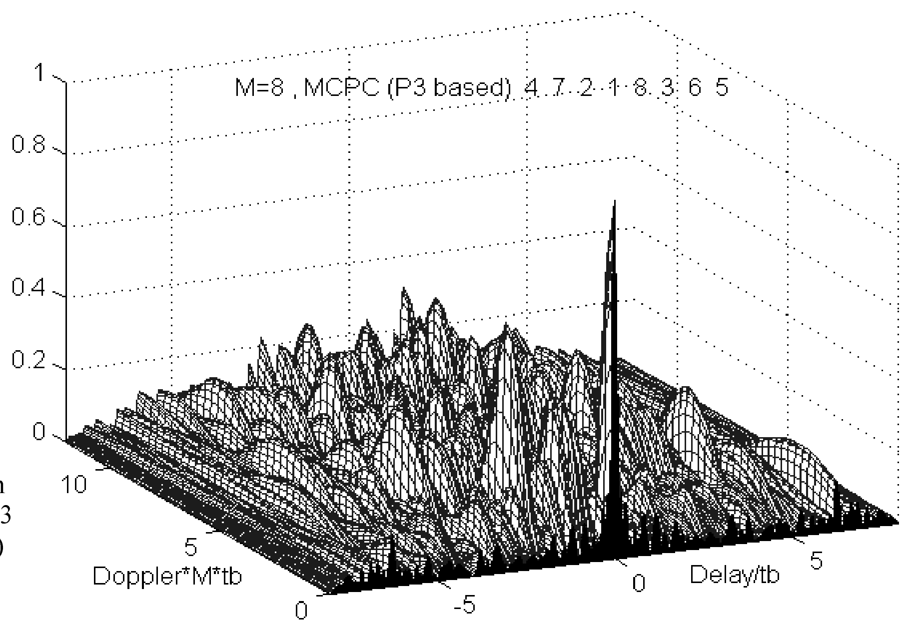

Fig. 7 Ambiguity function of an 8x8 MCPC pulse (P3 based, no freq. weighting)

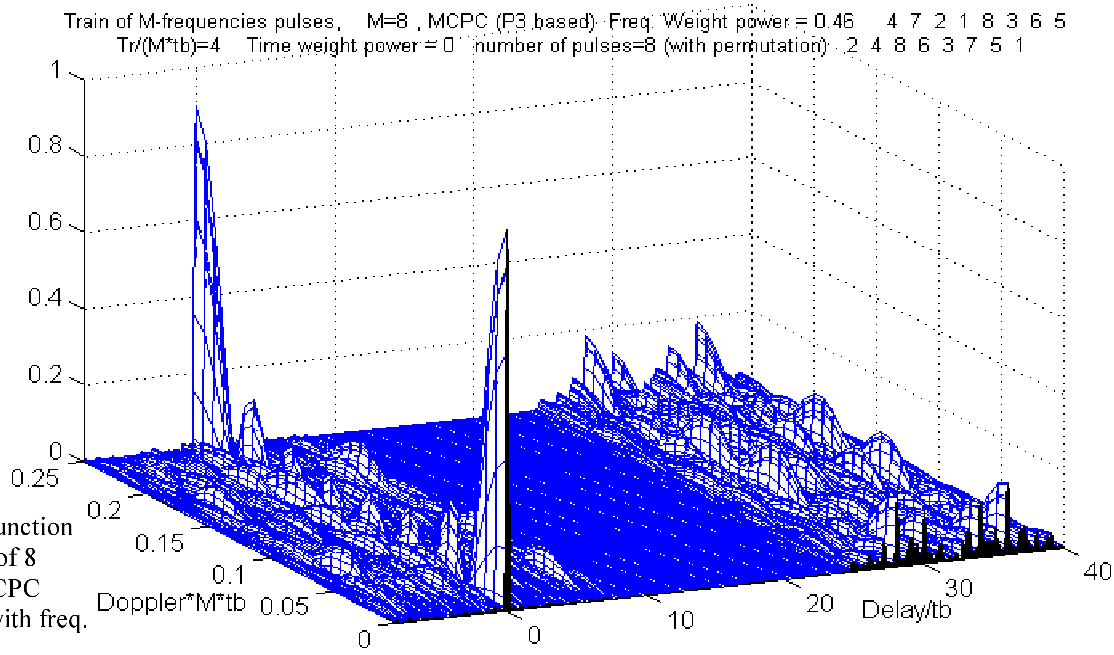

Fig. 8 Ambiguity function of a coherent train of 8 complementary MCPC pulses (P3 based, with freq. weighting)

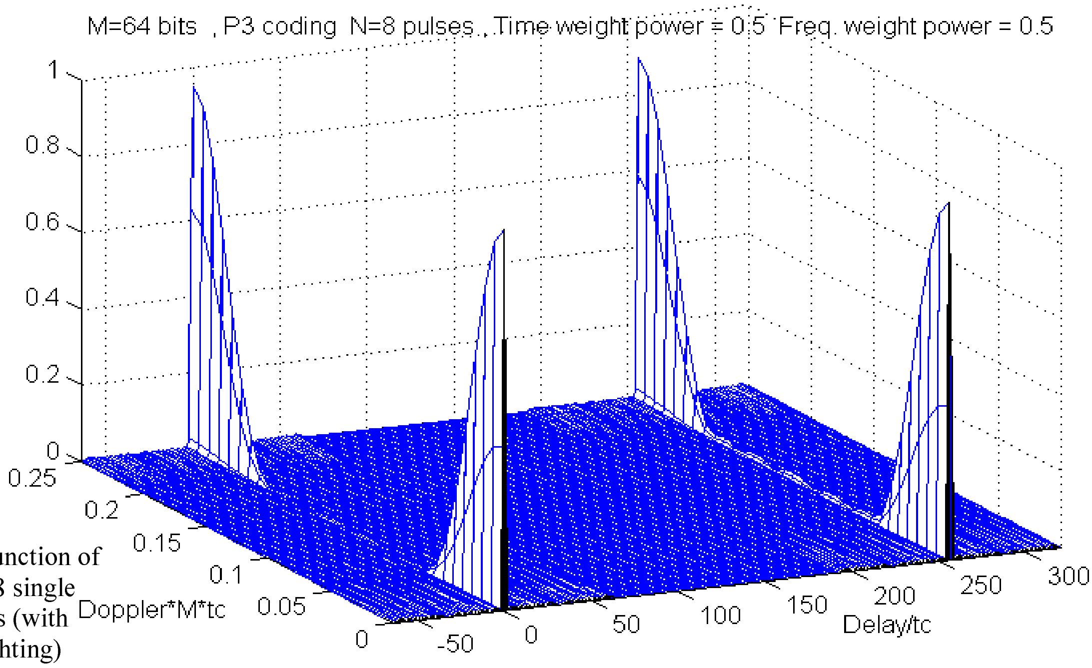

Fig. 9 Ambiguity function of a coherent train of 8 single frequency P3 pulses (with freq. and time weighting)

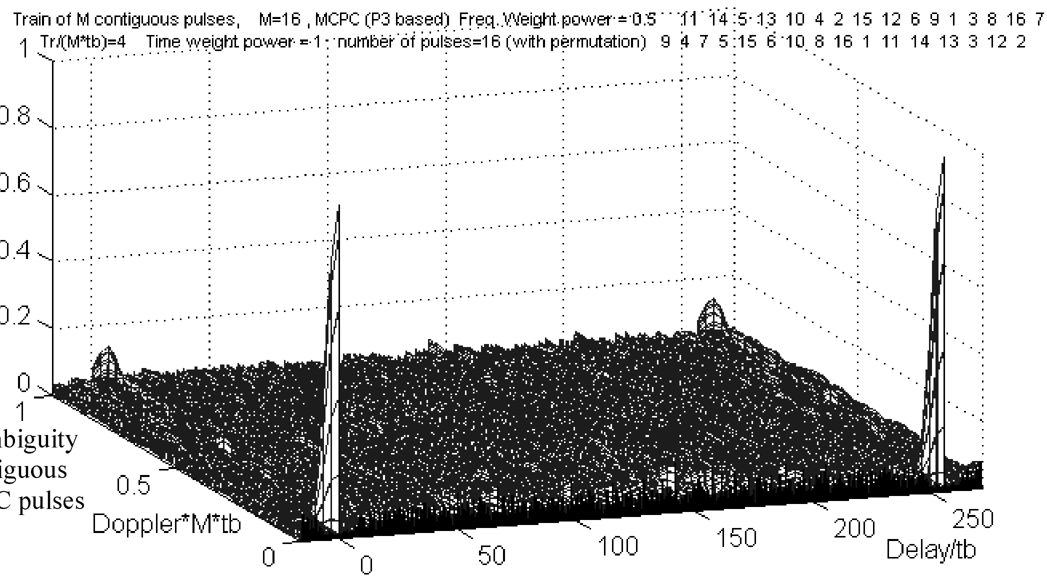

Fig. 10 Periodic ambiguity function of 16 contiguous (CW) 16x16 MCPC pulses (P3 based)