The Virtual Infinite Capacitor

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Abstract— We define the virtual infinite capacitor (VIC) as a nonlinear capacitor that has the property that for an interval of the charge $Q$ (the operating range), the voltage $V$ remains constant. We propose a lossless approximate realization for the VIC as a simple circuit with two controllers: a voltage controller acts fast to maintain the desired terminal voltage, while a charge controller acts more slowly and maintains the charge $Q$ in the desired operating range by influencing the incoming current. The VIC is useful as a filter capacitor for various applications, for example power factor compensators (PFC), as we describe. In spite of using small capacitors, the VIC can replace a very large capacitor in applications that do not require substantial energy storage. We give simulation results for a PFC working in critical conduction mode with a VIC for output voltage filtering.

Index Terms—nonlinear capacitor, power filtering, switched power converter, sliding mode control, power factor compensator.

I. INTRODUCTION

MOST power converters require input and output filters with capacitors for the purpose of voltage smoothing, in particular switching noise suppression. Unfortunately, covering the low frequency range requires a large capacitance, which has two major disadvantages: (1) low reliability; (2) heavy and large capacitors. (There may also be an issue of high cost.) Ultracapacitors suffer from low reliability and low operating voltages. The need for high reliability and lightweight power converters is growing, as renewable energy sources, electric transportation and efficient DC lighting become more widespread. Thus, we think that it would be useful to have a high reliability circuit that can replace a large capacitor used for voltage smoothing.

In this paper we propose a circuit that behaves like a nonlinear capacitor. Whereas for a usual capacitor, the dependence of the voltage $V$ on the charge $Q(t) = \int_0^t i(\tau) d\tau$ is linear, the $Q$-$V$ plot of the proposed circuit element would have a flat region for $Q \in [Q_{\min}, Q_{\max}]$ as shown in Fig. 1.

For charges in the interval $[Q_{\min}, Q_{\max}]$ the voltage would remain at a predefined level $V_{\text{ref}}$. Thus, in this interval, our circuit would be equivalent to an infinite capacitor charged to the voltage $V_{\text{ref}}$.

A few words about nonlinear capacitors in general: By a nonlinear capacitor we mean a circuit element where the voltage $V$ is a function of the charge $Q$. The dynamic capacitance $C$ at a given point $Q$ can be defined by $\frac{dV}{dQ} = C$. The energy absorbed by this circuit element during an infinitesimal change of charge $dQ = i\, dt$ is $dE = V\, dQ$. It is easy to see that a nonlinear capacitor is energy-conserving, in the sense that when moving from a charge $Q_1$ to another one $Q_2 > Q_1$ (charging) and then back to $Q_1$ (discharging), we get back the same energy that we have stored. Assuming that the nonlinear capacitor cannot produce any energy, the stored energy at any point must be positive:

$$\int_0^{Q_1} V(Q)\, dQ \geq 0 \quad \text{for all } Q_1 \text{ in the operating range.} \quad (1)$$

The above restriction allows to have $V < 0$ for some positive values of $Q$, but we shall not explore this possibility. Nonlinear capacitors can be found in the early references Manley and Rowe (1956) and Rowe (1958). For the theory of nonlinear circuits and their synthesis we refer to Chua (1967, 1968). Other interesting nonlinear circuit elements with memory have been proposed more recently by Di Ventra, Pershin and Chua (2007).

A nonlinear capacitor for which $V$ (as a function of $Q$) has a flat region, namely $\frac{dV}{dQ} = 0$ for $Q \in [Q_{\min}, Q_{\max}]$, will be called a virtual infinite capacitor (VIC). A VIC should have an additional output through which $Q$ can be measured, allowing to keep it in the desired range $Q \in [Q_{\min}, Q_{\max}]$. Indeed, this is needed because in this range $Q$ cannot be estimated from $V$. In the region of interest $Q \in [Q_{\min}, Q_{\max}]$, the dynamic capacitance is infinite, but the amount of stored energy (the left side of (1)) is of course finite and not very large. Thus, our device is good for filtering or voltage regulation, but it is not meant for energy storage (see also the end of Section II). We emphasize that the $Q$-$V$ plot of the VIC does not have to look like in Fig. 1. Indeed, outside the flat region $[Q_{\min}, Q_{\max}]$ the plot may have any shape as long as the restriction (1) is satisfied. It is even possible to build VICs that have a $Q$-$V$ plot with hysteresis, and this may be useful, for instance, if a very fast power-up is needed (but we do not discuss this in this paper).

Fig. 1. $Q$-$V$ characteristics of a virtual infinite capacitor.

Fig. 2. A typical application of the VIC, stabilizing the voltage on a load.
A typical application of the VIC would be to create a constant voltage $V_{\text{ref}}$ on a load $R$ when the energy is coming from a variable current source $i_{\text{in}}$, as shown in Fig. 2 (which may be a simplified circuit based on Norton’s theorem). The value $V_{\text{ref}}$ may be adjustable, and $Q$ should be measurable, as explained before and indicated in Fig. 2. Fluctuations of the load current can be absorbed, conceptually, into the fluctuations of $i_{\text{in}}$. An external charge controller (not shown in the figure) may regulate $i_{\text{in}}$ in the low frequency range, such that $Q$ remains in the desired range. This regulation of $i_{\text{in}}$, via the control signal $u$, is indicated in Fig. 2. Thus, to operate a VIC, we need two controllers: the VIC controller (discussed in Section IV) which, together with the VIC circuit (discussed in Section II) implements a VIC, and the charge controller (discussed in Section III) whose role we have explained above. In Section V we describe a possible application of the VIC as the filter capacitor of a PFC.

There are several methods that deal with the problem explained in Fig. 2 (sometimes called active power filtering), and using circuits that are similar to what we propose (in Section II), see for instance Wang et al. (2011), Zhong et al. (2012) and the references therein. We emphasize the fundamental difference between the VIC and capacitor multiplier circuits, as in the latter the output voltage is linearly dependent on the accumulated charge, unlike the $Q$-$V$ curve in Fig. 1. As far as we are aware, the concept of VIC and its control algorithms are new and enable the stabilization of $V$ also in the face of a randomly changing current $i$.

For typical values see Section V.

Fig. 2 shows two different realizations of the VIC, as realized in the circuits of Figs. 3 and 4. In the first region, the VIC is operated at a constant voltage $V_{\text{ref}}$ (or $V_{\text{max}}$) and the circuit operates in the horizontal region, with charge $Q_{\text{min}}$ transferred to $C_S$. In the second region, $Q_{\text{min}}$ is transferred to $C_L$, and the circuit operates in the linear region, where charge $Q_{\text{min}}$ is transferred to $C_L$. In the third region, $Q_{\text{min}}$ is transferred to $C_S$, and the circuit operates in the horizontal region, with charge $Q_{\text{min}}$ transferred to $C_S$.

**II. A Realization of the VIC**

We propose an approximate realization for the virtual infinite capacitor from Fig. 1, partially shown in Fig. 3 (the controller and the drivers of the switches are omitted from the figure). This realization is only an approximation, as it exhibits additional undesirable effects: switching noise, switching losses and small oscillations of the voltage $V$. We emphasize that this is only one of the possible realizations of the VIC.

We recognize that the central part of the circuit in Fig. 3 is a bidirectional DC/DC converter (a canonical switching cell, as defined in Kassakian, Schlecht and Verghese (1992, Ch. 6)). The binary signals $q, \bar{q} \in \{0,1\}$ control the switches at a fast rate such that the upper switch is closed if and only if $q = 1$, and similarly for the lower switch. In a typical application, when the VIC is used to filter the output voltage of a power factor compensator (PFC), the current $i$ (and hence the charge $Q$) will oscillate at twice the grid frequency. Charge fluctuations in the range $[Q_{\text{min}}, Q_{\text{max}}]$ are transferred to the capacitor $C_S$ via the converter, as long as the frequency of the charge fluctuations is much lower than the converter sampling frequency $1/T_s$. Thus, the voltage $V_s$ will vary while keeping $V$ almost constant (close to $V_{\text{ref}}$), which is the desired filtering effect. Let us denote by $[V_{S_{\text{min}}}, V_{S_{\text{max}}}]$ the interval in which $V_S$ will vary when $Q \in [Q_{\text{min}}, Q_{\text{max}}]$. We impose that

$$0 < V_{S_{\text{min}}} < V_{S_{\text{max}}} < V_{\text{ref}}.$$  

$V_{S_{\text{min}}}$ cannot be chosen too small, because it would lead to the DC/DC converter working at a high voltage ratio and hence low efficiency. There is also another reason why $V_{S_{\text{min}}}$ cannot be chosen too small, and similarly $V_{\text{ref}} - V_{S_{\text{max}}}$ cannot be chosen too small; this will become clear when we discuss the control of the VIC in Section IV (see formulas (16) and (18)). For typical values see Section V.

We give a brief description of the circuit operation in the various regions that correspond to the linear segments of the plot from Fig. 1. In the **first region**, when the total charge of the system is small (e.g., during power-up, when our objective is to charge $C$ to $V_{\text{ref}}$ while simultaneously charging $C_L$ to the minimal voltage $V_{S_{\text{min}}}$ (as discussed in the operating region), the converter creates an almost constant ratio between its input and output voltages,

$$D = \frac{V_s}{V} = \frac{V_{S_{\text{min}}}}{V_{\text{ref}}}.$$  

This situation corresponds to the first (left) linear segment of the plot in Fig. 1. Thus, when $V$ reaches the value $V_{\text{ref}}$, then $V_s$ reaches the value $V_{S_{\text{min}}}$. The charge needed for $V$ to reach the voltage $V_{\text{ref}}$ is $Q_{\text{min}}$. The control of the converter in the first region can be, for example, pulse width modulation (PWM) with duty cycle $D$. The dynamic capacitance in this region is $C + D^2 C_S$ (this is valid for frequencies significantly lower than the resonant frequency of $L$ and $C_L$). The **second region** corresponds to the horizontal segment (for $Q \in [Q_{\text{min}}, Q_{\text{max}}]$) and this is where we want the circuit to be most of the time, because here the dynamic capacitance is infinite. In this region the controller works with a sampling period $T_s$ and keeps $q$ and $\bar{q}$ constant for the entire sampling period (this is not PWM). The circuit remains in the horizontal region until $V_s$ reaches the maximum allowable value $V_{S_{\text{max}}}$. The control of the VIC in the horizontal region will be discussed in Section IV. In the first two regions, the switches change their state frequently, such that $\bar{q} = 1 - q$ (actually, small delays may have to be introduced to reduce switching losses). The **third region** corresponds to the rightmost segment of the plot in Fig. 1, when the capacitor $C$ is on its own, while $V_s$ remains at the constant level $V_{S_{\text{max}}}$. In the third region we have $q = \bar{q} = 0$ and the dynamic capacitance is $C$. This situation occurs if we overcharge the VIC.

We note that another possible realization of the VIC with $V_s > V_{\text{ref}}$ exists, by reversing the DC/DC converter. This alternative realization allows much better regulation of $V$, but the higher voltage required on $C_S$ may be a drawback for many applications. It is conceivable to use other types of switched...
power converters inside the VIC, for example, resonant switched-capacitor converters, see Cervera and Peretz (2014).

Finally, we derive a limitation on the current flowing through the VIC in sinusoidal regime, assuming that the VIC remains in the region $Q \in [Q_{\min}, Q_{\max}]$. Most of the energy stored in the VIC (if realized as in Fig. 3) is stored in the capacitor $C_S$. Thus, the operating range $Q \in [Q_{\min}, Q_{\max}]$ corresponds to $C_S$ holding energy in the range

$$\frac{1}{2} C_S V_{S,\min}^2 \leq E \leq \frac{1}{2} C_S V_{S,\max}^2. \quad (2)$$

As power regulation is a typical application of the VIC, it is useful to consider a sinusoidal input current $i = I_0 \sin(2\omega_B t)$, as would be the case in a PFC working in steady state on a grid with frequency $\omega_B$, ignoring the ripple current (see Section V). We assume that $V = V_{\text{ref}}$ is constant. During the first half of the period (when $2\omega_B t \leq \pi$) the VIC is storing energy, and during the second half the same energy is returned. The amount of energy stored during the half period (the increase in the total stored energy) is

$$\Delta E = V \int_0^{\pi / 2\omega_B} I_0 \sin(2\omega_B t) \, dt = \frac{V I_0}{\omega_B}.$$ 

According to (2), we have the upper bound $\Delta E \leq \frac{1}{2} C_S V_{S,\max}^2 - \frac{1}{2} C_S V_{S,\min}^2$. It follows that in order to maintain the VIC energy within the required range, the steady state current amplitude must satisfy:

$$I_0 \leq \frac{\omega_B C_S}{2V} \left[ V_{S,\max}^2 - V_{S,\min}^2 \right]. \quad (3)$$

Knowing that $V_{S,\min}$ is about a quarter of $V$ and $V_{S,\max}$ is almost equal to $V$, this formula enables us to choose the correct size for $C_S$.

III. Controlling the Charge in a VIC

Suppose that the VIC is a part of the general circuit from Fig. 2. As mentioned at the end of Section I, an external charge controller usually affects $i_m$ in the low frequency range, in order to maintain $Q \in [Q_{\min}, Q_{\max}]$, or equivalently, to maintain $V_S \in [V_{S,\min}, V_{S,\max}]$. We give an example of a simple charge controller for the circuit in Fig. 2, which can be easily adapted to most applications.

A useful way of representing the operation of the controlled current source $i_m$ in Fig. 2 would be

$$i_m(t) = m(t) \cdot \Theta(t), \quad \Theta(s) = \delta(s) + W(s) \cdot \bar{u}(s), \quad (4)$$

where a hat denotes the Laplace transformation, $d$ is a continuous and bounded disturbance signal (mostly the error introduced by the switching mechanism controlling $i_m$), $u$ is the control signal, $W$ is a low-pass filter used to limit the bandwidth of the control and $m(t)$ is a positive and bounded function, representing the time-dependency of $i_m$ on the control signal. Usually, $m(t)$ is periodic, for instance, in a PFC, $m(t)$ is approximately proportional to $\sin^2 \omega_B t$, see Section V (the text after (25)). The presence of $W$ means that the control can influence $i_m$ only in the low frequency range, corresponding to the bandwidth of $W$. This corresponds to typical system constraints, such as a unity power factor in the case of a PFC, where the control should not influence $i_m$ at the grid frequency and its harmonics, but only at lower frequencies (see Section V). For simplicity we take $W(s) = \frac{1}{1+s^2}$, with $\tau > 0$.

The total energy stored in the VIC (neglecting the inductor) is $E = \frac{1}{2} [V_S^2 C_S + V^2 C]$. In the operating region $Q \in [Q_{\min}, Q_{\max}]$ the voltage $V$ is constant, hence $dE = V_S C_S dV_S$. Equating this with $dE = V dQ$ we obtain $\frac{dV_S}{dQ} = \frac{V}{V_S C_S}$. The solutions of this differential equation are of the form

$$V_S = \sqrt{\frac{2V}{C_S}} Q + \beta,$$ 

where $\beta$ is a real constant. Applying the above relation for $Q = Q_{\min}$, we obtain that

$$\beta = \frac{V_{S,\min}^2 - 2}{C_S} \frac{Q_{\min} V}{C}. \quad (5)$$

Using (4), we also have

$$i(t) = (d(t) + (w * u)(t)) \cdot m(t) - i_{\text{out}}, \quad (6)$$

where a star denotes convolution, and $w$ is the impulse response of $W$.

Since the operation of this system introduces fluctuations in $V_S$, our control objective is not to achieve stability in the classical sense, but rather that the state trajectories of the system should converge to a limit cycle. What we control is the deviation of $V_S$ from its desired average value, denoted by $V_{S,\text{ref}}$. In order to allow a maximal dynamic range for $E$ in (2), we choose the reference voltage $V_{S,\text{ref}}$ to correspond to the midpoint of the VIC energy range, hence

$$V_{S,\text{ref}} = \frac{1}{2} \left( V_{S,\max}^2 + V_{S,\min}^2 \right),$$

and we work with the error $e$ defined as $e = V_{S,\text{ref}} - V_S$. We propose to use a PI controller $K_p \left( 1 + \frac{1}{\tau c} \right)$, with $K_p > 0$ and $\tau_c > 0$, acting on $e$, as shown in Fig. 4. A feed-forward term $K$ is present in order to respond to fast changes in $i_{\text{out}}$ (such as sudden load changes), and is application-specific, with an example discussed in Section V. There is also a squaring block in the loop, which is meant to cancel the square root introduced by (5).

We denote by $\Phi$ the state variable of the filter $W$ and by $\Psi$ the state of the integrator in the PI controller. Recalling that $Q$ is the charge stored in the VIC, the equations describing the charge control loop in Fig. 4 are
\[
\frac{d}{dt} \begin{bmatrix} \psi \\ Q \\ \phi \end{bmatrix} = \begin{bmatrix} 0 & -2V & 0 \\ 0 & K_p & m \\ \frac{1}{\tau c} & 2K_pV & -\frac{1}{\tau} \end{bmatrix} \begin{bmatrix} \psi \\ Q \\ \phi \end{bmatrix} + \begin{bmatrix} V_{S,\text{ref}} - \beta \\ m\tau - i_{\text{out}} \\ \frac{1}{\tau}(K_{\text{out}} + K_pV_{S,\text{ref}} - K_p\beta) \end{bmatrix}
\]

which we can write compactly as

\[
\dot{x} = A(m)x + f,
\]

Here \(m\) is a positive and \(T\)-periodic function of \(t \geq 0\), the 3x3 matrix \(A(m)\) is an affine function of \(m\), and \(f\) is continuous and bounded function of time.

We would like to find conditions for the system (7) to be convergent, which means that the solutions of the system (corresponding to different initial states \(x(0)\)) converge to each other (i.e. the difference between any two solutions tends to zero). We refer to Pavlov et al. (2004, 2007) and Russo, di Bernardo and Sontag (2010) for the background on convergent systems and further references. It is easy to show that (7) is convergent if and only if the corresponding linear homogeneous system

\[
\dot{x} = A(m)x
\]

is exponentially stable. Establishing the stability of time-varying linear systems is not easy, and has been the focus of extensive research, see for instance Farkas (1994), Hinrichsen and Pritchard (2005), Apkarian, Gahinet and Becker (1995), and Allwright (1999).

We know of two methods to determine an interval \([m_{\text{min}}, m_{\text{max}}]\) such that (8) is exponentially stable for any measurable function \(m\) with values in this interval. An obvious necessary condition for this is that (8) is exponentially stable for any constant \(m \in [m_{\text{min}}, m_{\text{max}}]\). For our specific matrix, the characteristic polynomial is

\[
\det(sI - A(m)) = s^3 + \frac{1}{\tau}s^2 + \frac{2K_pV}{\tau c}s + \frac{2V}{\tau c}\tau_c.
\]

The Routh test yields that \(A(m)\) (for any fixed \(m > 0\)) is exponentially stable if and only if

\[
\tau_c > \tau.
\]

Remarkably, this is independent of \(m\). In the sequel, we assume that (9) holds.

The first method for finding \([m_{\text{min}}, m_{\text{max}}]\) is to find a common Lyapunov function for all the systems of type (8) with constant \(m\) in this interval. For this, we solve the Lyapunov equation

\[
PA(m_0) + A(m_0)^TP = -I
\]

for some \(m_0 > 0\) in the middle of the range of interest. This is possible thanks to (9), and we get \(P > 0\), see for instance Hinrichsen and Pritchard (2005, Ch. 3.3). Then we search for \(m_{\text{min}} \in (0, m_0)\) such that \(PA(m_{\text{min}}) + A(m_{\text{min}})^TP < 0\) (such \(m_{\text{min}}\) certainly exists) and similarly we search for \(m_{\text{max}} \in (m_0, \infty)\) such that \(PA(m_{\text{max}}) + A(m_{\text{max}})^TP < 0\). Then \(V(x) = xP, x > 0\) is the desired common Lyapunov function, as it is easy to verify (using the affine nature of \(A(\cdot)\)). It is now easy to see that if \(m\) is a measurable function with values in \([m_{\text{min}}, m_{\text{max}}]\), then there exists \(\lambda > 0\) such that \(V(x) \leq -\lambda\|x\|^2\), which implies the exponential stability of (8). This first method can also be formulated as an LMI problem in the sense of Apkarian et al. (1995), the details are easy and we omit them.

The second method for finding \([m_{\text{min}}, m_{\text{max}}]\) is based on structured perturbation theory (Hinrichsen and Pritchard, 2005, Ch. 4.4). We write

\[
A(m) = A(m_0) + (m - m_0)BC
\]

where \(m_0 > 0\) is in the middle of the range of interest, \(B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}\) and \(C = [0 \ 0 \ 1]\). Consider the stable transfer function

\[
G(s) = C(sI - A(m_0))^{-1}B
\]

and compute its \(H^\infty\) norm \(\|G\|\) by any of the numerical methods available. Then we may take any \(m_{\text{min}} > m_0 - \frac{1}{\|G\|}\), \(m_{\text{max}} < m_0 + \frac{1}{\|G\|}\). Both methods described above are conservative, i.e., the true interval of stability may be larger than what these methods give. Note that in our application, \(m\) is not an arbitrary bounded measurable function, but a well defined continuous and periodic function. In this case, the exponential stability of (8) may be true also if the range of \(m\) is not contained in the interval \([m_{\text{min}}, m_{\text{max}}]\) discussed earlier.

IV. CONTROLLING THE VIC WHEN \(Q\) IS IN THE DESIRED RANGE

Let us discuss in more detail the operation of our circuit from Fig. 3 for \(Q \in [Q_{\text{min}}, Q_{\text{max}}]\). This looks like a reversible boost converter with output voltage \(V\) that should be kept as
close as possible to a given value $V_{\text{ref}}$. The state equations of this system are:

$$\begin{align*}
CV' &= -q \cdot i_S + i, \\
Li_S' &= q \cdot V - V_S, \\
C_S V_S &= i_S.
\end{align*}$$

(10)

These equations are considered only in the operating range $\Omega$ defined by

$$|i_S| \leq i_{S,\text{max}}, V_S \in [V_{S,\text{min}}, V_{S,\text{max}}], V > V_{\text{min}},$$

(11)

where $0 < V_{S,\text{min}} < V_{S,\text{max}} < V_{\text{min}} < V_{\text{ref}}$. The average model of the circuit (that corresponds to infinite switching frequency) is obtained by replacing in the equations (10) $q$ with $D$, where $D$ is the short-time average of $q$ (this would be the duty cycle if $q$ is a PWM signal). The control input of this system is $D$ and clearly $0 \leq D \leq 1$. The average model of the circuit is shown in Fig. 5 (following Kassakian et al. (1992, Ch. 11)). There is no current flowing to the measurement terminals indicated on the right side of the figure.

Fig. 5. The average model of the VIC.

The average model is a third order nonlinear system. We think that for this system, the aim of keeping exactly $V = V_{\text{ref}}$ (in steady state operation, with a fluctuating current $i$) cannot be achieved, no matter what control strategy we use, but we have no rigorous proof for this. A similar fundamental limitation for boost converters (that are of second order) has been discussed also in Sira-Ramirez (1987) and Ortega et al. (1998, Ch. 7) and it is due to the unstable zero dynamics. This difficulty is present even if we linearize the equations (we obtain a zero in the right half-plane when $i_S < 0$). In the linear case, this problem has been addressed by various techniques, including $H^\infty$ control, see for instance Naim, Weiss and Ben-Yaakov (1997). The approaches based on linearization are completely inadequate when the input voltage of the boost converter ($V_S$ in our case) varies significantly, since the small signal assumption of the linearization is violated.

We now introduce a sliding mode controller for our circuit from Fig. 3, working in the region $Q \in [Q_{\text{min}}, Q_{\text{max}}]$. Sliding mode control of boost converters, which seems to be a very attractive approach, has been explored by several researchers, among them Ortega et al. (1998), Cortez, Alvarez and Alvarez (2002), Escobar and Sira-Ramirez (1998), Gee, Robinson and Dunn (2011), Guo et al. (2010), Tan, Lai and Tse (2008) and Wai and Shih (2011). Good references on sliding mode control in general are Edwards and Spurgeon (1998), Levant (1993) and Tan, Lai and Tse (2008). Our sliding mode controller is inspired by the one in Hijazi et al. (2009), but is not the same. Indeed, the sliding function in Hijazi et al. (2009) (a function of two state variables) is designed for a boost converter with a constant input voltage, whereas our circuit requires a modified sliding function (a function of three state variables and the disturbance $i$). For our oscillating (but positive) voltage $V_S$ and the oscillating input current $i$ we propose to use the following sliding function:

$$S = V_{\text{ref}} - V - k(V_{\text{ref}} \cdot i - V_S \cdot i_S),$$

(12)

where only $V_{\text{ref}}$ is constant. The sliding surface $\Gamma$ is the set of all possible states $x = \left[\begin{array}{c} V_S \\ i_S \end{array}\right] \in \Omega$ for which $S(x) = 0$.

Intuitively, we can understand that it is desirable to keep the state $x$ on $\Gamma$ (or very close to it). Indeed, if $V = V_{\text{ref}}$, then the expression in the brackets $V_{\text{ref}} \cdot i - V_S \cdot i_S$ is the power entering the converter which (assuming no switching losses) is the power entering the inductor, and this power is very small. Hence, on the sliding surface $\Gamma$ we nearly have $V = V_{\text{ref}}$. The proposed sliding mode control is:

$$q = \begin{cases} 1 & \text{if } S < 0, \\ 0 & \text{if } S \geq 0, \end{cases} \quad \bar{q} = 1 - q.$$  

This control law is written in an ideal form that would lead to infinitely fast switching of $q$ and $\bar{q}$. In reality the switching frequency is limited, since the controller has a finite sampling frequency $1/T_s$, but we ignore this fact in our analysis, because $1/T_s$ is assumed to be high.

In order to ensure that the state trajectories will converge to the sliding surface $\Gamma$ for any initial conditions in $\Omega$, we must derive a sufficient condition for the sliding mode coefficient $k$. We require that $S \cdot \dot{S} < 0$ for every $x \in \Omega \setminus \Gamma$. This condition is referred to as the hitting condition in Edwards and Spurgeon (1998), and it means that every state trajectory approaches $\Gamma$.

We impose limitations on the input $i$:

$$|i| \leq i_{\text{max}}, \quad |\bar{i}| \leq i_{\text{max}},$$

(13)

where $i_{\text{max}} > 0, i_{\text{max}} > 0$. Imposing a bounded $i$ might be problematic for many applications, but in fact the circuit will tolerate short times when $|i|$ exceeds $i_{\text{max}}$, for example due to sudden changes in the load ($R$ in Fig. 2). During these short times, the hitting condition may be violated, so that $|S|$ may grow, but this will be corrected later by the controller. For reasons that will become clear later, we assume that

$$V_{\text{min}} > Li_{\text{max}} V_{\text{ref}} V_{S,\text{max}}.$$ 

(14)

The hitting condition should be derived for both sides of the sliding surface, namely $S > 0$ and $S < 0$. Suppose first that $S > 0$ (hence $q = 0$), then from (10) and (12),

$$S = -\frac{i}{C} + k \left(\frac{i_S^2}{C_S} - \frac{V_S^2}{L}\right) - kV_{\text{ref}} \bar{i}.$$ 

(15)

Using (15), the condition $\dot{S} < 0$ becomes a condition on $k$, which in the worst case is

$$k \left(\frac{V_{S,\text{min}}^2}{L} - \frac{i_{S,\text{max}}^2}{C_S} - V_{\text{ref}} i_{\text{max}}\right) > \frac{i_{\text{max}}}{C}.$$ 

(16)
Now suppose that $S < 0$ (hence $q = 1$). Then from (10), (12),
\[
\dot{S} = \frac{i_s - i}{C} + k \left( \frac{i_s^2}{C_s} + \frac{V_s V_s}{L} - \frac{V_i^2}{L} \right) - k V_{\text{ref}} i.
\]  
(17)
Similarly as in the previous case, the condition $\dot{S} > 0$ becomes:
\[
k \left[ \frac{V_{S,\text{max}} (V_{S,\text{min}} - V_{S,\text{max}})}{L} - V_{\text{ref}} i_{\text{max}} \right] > \frac{i_{\text{max}} + i_{S,\text{max}}}{C}.
\]  
(18)
From (14) we know that the coefficient of $k$ in (18) is positive.

Choosing $k$ that satisfy both the conditions (20), (21) will fulfill the existence condition and will ensure that the state trajectory near the sliding surface will always be directed towards the sliding surface. Finding the range of $k$ that will satisfy (20) and (21) for all possible $x \in \Gamma$ and for all $\dot{i}$ satisfying $|\dot{i}| < \dot{i}_{\text{max}}$ is a numerical problem that is best handled by a computer. One usually obtains a condition of the type $k > k_{\text{min}}$, where $0 < k_{\text{min}} < k_{\text{min}}$. In order to get reasonable results, we must impose an upper bound $V_{\text{max}}$ on $V$, otherwise (20) would require an infinite $k$. When choosing $k$ based on the existence condition, we choose $V_{\text{min}}$ and $V_{\text{max}}$ close to $V_{\text{ref}}$ to get a better (i.e., lower) value for $k_{\text{min}}$.

Formula (19) imposes an upper bound for the ripple of $V$ when $x$ stays close to $\Gamma$ and the existence condition is satisfied. Indeed, expressing $V$ from (19) and then applying the first two constraints from (11) and the first constraint from (13) we obtain
\[
|V - V_{\text{ref}}| < k (V_{S,\text{max}} \cdot i_{S,\text{max}} + V_{\text{ref}} i_{\text{max}}).
\]

Computing $\dot{S}$ as a function of $q$, the state variables, $i$ and $\dot{i}$ and setting $\dot{S} = 0$, we can derive the short-time average of $q$:
\[
\bar{q} = \frac{L i + k C \left( \frac{V_s^2}{C_s} - \frac{L}{C_s} i_{S,\text{max}}^2 + Li_{\text{ref}} \right)}{L i_s + k C V_s V_{\text{ref}}}.
\]  
(22)
By setting $S = 0$ and substituting (19) into (22), we get
\[
\bar{q} = \frac{L i + k C \left( \frac{V_s^2}{C_s} - \frac{L}{C_s} i_{S,\text{max}}^2 + Li_{\text{ref}} \right)}{L i_s + k C V_s V_{\text{ref}}}. \]

We note that the sliding mode control can also be achieved by an equivalent PWM control with the duty cycle $D = \bar{q}$. However, we think that this equivalent PWM control is not practical, because it requires the measurement of $\dot{i}$ and a complicated computation.

V. A PFC WITH A VIC – THEORY AND SIMULATION RESULTS

The performance of the VIC has been examined in a typical application as the output capacitor of a boost converter used in a PFC, shown in Fig. 6. For the boost converter in a PFC there
are two control objectives, namely: (1) attaining a nearly constant output voltage $V = V_{\text{ref}}$; (2) keeping the (short-time) average value of the input current $i_g$ nearly proportional to the input voltage $u_g = A \sin(\omega t)$, thus obtaining a close to unity power factor.

There are several methods to build a PFC boost converter in continuous or discontinuous conduction modes. The boost converter depicted here operates in the critical conduction mode (CRM), also called border-line mode, as described in Gotfryd (2003) and in Lai and Chen (1993). In the CRM operation, each current pulse of $i_L$ has the form shown in Fig. 7. During $0 \leq t < t_{on}$ in each pulse, the switch will be closed and the inductor current $i_L$ will rise. During $t_{on} \leq t < T$ the switch will be open and the energy in the inductor will be released to the load and the VIC, causing $i_L$ to fall to zero, since $V > u_r = |u_g|$. When $i_L = 0$ is detected, the switch will close again, starting the next triangular pulse. Note that the switching frequency $1/T$ (which is variable) should be much greater than the grid frequency. The average current in a triangle is

$$i_L = \frac{u_r}{2L_b}t_{on}.$$  \hspace{1cm} (23)

Thus, to create an average current that is proportional to $u_r$, we hold $t_{on}$ constant during each semi-cycle of the grid (from one zero crossing of the grid to the next). Then (according to (23)) the boost converter is seen from the grid like a resistor with resistance $\frac{2L_b}{t_{on}}$, which is a desirable behavior.

The diode is conducting only when the switch is open, hence its current $i_D$ corresponds to the descending part of $i_L$ in Fig. 7 (for $t_{on} \leq t < T$), and $i_D = 0$ otherwise. The average current of the diode is

$$i_D = \frac{1}{T}(T - t_{on}) \frac{u_r}{2L_b}t_{on}.$$  \hspace{1cm} (24)

From elementary considerations we have $T - t_{on} = \frac{u_r}{V - u_r}t_{on}$, from which $T = \frac{V}{V - u_r}t_{on}$. Substituting this into (24) we obtain the (short-time) average current of the diode:

$$i_D = \frac{u_r^2t_{on}}{2L_bV}.$$  \hspace{1cm} (25)

Identifying $i_D$ as $i_{in}$ from Fig. 2, and $t_{on}$ as $\theta(t)$ from (4), we obtain $m(t) = \frac{u_D(t)}{2L_bV}$. Note that when the VIC is in the desirable second region (the flat segment in Fig. 1), then $V \approx V_{\text{ref}}$, so that $m$ is approximately proportional to $u_r^2$. The expressions (23), (25) enable us to obtain an average model of the boost converter using a resistor and a current source that both depend on $t_{on}$, as shown in Fig. 8.

The current flowing into the VIC is $i = i_D - i_{out}$. This current is fluctuating at twice the grid frequency, and is linearly dependent on $t_{on}$. We recall that the purpose of the VIC is to hold $V$ constant. Thus, keeping $Q$ in the region $Q \in [Q_{\text{min}}, Q_{\text{max}}]$ (which is equivalent to keeping $V_S \in [V_S_{\text{min}}, V_S_{\text{max}}]$) involves regulating $t_{on}$ using a variation of the charge controller presented in Section III. The value $t_{on}$ that the boost converter receives is updated when the grid voltage crosses zero, hence at twice the grid frequency, because $t_{on}$ should be constant in each semi-cycle to guarantee a nice sinusoidal shape for $i_g$. In addition, a feed-forward term from $i_{out}$ is used to improve the transient response to changes in the load:

$$\hat{t}_{on}(s) = W(s) \cdot [\hat{u}(s) + K\hat{i}_{out}(s)],$$

where $u$ is the output of the PI controller, as in Fig. 4, and $K$ is the feed-forward gain:

$$K = \frac{4L_bV_{\text{ref}}}{u_{g,\text{max}}},$$

where $u_{g,\text{max}}$ is the amplitude of the grid voltage. This gain is

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**Fig. 7:** The waveform of the inductor current, as required for critical conduction mode.

**Fig. 8:** The PFC from Fig. 6 with the average model of the boost converter. The VIC charge controller affects both the PFC input current and the VIC input current via $t_{on}$. Since $C_{in}$ is small, the circuit looks from the grid like a resistor with resistance $\frac{2L_b}{t_{on}}$. Note the similarity of the right side of this circuit to the circuit in Fig. 2.
derived from the steady-state criterion for zero energy change during one grid semi-cycle. Indeed, integrating (25) and a constant $i_{out}$ over one grid semi-cycle $\left[0, \frac{\pi}{\omega_g}\right]$ we obtain

$$u_{g,\text{max}}^2 t_{on} = \frac{2L_i V}{\pi/\omega_g} \int_0^{\pi/\omega_g} \sin^2(\omega_g t) dt = i_{out} \frac{\pi}{\omega_g}.$$  

Expressing from here $t_{on}$, we obtain the factor $K$.

The load changes are supposed to be rare events (when a sudden change in the load occurs, the first priority is to maintain a constant $V$, even at the price of temporarily causing a distortion in $i_g$). The feed-forward term may instantaneously affect $t_{on}$, thus violating the PFC objective of sinusoidal $i_g$, in case the VIC does not have enough energy reserve. The internal sliding mode controller of the VIC is not shown in Fig. 6 and 8.

Fig. 9: Simulation results for a PFC boost converter using a VIC. Two grid cycles are shown in steady state operation. All the signals shown, except for the last (d), are filtered via a low-pass filter with a cut-off frequency of 1kHz, to eliminate the ripple noise of the boost converter. (a) the converter input current $i_c$; (b) the VIC input current $i$; (c) the voltage error $V_{ref} - V$; (d) the VIC inner capacitor voltage $V_C$, used as a measure of $Q$.

The VIC performance in the circuit from Fig. 6 has been examined by MATLAB simulations using the SimPowerSystems toolbox. The parameters of the VIC (shown in Fig. 3 and the sliding function (12)) are $k = 0.002, C = 4\mu F, C_S = 40 \mu F$ and $L = 60 \mu H$. We choose $V_{ref} = 400 \sqrt{2}, V_{s,\text{min}} = 100 V, V_{s,\text{max}} = 390 V$ and $V_{min} = 395 V$. From (16) and (18) we get $k_{min} \approx 0.008$ (this value strongly depends on $V_{min}$) and from (20) and (21) we get $\hat{k}_{min} \approx 0.002$ (using $398 < V < 402$). The sampling frequency of the VIC controller is 2MHz. The boost converter circuit parameters (shown in Fig. 6) are $R = 320 \Omega$ (thus, the power is 500W), $L_B = 60 \mu H$ and $C_in = 100 \mu F$.

The grid voltage $u_g$ has the amplitude $u_{g,\text{max}} = 340 V$ and the frequency 50Hz. The results are shown in Fig. 9. The calculated total harmonic distortion (THD) of the input current $i_g$ (in the range up to 1kHz) is less than 0.7%. The low frequency voltage ripple seen in Fig. 9(c) is 1.63V.

### VI. AN IMPROVED FULL-ORDER CONTROLLER

As we have seen, small perturbations of the output voltage $V$ still remain in a PFC regulated by the VIC. We can try improving the steady state performance by increasing the order of the controller. This practice is advised in Fridman, Moreno and Iriate (2011, Ch. 1) and in practical design guides such as Tan et al. (2008), especially if a fixed-frequency SMC is employed (which is the case in our implementation). This kind of full order SMC is called Integral Sliding Mode (ISM) by Utkin and Shi (1996) and in Fridman et al. (2011, Ch. 1), but we do not follow their approach, since it requires a non-singular state dependent input matrix, which is not our case (see (10)). Instead, we add a simple integrator of the output voltage error to the sliding function $S(x)$ (following Tan et al. (2008)). The improved sliding function will be

$$S = V_{ref} - V - k_s (V_{ref} \cdot i - V_S \cdot i_S) + k_I \int_0^t (V_{ref} - V) dt,$$

where $k_I > 0$ is a second sliding coefficient. Since the implementation of the controller is discrete, the integral is replaced by an appropriate summation. Intuitively, such an integrator reduces the accumulated error in $V$, and gives added weight to the low-frequency fluctuations over the high-frequency ripple which is easily filtered.

Fig. 10: Simulation results for a PFC boost converter using a VIC with the improved full order sliding function (26). Three grid cycles are shown in steady state operation. The signals (except $V_C$) pass through an LPF with cut-off frequency of 1kHz. This filter is insufficient to smooth the signal $V_{error} = V_{ref} - V$, yet it is evident that the low frequency ripple is of the order of 50mV. The meaning of all the signals is as in Fig. 9.

As a result of the increased order, the existence condition of the sliding mode will become a complex relation between $k$ and $k_I$. Although it is possible to formulate this relation analytically and use numerical methods to find the domain
where the existence condition holds, it is more practical to use the $k$ appropriate for the second order sliding function (12), and a small $k_1$ that would not greatly influence the existence condition. The hitting condition is not a concern in our implementation, since the reaching phase is done using a constant PWM and not SMC, as explained in Section II.

The performance of the VIC with the improved sliding function (26) is shown in Fig. 10. In this simulation, the sliding coefficients were $k = 0.008, k_1 = 0.1$. All other parameters were as in the previous section. As seen in the simulation results, this controller indeed provides a significant improvement in the steady state error, with a low-frequency output voltage ripple of about 50mV, in comparison to 1.63V using the sliding function (12). The THD of $i_p$ is not changed by much, with a value of 0.72%. 

The capacitance of a conventional capacitor, if used to achieve the same output voltage ripple instead of the VIC, would have been $C = \frac{i_{pp}}{V_{pp}} = \frac{2.72}{0.052r + 100} \approx 86\, \text{mF}$, more than 2000 times larger than $C_S$ (we have denoted by $i_{pp}$ the peak-to-peak variation of $i$, as observed in Fig. 10(b), and similarly $V_{pp}$ is the peak-to-peak variation of $V$).

VII. CONCLUSION

In this paper we have introduced the VIC, an electronic circuit that replaces a large filter capacitor. A realization of the VIC using a bidirectional DC/DC converter with sliding mode control was proposed. An application of the VIC as a component in the boost converter of a PFC was studied and simulated, using a capacitor 2000 times smaller than an equivalent filter capacitor required to achieve the same output voltage ripple.

The use of much smaller capacitors instead of electrolytic capacitors contributes to higher reliability and smaller devices. Therefore, the VIC can replace the input or output capacitors in various power converters, where there is a significant low frequency current flowing through the capacitor. Typical applications are in PFCs and in single phase inverters.

Further work is needed to develop a zero-voltage switching version of the VIC. Suggested future work is a robustness analysis of the VIC sliding mode controller, including a comparison with the total sliding mode controller in Wai and Shih (2011).

REFERENCES


