The Jacobi MIMO Channel: Achieving The No-Outage Promise

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Abstract—In the Jacobi fading model $H$, the transfer matrix which couples the $m_t$ inputs into $m_r$ outputs, is a sub-matrix of an $m \times m$ random (Haar-distributed) unitary matrix. The (squared) singular values of $H$ follow the law of the classical Jacobi ensemble of random matrices. In the case where the model parameters satisfy $k = m_t + m_r - m > 0$, at least $k$ singular values are guaranteed not to fade for any channel realization, enabling an achievable zero outage probability at the corresponding rates. A simple scheme utilizing (a possibly outdated) channel state feedback is provided, attaining the no-outage promise.

I. INTRODUCTION

In Multiple-Input Multiple-Output (MIMO) channels the vector $\mathbf{y}$ containing $m_r$ complex components, represents the received signal and can be described as $\mathbf{y} = \mathbf{Hx} + \mathbf{z}$, where $\mathbf{x}$, containing $m_t$ complex components, represents the transmitted signal, $\mathbf{z}$ accounts for the presence of additive Gaussian noise. The vector $\mathbf{H}$ is a block of size $m_r \times m_t$ within an $m \times m$ uniformly drawn unitary matrix. The (squared) singular values of $\mathbf{H}$ follow the law of the classical Jacobi ensemble of random matrices, hence the name of the channel.

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In [1], [3] the Jacobi channel is further analyzed and analytical expressions are given for the ergodic capacity, outage probability and diversity multiplexing tradeoff. An interesting result is that when the model’s parameters satisfy $k = m_t + m_r - m > 0$, $k$ singular values of $\mathbf{H}$ are 1, resulting in an achievable zero outage probability for any transmission rate below $k \log(1 + \text{SNR})$, where SNR is the signal-to-noise ratio in the unfaded Single-Input Single-Output (SISO) case ($m = 1$). Here we provide a coding scheme that uses channel state (CS) feedback to achieve the highest rate possible with no-outage. It is important to mention that the scheme works even when the CS information at the transmitter is “outdated”, allowing simple decoding with no complicated MIMO signal processing, making it plausible for optical communication.

The paper is organized as follows. We start by defining the system model and presenting the channel statistics in Section II. In Section III we consider the slow-fading channel scenario and analyze the outage probability, showing that for $k > 0$ a strictly zero outage probability is obtainable for $k$ degrees of freedom. Following this finding, we present in Section IV a new communication scheme which exploits a channel state feedback to achieve zero outage probability. Section V briefly summarizes the results.

II. SYSTEM MODEL AND CHANNEL STATISTICS

We consider a space-division multiplexing (SDM) system that supports $m$ spatial propagation paths. In tribute to optical communication, in particular multi-mode optical fibers, the initial motivation for this work, we shall refer to these links as modes. Assuming a unitary coupling among all transmission modes the overall transfer matrix $\mathbf{H}$ can be described as an $m \times m$ unitary matrix, where each entry $h_{ij}$ represents the complex path gain from transmitted mode $i$ to received mode $j$. We further assume a uniformly distributed unitary coupling, that is, $\mathbf{H}$ is drawn uniformly from the ensemble of all $m \times m$ unitary matrices (Haar distributed). Considering a communication system where $m_t \leq m$ and $m_r \leq m$ modes are being addressed by the transmitter and receiver, respectively, the effective transfer matrix is a truncated version of $\mathbf{H}$. Under these conditions the channel can be described as

$$\mathbf{y} = \sqrt{\rho} \mathbf{H}_{11} \mathbf{x} + \mathbf{z},$$

where the vector $\mathbf{x}$ containing $m_t$ complex components, represents the transmitted signal, the vector $\mathbf{y}$ containing $m_r$ complex components, represents the received signal, and $\mathbf{z}$ accounts for the presence of additive Gaussian noise. The $m_r$ components of $\mathbf{z}$ are statistically independent, circularly symmetric complex zero-mean Gaussian variables of unit energy $\mathbb{E}(\lvert z_j \rvert^2) = 1$. The components of $\mathbf{x}$ are constrained such that the average energy of each component is equal to 1, i.e., $\mathbb{E}(\lvert x_j \rvert^2) = 1$ for all $j$. The term $\rho \geq 0$ is proportional to the constant per-mode power constraint, as opposed to the constant total power constraint often used in wireless communication, is motivated by the optical fiber nonlinearity limitation. Nevertheless, the total power constraint will be considered as well when needed.
to the power per excited mode so that it equals to the signal-to-noise ratio in the single mode case \((m = 1)\). The matrix \(H_{11}\) is a block of size \(m_r \times m_t\) within the \(m \times m\) random unitary matrix \(H\)

\[
H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}.
\] (2)

As a first stage in our work we present the relation between the transfer matrix \(H_{11}\) and the Jacobi ensemble of random matrices [4]-[6]. Limiting our discussion to complex matrices we state the following definitions:

**Definition 1** (Gaussian ensemble). \(G(m,n)\) is \(m \times n\) matrix of i.i.d complex entries distributed as \(CN(0,1)\).

**Definition 2** (Wishart ensemble). \(W(m,n)\), where \(m \geq n\), is \(n \times n\) Hermitian matrix which can be constructed as \(A^\dagger A\), where \(A\) is \(G(m,n)\).

**Definition 3** (Jacobi ensemble). \(J(m_1,m_2,n)\), where \(m_1, m_2 \geq n\), is \(n \times n\) Hermitian matrix which can be constructed as \(A(A + B)^{-1}\), where \(A\) and \(B\) are \(W(m_1,n)\) and \(W(m_2,n)\), respectively.

The joint probability density function (PDF) of the eigenvalues of these three ensembles is well-known [4]. Here, we quote the joint PDF of the ordered eigenvalues \(0 \leq \lambda_1 \leq \ldots \leq \lambda_n \leq 1\) of the Jacobi ensemble \(J(m_1,m_2,n)\)

\[
f(\lambda_1^n) = K_{m_1,m_2,n}^{-1} \prod_{i=1}^{n} \lambda_i^{m_1-i}(1-\lambda_i)^{m_2-n} \prod_{i<j}(\lambda_i - \lambda_j)^2 ,
\] (3)

where \(K_{m_1,m_2,n}\) is a normalizing constant. We shall say that \(n\) variables follow the law of the Jacobi ensemble \(J(m_1,m_2,n)\) if their joint distribution follows (3).

Now, the first two ensembles relate to wireless communication [7]. The third classical ensemble, the Jacobi ensemble, is relevant to this channel model, where we need to distinguish between the following two cases:

**A. Case I** - \(m_t + m_r \leq m\)

In [8, Theorem 1.5] it was shown that for \(m_t, m_r\) satisfying the conditions \(m_t \leq m\), and \(m_t + m_r \leq m\), the eigenvalues of \(H_{11}^1H_{11}\) have the same distribution as the eigenvalues of the Jacobi ensemble \(J(m_t, m_r - m_t, m_t)\). For \(m_t, m_r\) satisfying \(m_t > m_r\) and \(m_t + m_r \leq m\), since \(H_{11}^1\) share the same distribution with \(H\), the eigenvalues of \(H_{11}^1H_{11}\) follow the law of the Jacobi ensemble \(J(m_t, m_r - m_t, m_r)\). Combining these two results, we can say that the squared non-zero singular values of \(H_{11}\) have the same distribution as the eigenvalues of the Jacobi ensemble \(J(m_{max}, m - m_{max}, m_{min})\), where here and throughout this paper we denote \(m_{max} = \max\{m_t, m_r\}\) and \(m_{min} = \min\{m_t, m_r\}\).

**B. Case II** - \(m_t + m_r > m\)

When the sum of transmit and receive modes, \(m_t + m_r\), is larger than the total available modes, \(m\), the statistics of the singular values change. Having in mind that the columns of \(H\) are orthonormal, one can think of \(m_t + m_r > m\) as a transition threshold in which the size of \(H_{11}\) is large enough with respect to \(m\) to change the singularity statistics. The following Lemma provides the joint distribution of the singular values of \(H_{11}\), showing that for any realization of \(H_{11}\) there are \(m_t + m_r - m\) singular values which are 1.

**Lemma 1.** Suppose \(H\) is an \(m \times m\) unitary matrix, divided into blocks as in (2), where \(H_{11}\) is an \(m_r \times m_t\) block with \(m_t + m_r > m\). Then \(m_t + m_r - m\) eigenvalues of \(H_{11}^1\) are 1, \(m_t - m_{min}\) are 0, and \(m - m_{max}\) are equal to the non-zero eigenvalues of \(H_{22}^1\): thus, if \(H\) is Haar distributed these \(m - m_{max}\) eigenvalues follow the law of the Jacobi ensemble \(J(m - m_{min}, m_{min}, m - m_{max})\).

**Proof:** See [1].

Lemma 1 reveals an interesting algebraic phenomenon: \(k = \max\{m_t + m_r - m, 0\}\) singular values of \(H_{11}\) are 1 for any realization of \(H\). This provides some powerful results in the context of Jacobi fading channels. For example, the channel’s power \(|H_{11}^1|^2\), where \(|A|_F\) denotes the Frobenius norm of \(A\), is guaranteed to be at least \(k\). Furthermore, \(H_{11}\) always comprises an unfaded \(k\)-dimensional subspace. In what follows we show that this implies an achievable zero outage probability for certain rates.

### III. Slow-Fading Channel

In the slow-fading case the channel matrix is drawn randomly but rather assumed to be constant within the entire transmission period of each code-frame. Assuming that the channel instantiation is known only at the receiver end, the figure of merit in this non-ergodic scenario is the **outage probability** defined as the probability that the mutual information induced by the channel realization is lower than the rate \(R\) at which the link is chosen to operate. By taking an input vector of circularly symmetric complex zero-mean Gaussian variables with covariance matrix \(Q\) the mutual information is maximized and the outage probability can be expressed as

\[
P_{out}(m_t,m_r, m; R) = \inf_{Q: Q \succeq 0} P_r[\log \det(I_{m_r} + \rho H_{11}Q H_{11}^1) < R],
\] (4)

where the minimization is over all covariance matrices \(Q\) satisfying the power constraints. Since the statistics of \(H_{11}\) is invariant under unitary permutations, the optimal choice of \(Q\), when applying constant per-mode power constraint, is simply the identity matrix. We note that when imposing total power constraint, the optimal choice of \(Q\) may depend on \(R\) and \(\rho\) and in general is unknown, even for the Rayleigh channel. Nevertheless, when \(\rho \gg 1\) the identity matrix is approximately the optimal \(Q\) (see [1]). Thus, in what follows we make the simplifying assumption that the transmitted covariance matrix is the commonly used choice \(Q = I_{m_r}\).

Now, let the transmission rate be \(R = r \log(1+\rho)\) (bps/Hz) and let \(\lambda = \{\lambda_i\}_{i=1}^{m}\) be the ordered non-zeros eigenvalues of
Consider $H_{11}^\dagger H_{11}$; we can write

$$P_{\text{out}}(m_t, m_r, m; R) = Pr \left[ \prod_{i=1}^{\min(m_t, m_r)} (1 + \rho \lambda_i) < (1 + \rho)^{r} \right], \quad (5)$$

and evaluate this expression by applying the statistics of $\Lambda$.

**A. Case I - $m_t + m_r \leq m$**

Using (3) we can apply the joint distribution of $\Lambda$ into (5) to get

$$P_{\text{out}}(m_t, m_r, m; r \log(1 + \rho)) = K_{m_t, m_r, m}^{-1} \int_B \prod_{i=1}^{\min(m_t, m_r)} \lambda_i^{m_r - m_t} \times (1 - \lambda_i)^{m - m_r - m_t} \prod_{i<j} (\lambda_i - \lambda_j)^2 d\lambda,$$

where $K_{m_t, m_r, m}$ is a normalizing factor and $B$ describes the outage event

$$B = \left\{ \Lambda : \prod_{i=1}^{\min(m_t, m_r)} (1 + \rho \lambda_i) < (1 + \rho)^{r} \right\}.$$

This gives an analytical expression to the outage probability. See Fig. 1.

**B. Case II - $m_t + m_r > m$**

Applying Lemma 1 into (5) results the following Theorem.

**Theorem 1.** The outage probability of the channel defined in (1), with $m_t, m_r$ satisfying $m_t + m_r > m$, satisfies

$$P_{\text{out}}(m_t, m_r, m; r \log(1 + \rho)) = P_{\text{out}}(m_t, m_r, m; r \log(1 + \rho)) \times \prod_{i=1}^{\min(m_t, m_r)} (1 + \rho \lambda_i) < (1 + \rho)^{r - (m_t + m_r - m)}$$

where $\{\lambda_i\}_{i=1}^{\min(m_t, m_r)}$ are the eigenvalues of $H_{22}^\dagger H_{22}$.

When $r = r - (m_t + m_r - m)$ and $0$, we have $P_{\text{out}}(m_t, m_r, m; r \log(1 + \rho)) = 0$.

Note that the right-hand-side of (7) drops to 0, when $m_r$, or $m_t$ equals $m$. Most importantly, when $r < m_t + m_r - m$, $\hat{r} = 0$, implying that for such rates zero outage probability is achievable. In addition, when $r \geq m_t + m_r - m > 0$, Eq. (7) implies that the outage probability is identical to that of a channel with $m - m_r$ modes addressed by the transmitter and $m - m_t$ modes addressed by the receiver, which is designed to support a transmission rate equivalent to $\hat{r}$ single-mode channels. Thus the right-hand-side of (7) applies to Eq. (6). In Fig. 1 we show an exemplary calculation of the outage probability. Note how the outage probability abruptly drops to 0 whenever $r$ becomes smaller than $m_t + m_r - m$. Also note that the outage probability is symmetric in $m_t, m_r$ since we applied a constant per-mode power constraint, thus all combinations of $m_t, m_r$ are plotted in Fig. 1.

![Outage probability vs. normalized rate for $\rho = 20$dB. The number of supported modes is fixed $m = 4$, various numbers of transmit $\times$ receive modes.](image)

**IV. ACHIEVING THE NO-OUTAGE PROMISE**

In the previous section we saw that for systems satisfying $k = m_t + m_r - m > 0$, a zero outage probability is achievable for any transmission rate below $R = k \log(1 + \rho)$. In this section we present a new communication scheme that achieves this promise with a transmission rate arbitrarily close to $R = k \log(1 + \rho)$. Using simple manipulations, the scheme exploits a (delayed) channel state information (CSI) feedback to transform the channel into $k$ independent SISO channels, supporting $k$ streams (degrees of freedom) with zero outage probability.

Let

$$H^{(i)} = \begin{bmatrix} H_{11}^{(i)} & H_{12}^{(i)} \\ H_{21}^{(i)} & H_{22}^{(i)} \end{bmatrix}$$

be the unitary matrix realization at channel use $i$ and let

$$\mathbf{y}^{(i)} = \sqrt{\rho} H_{11}^{(i)} \mathbf{x}^{(i)} + \mathbf{z}^{(i)}$$

be the received signal. We assume a perfect knowledge of $H_{11}^{(i)}$ at the receiver and a noisless CSI feedback with a delay of a single channel use. Since $H^{(i)}$ unitary, $H_{21}^{(i)}$ can be computed from $H_{11}^{(i)}$ and we assume that the receiver noiselessly communicates $H_{21}^{(i)}$ to the transmitter. Note that $H_{21}^{(i)}$ completes $H_{11}^{(i)}$ into orthonormal vectors, thus for $m_t + m_r - m > 1$ and certain matrix instantiations, the computed $H_{21}^{(i)}$ is not unique and can be chosen wisely (see Remark 4).

Now, let the transmitter excites the following signal from
the addressed modes at each channel use \( i = 1, \ldots, n \)

\[
\mathbf{x}^{(i)} = \begin{bmatrix}
x_1^{(i)} \\
\vdots \\
x_{m_l+m_r-m}^{(i)} \\
\mathbf{H}_{21}^{(i-1)} \mathbf{x}^{(i-1)}
\end{bmatrix}.
\]

That is, the transmitter conveys \( m_l + m_r - m \) new information bearing symbols and \( \mathbf{H}_{21}^{(i-1)} \mathbf{x}^{(i-1)} \), a linear combination of the signal that was transmitted in the previous channel use (\( \mathbf{x}^{(0)} \) is a vector of zeros). Note that \( \mathbf{H} \) is unitary, thus the power constraint is left satisfied.

We shall now assume that after the last signal \( \mathbf{y}^{(n)} \) is received, the receiver gets as a side information the following noisy measures

\[
\mathbf{y}_{i} = \sqrt{\rho} \mathbf{H}_{21}^{(n)} \mathbf{x}^{(n)} + \mathbf{z}_{i},
\]

where the components of \( \mathbf{z}_{i} \) are i.i.d. \( \mathcal{CN}(0,1) \). Thus the receiver can linearly combine \( \mathbf{y}_{i}^{(n)} \) and \( \mathbf{y}_{i} \) in the following manner

\[
\hat{\mathbf{y}}^{(n)} = \left[ \mathbf{H}^{(n)}_{11} \mathbf{H}^{(n)}_{21} \right] \begin{bmatrix} \mathbf{y}_{i}^{(n)} \\ \mathbf{y}_{i} \end{bmatrix}
\]

(10)

to yield

\[
\hat{\mathbf{y}}^{(n)} = \sqrt{\rho} \mathbf{H}^{(n-1)}_{21} \mathbf{x}^{(n-1)} + \tilde{\mathbf{y}}
\]

(11)

where the entries of \( \tilde{\mathbf{y}} \) are i.i.d. \( \mathcal{CN}(0,1) \). We remind that the first \( m_l + m_r - m \) entries of \( \mathbf{x}^{(n)} \) are new information bearing symbols and the last entries are equal to \( \mathbf{H}^{(n-1)}_{21} \mathbf{x}^{(n-1)} \). Thus, the last \( m_l + m_r - m \) entries of \( \hat{\mathbf{y}}^{(n)} \), denoted \( \hat{\mathbf{y}} \), satisfy

\[
\hat{\mathbf{y}} = \sqrt{\rho} \mathbf{H}^{(n-1)}_{21} \mathbf{x}^{(n-1)} + \tilde{\mathbf{y}}
\]

where \( \tilde{\mathbf{y}} \) are the last \( m_l + m_r \) entries of \( \tilde{\mathbf{y}} \). Again, the receiver can linearly combine \( \hat{\mathbf{y}}^{(n-1)} \) and \( \hat{\mathbf{y}} \) as

\[
\hat{\mathbf{y}}^{(n-1)} = \left[ \mathbf{H}^{(n-1)}_{11} \mathbf{H}^{(n-1)}_{21} \right] \begin{bmatrix} \mathbf{y}_{i}^{(n-1)} \\ \hat{\mathbf{y}} \end{bmatrix}
\]

(12)

to yield measures of \( \hat{\mathbf{y}}^{(n-1)} \) as in Eq. (11). Repeating this procedure for \( i = n - 1 \to 1 \) results in \( m_l + m_r - m \) independent streams of measures

\[
\begin{bmatrix}
\hat{\mathbf{y}}^{(1)} \\
\vdots \\
\hat{\mathbf{y}}^{(n-1)} \\
\end{bmatrix}, \ldots, \begin{bmatrix}
\hat{\mathbf{y}}^{(1)} \\
\vdots \\
\hat{\mathbf{y}}^{(n)} \\
\end{bmatrix}
\]

\( \hat{\mathbf{y}}^{(1)}_{m_l+m_r-m} \ldots \hat{\mathbf{y}}^{(n)}_{m_l+m_r-m} \)

The scheme above is feasible if the side information after channel use \( n \) is being conveyed by the transmitter through a negligible number of channel uses (with respect to \( n \), see Remark 3). In that case the receiver can construct \( m_l + m_r - m \) independent SISO channels, each with signal-to-noise ratio \( \rho \). Thus the scheme supports a rate arbitrarily close (as \( n \) is larger) to \( (m_l + m_r - m) \log(1 + \rho) \) with zero outage probability. Note that the scheme essentially completes the singular values of the channel to 1. This is feasible since \( m - m_r < m_l \), thus at each channel use the transmitter can transmit \( \mathbf{H}_{21}^{(i-1)} \mathbf{x}^{(i-1)} \), a signal of \( m - m_r \) entries, and new symbols.

The scheme presented above can be easily expanded to the case where the feedback delay is \( l \) channel uses. In that case the transmitter conveys at each channel use \( m_l + m_r - m \) new information bearing symbols and \( \mathbf{H}_{21}^{(i-l)} \mathbf{x}^{(i-l)} \), a linear combination of the signal that was transmitted \( l \) channel uses before. After channel use \( n \), the transmitter would have to convey \( l \) noisy measures of the last \( l \) signals, so that the receiver could construct \( m_l + m_r - m \) independent SISO channels. This can be done in a fixed number of channel uses (see Remark 3), thus as \( n \) is larger, the transmission rate of the scheme approaches \( (m_l + m_r - m) \log(1 + \rho) \).

**Remark 1 (Outdated feedback).** Our scheme exploits a noiseless CSI feedback system to communicate a (possibly) outdated information - the channel realization in previous channel uses. Thus, the feedback is not required to be fast, that is, no limitations on the delay time \( l \). However, if \( l \) is smaller than the coherence time of the channel, the feedback may carry information about the current channel realization. Thus, the transmitter can exploit the up-to-date feedback to use more efficient schemes. Nevertheless, for systems with a long delay time (e.g., relatively long distance optical fibers), the channel can be regarded as non-ergodic with an outdated feedback. In these cases our scheme efficiently achieves zero outage probability.

**Remark 2 (Simple decoding).** The scheme linearly process the received signals to construct \( m_l + m_r - m \) independent streams of measures, each with signal-to-noise \( \rho \). This allows the decoding stage to be simple, where a SISO channel decoder can be used, removing the need for further MIMO signal processing.

**Remark 3 (Side information measures).** For a feedback with a delay of \( l \) channel uses, the transmitter has to convey \( \mathbf{H}_{21}^{(i)} \mathbf{x}^{(i)} \), for each \( i = n - (l - 1), \ldots, n \), such that the receiver can extract a vector of noisy measures with signal-to-noise ratio that is not smaller than \( \rho \). This is feasible with a finite number of channel uses. For example, the repetition scheme can be used to convey these measures (see Section ?? Example ??). Suppose each \( \mathbf{H}_{21}^{(i)} \mathbf{x}^{(i)} \) is conveyed to the receiver within \( N_{si} \) channel uses (e.g., for the repetition scheme \( N_{si} = m_l (m_l - m_r) \)). By taking large enough \( n \) (with respect to \( l \cdot N_{si} \)) one can approach the rate \( (m_l + m_r - m) \log(1 + \rho) \).

**Remark 4 (Uniqueness of \( \mathbf{H}_{21} \)).** The scheme can be further improved to support even an higher data rate with zero outage probability. For example, the last \( m - m_r \) entries of the transmitted signal at the first channel use can be used to excite information bearing symbols instead of the zeros symbols. Furthermore, as was mentioned above, when \( m_l + m_r - m > 1 \), \( \mathbf{H}_{21}^{(i)} \) is not unique; there are many \((m-m_r)xm_l\) matrices that complete the columns of \( \mathbf{H}_{21}^{(i)} \) into orthonormal vectors. Thus, the receiver can choose \( \mathbf{H}_{21}^{(i)} \) to be the one with the largest number of zeros rows. Now, at time \( i+1 \) the transmitter excites
\( m_t + m_r - m \) new symbols and \( \mathbf{H}_{21}^{(i)}, \mathbf{x}^{(i)} \), a retransmission of \( \mathbf{x}^{(i)} \), the transmitted signal at time \( i \). With an appropriate choice of \( \mathbf{H}_{21}^{(i)}, \mathbf{H}_{21}^{(i)}\mathbf{x}^{(i)} \) contains entries that are zero. Instead, these entries can contain additional new information bearing symbols. An open question is how to further enhance the data rate. One would like to exploit the feedback to approach the empirical capacity for any realization of \( \mathbf{H}_{11} \). Note that this rate is achievable with an up-to-date feedback. Further approaching this rate with an outdated feedback system (and with zero outage probability) is left for future research.

V. Summary

The Jacobi MIMO channel is defined by the transfer matrix \( \mathbf{H}_{11} \), a truncated \( m_r \times m_t \) portion of an \( m \times m \) Haar distributed unitary matrix. An interesting phenomenon is observed when the parameters of the model satisfy \( m_t + m_r > m \): for any realization of \( \mathbf{H}_{11}, m_t + m_r - m \) singular values are 1. This results in a promise for strictly zero outage probability and an exponentially decaying error probability for any transmission rate below \((m_t + m_r - m) \log(1 + \rho)\). A simple communication scheme that exploits a (possibly “outdated”) channel state feedback, is provided. The theoretical findings indicate that the no-outage promise can be attained with no feedback, yet the quest for such simple schemes is open.

References