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DESIGN THROUGH COMMON GRAPH REPRESENTATIONS

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ABSTRACT

Current paper introduces a new technique that enables to solve design problems through their discrete mathematical models called – graph representations. When different engineering fields are represented by the same (common) graph representation, channels for knowledge transformation are paved between these fields. Current paper employs these knowledge transformation channels for design, by transforming a design problem into a design problem in another (secondary) engineering domain. Then, a search is performed in the secondary domain for existent solution. Once such solution is found, it is transformed back to the original domain through the same graph representation based channel. The paper provides a thorough design case study demonstrating the idea behind the proposed technique.

Keywords: Graph Representations, engineering design, graph theory, systematic design.

INTRODUCTION

The work reported in this paper is a part of a general approach providing a global perspective over various engineering systems. The approach is focused on developing global mathematical models, called Graph Representations (GR) and associating them with different engineering systems [1]. Once the engineering system is associated with a specific GR, design, analysis and other forms of engineering reasoning can be conducted solely upon the GR.

A Graph Representation (GR) is basically a graph that is augmented by additional mathematical properties and rules, while different sets of such properties and rules yield different types of graph representations. The approach introduced in the paper uses graph representations to model the behavior of engineering systems. Accordingly, graph representations are characterized by variety of engineering systems that they can represent, the knowledge on how to construct them from these engineering systems and mathematical relations to other graph representations.

A multidisciplinary perspective over engineering systems has widely been discussed in the literature, while the foundation in the field was established in the works of Kron [2] who used his experience and knowledge in electricity to represent different engineering systems belonging to remote engineering domains by electric circuits. In this approach, all the engineering systems are represented by the graph representations.

Graph theory has already been widely employed in engineering, mostly for analysis. In 1955, Trent [3] was one of the first to establish a relationship between physical systems and graph theory, since then it has been applied in many engineering fields. The widest usage of graph theory till now is in electrical networks [4]. Andrews has developed a methodology for applying graphs to multidimensional dynamic systems [5,6]. Significant work has been carried out in structural mechanics [7,8]. In machine theory the pioneer was Freudedstein, known as “the father of the kinematics”, who used graphs as an abstract model of kinematic chains, to aid in the creative stage of mechanism design [9]. More recent works include works on dynamic analysis of geared mechanisms [10]. In simulation, a technique at the level of the composition of system components has been developed [11]. Furthermore, graph theory based analogies between different one dimensional systems such as dynamical, electrical, and heat transfer systems are thoroughly studied in many books dealing with system modeling [12].

In engineering design the graph theory was employed mostly as a tool for systematic enumeration of systems like mechanisms [13], or gear trains [14].

Several aspects underlying the methodology suggested in the current paper relate to well-known topics of the contemporary engineering design research. Following is a brief literature review on these topics, including ‘design by analogy’, ‘systematic design’, ‘schematic synthesis’, 'representation of
The new designs obtained by means of the suggested methodology are mathematically isomorphic to already known designs in other engineering fields. Because of that, there is a certain correlation between the techniques proposed in this paper and the ‘design by analogy’ approach that has recently gained a wide appreciation among the engineering design and AI communities.

Nevertheless, unlike the suggested approach, where mathematical foundation of graph theory underlies the process of deriving the design, a majority of the works in the field of design by analogy attempt to simulate the process of how the designers and engineers arrive at their solutions. Particularly, in this regard, one should mention the works of Gero dealing with the systems to produce new engineering designs. Given a design schematic description describing the topology of engineering systems for design, analysis, and other forms of reasoning upon the problem requirements.

The issue of representing the design specification has also been dealt in the design community, while in most of the works the representations are used to express explicitly both the geometry and the behavior of the design. One of the works related to this issue is due to Finger and Rinderle [20] who used the bond graphs [21] for that purpose. In the current paper the behavior of the system is inherent in the representation, and can be derived upon applying the representation rules. This issue makes the representations more compact and more convenient to reason over.

The technique appearing in this paper is only a part of a general approach underlying it, called Multidisciplinary Combinatorial Approach (MCA) [1]. In this approach the representations are first to be developed, including thorough investigation of embedded theorems, methods and special properties from discrete mathematics (specifically for this paper - the graph theory). On the next stage, the relations between the representations are established and only then diverse engineering domains are associated with these representations. Proceeding in this direction enabled achieving following results: establishing the duality relation between static and kinematical systems [21]; revealing new relations between known methods - proving the duality relation between force and displacement methods in structural mechanics through matroid theory [23], proving the duality relation between image velocity in mechanisms and Maxwell-Cremona diagram in structural mechanics [24]; deriving known theorems and methods in engineering – yielding the equation of joint displacement in trusses from Tellegen’s theorem [25]; Deriving new methods in engineering – decomposition of trusses to Assur’s groups [24], methods for checking the stability of trusses [24,26], and mobility of mechanisms [24], developing new types of cooperations between engineers from different fields – “infused design” [27]. Extending the approach for design is done by employing the knowledge transfer channels that were already paved for other applications, such as analysis, to transform methods and theoretical knowledge, some of which are listed above. The paper introduces a design technique based on transforming design solutions through a common representation, a channel that have already been applied to other engineering applications such as applying Tellegen’s theorem from electricity to structural mechanics through a common graph representation in order to derive the unit force method in mechanics [25]. Thus, it is important for the reader to comprehend that knowledge transfer channels employed in this paper have already been used for other purposes, such as analysis, making additional knowledge available for use in the design process.

NOMENCLATURE

FGR - Flow Graph Representation.
GR - Graph Representation.
PGR - Potential Graph Representation.

GRAPH REPRESENTATIONS

Graph Representation is an isomorphic graph-theoretical substitute of an engineering system, which can be used for design, analysis, and other forms of reasoning upon the engineering system. The purely mathematical essence of Graph Representations makes them convenient for computerization and enables them to provide tools for a generalized treatment of the systems. Different types of Graph Representations are characterized by four main parts, as is outlined in the diagram of Figure 1.
In more detail, the aspects characterizing a Graph Representation constitute the following:

1) Embedded Knowledge – This primary component contains mathematical knowledge underlying the representation, including mathematical laws, theorems and methods. Once an engineering system is associated with a representation, the embedded knowledge becomes available for the engineering system.

2) Relations to other Graph Representations – mathematical relations in graph theory yield relations between different types of graph representations. Current paper employs the duality relation between graph representations based on the known graph theory duality [28]. The relations between graph representations were found to be of crucial importance since they enable to transfer knowledge between different graph representations. In the paper, such a relation is used to transfer design knowledge from statics to kinematics through the corresponding graph representations interrelated by the duality relation.

3) Represented Engineering Domains - Each Graph Representation can be applied to represent a number of engineering domains. Choosing a suitable Graph Representation for an engineering system being treated has a crucial impact on the effectiveness of the subsequent process. This part contains decision rules to determine to which engineering disciplines the specific representation is available.

4) Rules for construction of the Graph Representation – The algorithmic steps for constructing the graph representation corresponding to an engineering system.

**Types of Graph Representations**

Till now several types of graph representations were reported and employed to represent different engineering domains. Table 1 summarizes some of these representations, their basic properties and the engineering systems to which these representations have already been applied. For more details on specific representations, the reader is referred to publications listed in the last column of the table.

**DESIGN THROUGH GRAPH REPRESENTATIONS.**

When an engineering system is represented by a Graph Representation, it is actually brought up to a higher mathematical level, comprised of graph representations partly appearing in Table 1. At this level, the design problem can be solved with the help of the knowledge embedded in the representation, after which the obtained solution can be returned to the engineering domain. Consequently, the connections between the engineering domains and graph representations can be seen as bi-directional knowledge transfer channels. During the representation process, the knowledge flows from the engineering domain up to the representation level, and after the Graph Representation is analyzed at that level, the solution flows back to the engineering domain.

Observing the Graph Representations presented in the previous sub-section leads to a conclusion that all the Graph Representations form a highly interconnected system of knowledge transfer channels both between the engineering domains and the Graph Representations. Figure 2 summarizes all the relations in a "hierarchic map" of different Graph Representations and engineering domains.

**Figure 1. Schematic structure of a graph representation.**

**Figure 2. Hierarchic map of graph representations, their interrelations and association with engineering systems.**
<table>
<thead>
<tr>
<th>Type of graph representation</th>
<th>General Description</th>
<th>Related engineering disciplines</th>
<th>Example of engineering system</th>
<th>The representation of the example engineering system</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow Graph Representation (FGR)</td>
<td>Each edge in FGR is associated a vector called ‘flow’. Flows in FGR satisfy the “flow law”, stating that sum of flows in each cut-set is equal to zero.</td>
<td>Determinate structures, static systems, electric circuits, network flows.</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td>[1, 22,24]</td>
</tr>
<tr>
<td>Potential Graph Representation (PGR)</td>
<td>Each vertex in PGR is associated a vector, called 'potential' that satisfies the 'potential law', saying that the sum of potential differences in each circuit is equal to zero.</td>
<td>Mechanisms, electric circuits, conjugate structures, serial robots, Stewart platforms.</td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
<td>[1, 22,24]</td>
</tr>
<tr>
<td>Resistance Graph Representation (RGR)</td>
<td>Generalizes properties of FGR and PGR. Additionally, each edge is associated with a terminal equation defining the relation between its flow and potential difference.</td>
<td>Electronic circuits, indeterminate structures hydraulic and dynamic systems.</td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
<td>[1]</td>
</tr>
<tr>
<td>Line Graph Representation (LGR)</td>
<td>Each vertex corresponds to an element of the engineering system and edge to a relation between the elements.</td>
<td>Gear and planetary trains.</td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td>[1]</td>
</tr>
<tr>
<td>Flow Line Graph Representation (FLGR)</td>
<td>Possesses the same embedded knowledge as FGR, augmented with the embedded knowledge of LGR.</td>
<td>Static pillar systems, Stewart platforms, determinate beams.</td>
<td><img src="image9.png" alt="Image" /></td>
<td><img src="image10.png" alt="Image" /></td>
<td>[29]</td>
</tr>
<tr>
<td>Potential Line Graph Representation (PLGR)</td>
<td>Possesses the same embedded knowledge as PGR, augmented with the embedded knowledge of LGR.</td>
<td>Mechanisms, serial robots.</td>
<td><img src="image11.png" alt="Image" /></td>
<td><img src="image12.png" alt="Image" /></td>
<td>[29]</td>
</tr>
</tbody>
</table>

Table 1. Graph Representations developed in previous publications.

In Figure 2 it can be seen that there are several couples of engineering systems that can be related to the same graph representation, thus knowledge transfer route between these systems is immediately laid out through this common representation. Such a route yields the most basic design
technique of the proposed approach, which is the topic of the current paper.

Following are the steps of the algorithm for obtaining new designs by means of this technique (Figure 3):

1. Originally, the requirements from the engineering system design are formulated in the terminology of the relevant engineering domain (original engineering domain).
2. Transforming the engineering problem formulation into the terminology of the representation (common graph representation) through the representation construction rules (component 4 of a graph representation – Figure 1).
3. Transforming the problem in the graph representation to the terminology of another engineering domain (secondary engineering domain) through the representation construction rules.
4. Finding a known design or solving the design problem in the secondary engineering domain for the transformed problem.
5. Transforming the design solution from the secondary domain into terms of the common graph representation through the representation construction rules.
6. Constructing the desired system in the original engineering domain from the corresponding graph obtained in step 5. The process can be performed gradually, by augmenting one element of the system at a time.

Figure 3 presents the flowchart describing the above process.

Figure 3. Flowchart outlining the design method.

Due to mathematical basis of the isomorphism between the graphs and the engineering systems, the system obtained in step 6 possesses the properties that were originally requested.

Following subsection reviews a design case of applying the common representation technique. In these cases, the common representations are Potential Graph Representation (PGR) or Flow Graph Representation (FGR) [22] through which routes are laid out and engineering knowledge is transferred between electronic circuits, static systems and gear trains.

**Applying common representation technique for design of unidirectional gear train.**

Both planetary gear trains and electronic circuits can be represented by a Potential Graph Representation, as is outlined in Figure 3. This opens up a possibility to apply the common representation technique for designing of a gear train in which for any arbitrary rotation direction of the driving shaft, the output shaft rotates in a sole direction.
Applying the common representation technique for this case yields the terms summarized in Table 2.

<table>
<thead>
<tr>
<th>Common Representation technique</th>
<th>Current example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original engineering domain.</td>
<td>Gear trains.</td>
</tr>
<tr>
<td>Secondary engineering domain.</td>
<td>Electronic circuits.</td>
</tr>
<tr>
<td>Common graph representation.</td>
<td>Potential Graph Representation (PGR).</td>
</tr>
</tbody>
</table>

Table 2. Correspondence between the terminology of common representation technique and the case study.

<table>
<thead>
<tr>
<th>Terminology of gear trains</th>
<th>Terminology of Potential Graph Representation</th>
<th>Terminology of electronic circuits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Link (gear, planet carrier, shaft).</td>
<td>Edge.</td>
<td>Electronic element (resistor, capacitor, coil, etc.).</td>
</tr>
<tr>
<td>Link joint and the locus of points possessing the same linear velocity as such.</td>
<td>Vertex.</td>
<td>Junction.</td>
</tr>
<tr>
<td>Link relative linear velocity.</td>
<td>Edge potential difference.</td>
<td>Voltage on electrical element.</td>
</tr>
<tr>
<td>Driving link.</td>
<td>Source edge (the edge with preset potential difference value).</td>
<td>Voltage source.</td>
</tr>
<tr>
<td>Overrunning clutch.</td>
<td>Unidirectional edge - an edge that can have only negative potential difference, meaning that its tail vertex has a lower potential than its head vertex.</td>
<td>Diode.</td>
</tr>
</tbody>
</table>

Table 3. PGR construction rules.

Step 1. Stating the Design problem in the terminology of the original domain

Design a gear train whose output link rotates in a sole direction for any rotation direction of the input link, while the magnitudes of the velocities of both links are equal.

Step 2. Transforming the design problem into the terminology of the common graph

According to Table 3, the problem stated in step 1, obtains in the terminology of the common graph the following form: Find a potential graph that for any potential difference function at its source edge returns in the output edge an absolute value of the potential difference of the source.

Step 3. Translating problem statement from the terminology of the common graph to terminology of the secondary engineering domain - electronics.

Applying the transformation rules appearing in Table 3, the design case is transformed into the following form: Find an electronic circuit having a voltage source with arbitrary voltage function and an output element, the voltage in which is equal to the absolute value of the voltage at the input.

Figure 4. Transferring gear system design problem to the terminology of electronic circuits (Steps 1,2 and 3).

Step 4. Solving the problem in the secondary domain - electronics.
The search for electronic systems satisfying these requirements immediately yields several known electronic rectifier circuits [30]. One of these circuits is the “diode bridge circuit” (Figure 5).

Figure 5. Bridge rectifier circuit with voltage source.

Step 5. Building the graph representing the circuit.
In this step the potential graph isomorphic to the electric circuit (assuming ideal model of the diodes) found in Step 4 is built by means of the construction rules of Table 3 as shown in Figure 6.

Figure 6. PGR of the bridge rectifier.

Step 6. Building gear train from the Graph Representation.
Since the same type of graph representation is used to represent gear trains [1], the graph obtained in Step 5 can be interpreted as a representation of a gear train. Construction of the design in the original engineering domain is the most important step in the whole process, thus it will be done gradually by augmenting one element at a time in accordance to the potential graph of Figure 6. Following are the steps of the process of building the design of the mechanical rectifier from the graph representation.

Step-by-step explanation for the process of construction of the mechanical rectifier from the corresponding graph representation:

1) Potential Difference source edge 1 may represent a gear or a shaft. Since the potential difference of the edge is given, the velocity of the corresponding shaft is externally driven. The freedom of choice at this stage allows adapting the design to the specific requirements of the consumer, by allowing to adjust an appropriate: form, length suspension and other parameters of the shaft. Figure 7, shows that an engineering element corresponding to edge 1 can be chosen to be a long shaft connected to a driving wheel. To make the design clear, Figure 7 shows the system from two different perspectives. The end vertices - A and B of the edge correspond to the top and the bottom of the shaft, between which the relative linear velocity of the shaft (equal to the potential difference of edge 1) is measured.

Figure 7. First stage of constructing a mechanical rectifier – adding element corresponding to edge 1.

2) Edge 2 is unidirectional, namely, the value of the potential of its head vertex is higher than the potential of its tail vertex:

\[ \pi_C \geq \pi_A \quad (1) \]

This means that in the represented gear system - the velocity of point A is higher or equal than the velocity of point C.

\[ V_C \geq V_A \quad (2) \]

A is the top of shaft 1, which has already been added to the system, so in order to implement edge 2 we shall add locus of points, C, in such a way so that (2) is satisfied. One way to do so is to connect the two joints through a overrunning clutch forcing this relation, as shown in Figure 8.

Figure 8. Second stage of constructing a mechanical rectifier – adding element corresponding to edge 2.

Similarly to stage 2, implementing edge 3 yields addition of vertex D to the system. It should be noted that since the velocity of joint B is opposite in direction to that of the joint A, the direction of transmission of the new overrunning clutch is also opposite.

3) Figure 9 shows the augmented system.

Figure 9. Third stage of constructing a mechanical rectifier – adding element corresponding to edge 3.

4) Edge 4 corresponds to an element the relative velocity of which is equal to the relative velocity between joints C and D, which indicates that it is a shaft connected between these two joints, as is shown in Figure 10.
5) Edge 5 constitutes a connection between joints A and D, forcing a relation:

\[ V_A \geq V_D \quad (3) \]

Constraint (3) presents an obstacle to the construction process, since point D cannot be connected with A due to geometrical limitations. To resolve it, an expert engineering knowledge can be employed. The property that can be exploited is that points found in the same distance from the rotation axis but from its opposite sides possess the velocity of same magnitude but of the opposite sign. Here, the property yields that \( V_D = -V_C \). Thus one may implement (3) by connecting joint A to a joint with velocity equal to minus velocity of C. Inverting velocity of a joint by gear train is widely accustomed in mechanical engineering community, and can be easily implemented in several ways, one of which appearing in Figure 11.

6) Implementing edge 6, \( V_C \geq V_B \), can be done similarly to stage 5, by inverting the velocity of joint D and connecting it to the joint B, as shown in Figure 12.

We shall now examine the obtained device for performing the function stated in the original design problem. In addition we shall observe the equivalence between operation of the obtained mechanical device and the electronic circuit from which it has been obtained. From the design problem one can distinguish between two cases: one in which the input shaft is rotating in the counterclockwise direction, which corresponds to a negative potential difference in the potential difference source of the graph, and to a negative voltage in the voltage source in the circuit. The other case stands for clockwise rotation corresponding to positive potential difference and to a positive voltage.

The first case is depicted in Figure 13. Due to rotation of the input shaft, the locus of points - A moves in negative direction (out of the page) and thus is marked with a blue color, while B moves in the positive direction and is marked with red. Overrunning clutch 5(6) forces the velocity of D(C) be higher (lower) than the velocity of A(B) - thus the rotation of the shaft in this case would not engage this clutch. On the other hand, the clutch 2(3) force the velocity of C(D) be lower (higher) than the velocity of A(B) - thus the rotation of the shaft in this case would engage these clutched and will cause the velocity of C(D) be the equal to the velocity of A(B) and thus marked with red (blue). One may conclude now that the rotation of the output shaft 4 is in the same direction as the input shaft and possesses the same relative velocity.

The same behavior pattern can be observed in the electronic circuit: since the voltage source has negative voltage - the potential of joint A is negative (and thus is marked blue), while the potential of joint B is positive (and thus is marked red). Such potentials case diodes 5(6) to be disconnected, while diodes 2(3) are conducting. Conduction of diodes 2(3) cases the potential of joints C(D) be equal to the potential of joints A(B). Thus the voltage on the resistor 4 connected between these two points is the same as the voltage in the input voltage source.
The second case is depicted in Figure 14. Due to clockwise rotation of the input shaft 1, the A moves in positive direction (into the page) and thus is marked with a red color, while B moves in the negative direction and is marked with blue. Overrunning clutch 5(6) forces the velocity of D(C) be higher (lower) than the velocity of A(B) - thus the rotation of the shaft in this case will engage this clutch and will cause the velocity of D(C) be the equal to the velocity of A(B) and thus marked with red (blue). These velocities are transferred through the gears to the output shaft causing it to rotate in the direction opposite to the direction of the input shaft. One can see that the clutch 2(3) is not engaged in this case.

Again, the same behavior pattern can be observed in the electronic circuit: since the voltage source has positive voltage - the potential of joint A is positive (and thus is marked red), while the potential of joint B is negative (and thus is marked blue). Such potentials cause diodes 2(3) to be disconnected, while diodes 5(6) are conducting. Conduction of diodes 5(6) causes the potential of joints D(C) be equal to the potential of joints A(B). Thus the voltage on the resistor 4 connected between these two points is opposite to the voltage in the input voltage source.

Combining the results of the two cases yields the following picture:

1) In the gear train: the velocity of the output shaft is equal to the velocity of the input shaft in magnitude, but is always rotating in the counterclockwise direction.

2) In the electronic circuit: the value of the voltage in the resistor is equal to the value of the voltage in the voltage source, but its sign is always negative.

This design of mechanical rectifier obtained in this example has been built in the mechanism laboratory of Tel-Aviv University (see photograph in Figure 15) and successfully tested.

Figure 15. Photograph of the mechanical rectifier that has been built at Tel-Aviv University.

CONCLUSIONS AND FURTHER RESEARCH

The paper has employed graph representations that are mutually applicable to represent different engineering domains, in order to enable knowledge transfer between these domains in purposes of design. In general, the design technique developed in the paper first transforms the design problem into the terminology of the graph, and then, into another (secondary) engineering domain where a search is performed for an existent design. Once found, the design from the secondary domain is transformed through the graph representation back to the original domain, yielding the solution for the original design problem.

To clarify how the technique works, it was applied to an example, where a kinematical system was designed by transferring knowledge from electronics through the graph representation. It should be noted that the approach is not applicable only to these two but to any of the engineering domains which are representable by graph representations.

The design technique proposed in the paper is only one possible variation stemming from a general design approach in which other techniques based on graph representations are being developed, such as: design through duality.

Usage of the proposed approach contributes to engineering beyond the design technique. The approach opens up a new avenue of cooperation between engineers from different fields,
by providing them with a common language of graph representations so they can work together on multidisciplinary design projects. It enables them to share theoretical and practical knowledge, theorems, patents, and analysis methods. Additionally, in light of constantly developing modern technologies comprising more and more different fields, the approach has a potential to provide a powerful basis in design education. Studying the graph representations prior to specific disciplinary issues, would make subsequent studying on the basis of the representations more efficient and flexible [23].

The design techniques of this approach are based on graph theory, one of the branches of discrete mathematics, being the mathematical foundation of computer science. Thus the processes described in the paper are convenient for computerization which will lead in the future to establishment of computerized creativity methods in engineering.

REFERENCES