An experimental investigation of the quasisteady turbulent pulsating flow in a pipe

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Turbulent pulsating pipe flow is investigated at $3600 < \text{Re} < 9000$ and a period of forcing $0.55 \text{ sec} < T < 5.5 \text{ sec}$. Velocities at the exit plane of the pipe and the pressure drop along the pipe are measured simultaneously and processed by a computer. The conditions of the quasisteady behavior of the flow are defined, and some conclusions regarding the details of the steady turbulent pipe flow are drawn from the nonsteady measurements.

I. INTRODUCTION

In periodic turbulent flow it is common to distinguish between three components of any flow parameter $f$: the time-mean part $\bar{f}$, the time-dependent part $\tilde{f}$, and the random part $\phi f''$ (Hussain and Reynolds). The possibility of a straightforward definition of the regular time-dependent component of the flow makes periodic flow a very convenient unsteady flow for an investigation of the relation between the mean flow parameters and the corresponding phase-averaged turbulent quantities. The particular geometry of fully developed pipe flow provides an additional advantage of dependence of all flow parameters on a single spatial coordinate (radial). Harmonic pulsations of small amplitude of the bulk velocity (relative to the time-mean value) can be obtained by imposing harmonic pulsations of limited amplitude of pressure in the settling chamber.

Under these conditions the analysis of the experimental results may be further simplified: first, the time-mean properties of the flow become independent of the forcing, so that the time-dependent portion of the flow can be analyzed separately; and second, this time-dependent component of any flow parameter $\langle f \rangle$ can be assumed to behave harmonically, and therefore be represented by two functions of radius only, i.e., the amplitude of its Fourier component at the fundamental frequency of the forcing $\langle f(r) \rangle$ and the phase angle $\phi_{f'\bar{f}}(r)$ relative to an arbitrary datum (the phase of pressure drop pulsations along the pipe was chosen as a reference),

\[
\langle f(r) \rangle = [\langle f(r) \rangle] \exp\{i(\omega t - \phi_{f'\bar{f}}(r))\}.
\]

The validity of the assumption of the harmonic behavior is verified below.

At very low frequencies of pulsations, the turbulent structure has enough time to accommodate to the slowly varying flow rate and thus is in accordance with the instantaneous value of the Reynolds number. Such flow can be called quasisteady. At higher rates of change of the mean flow, the importance of the relaxation nature of turbulence emerges, and the flow behavior deviates from the quasisteady pattern. When the pulsating frequency becomes very high, the turbulence cannot respond to the rapid changes in the mean flow, and the turbulent structure becomes independent of the phase angle of the pulsations, or "frozen." Our aim in this paper is to find out under what conditions the behavior of the time-dependent portion of the turbulent structure can be regarded as quasisteady. In order to arrive at a conclusion based on the experimental data as to whether the flow is quasisteady or not, a criterion must be defined, which allows for comparison of the time-mean and time-dependent components of an unsteady flow. The criterion chosen here is based on a modified normalization procedure proposed in Ref. 2. In steady turbulent flow the growth of the rms value of longitudinal velocity fluctuations with increasing Re is approximately proportional to the friction velocity $u' = \sqrt{\tau_\theta/\rho}$. The friction velocity is, in turn, roughly proportional to the bulk velocity $\bar{U}$, so that normalization of $u'$ by either $u'_\infty$ or $\bar{U}$ is generally accepted. Assuming quasisteady behavior of pulsating flow, we obtain

\[
\bar{u}^2 + \langle u'^2 \rangle \sim \tau_\theta + \tau_w \sim (U + \langle U \rangle)^2 \approx \bar{U}^2 + 2\bar{U} \langle U \rangle.
\]

Keeping in mind that under the above-mentioned experimental limitations the mean flow is unaffected by the imposed pulsations, the following relation between the time-mean values of $\bar{u}^2$ and the amplitude of its time-dependent part is obtained:

\[
[\langle u'^2 \rangle]/2\bar{U} \langle U \rangle = \bar{u}^2/\bar{U}^2.
\]

In this derivation it was tacitly assumed that the radial distribution of $\bar{u}^2$ remains similar with increasing Re, i.e., $\bar{u}^2(r/R)/\bar{u}^2(0)$ is independent of the Reynolds number. This assumption obviously does not hold, since the maximum in the radial distribution of $\bar{u}^2$ shifts towards the wall.
with increasing Re (see Ref. 3). The radial distributions of the non-normalized values of \( \overline{u^2} \) at three close and relatively low values of Re must thus look qualitatively as shown in Fig. 1. It follows from the diagram that for sufficiently small changes in Re, the increase in the flow rate may lead to a decrease in the absolute values of \( \overline{u^2} \) at some radial positions (corresponding to \( r_1 < r < r_2 \) on the diagram). The higher the Reynolds number, the narrower and more adjacent to the maximum in the distribution (and, hence, to the wall) must be this region. The additional increase in Re results in the growth of \( \overline{u^2} \) at all radial positions (curve 3), but the rate of growth is obviously nonuniform and remains dependent on the radius. Thus it seems reasonable to introduce the increment in the turbulent intensity \( \Delta \overline{u^2} \) as the difference in \( \overline{u^2} \) for two steady flows with Reynolds numbers that differ by the amplitude of pulsations in the periodic flow under consideration. One can expect that the following relation holds in a quasi-steady turbulent pulsating flow:

\[
\frac{\langle u'^2 \rangle}{2\bar{U}} \left( \frac{\langle U \rangle}{\bar{U}} \right) = \overline{u^2}/2\bar{U} \Delta \bar{U}.
\]  

[The factor 2 is retained in the denominator in order to keep the values comparable with the normalization according to Eq. (5).]

II. EXPERIMENTAL TECHNIQUE

The experiments were carried out in a long, smooth pipe 33 mm in diameter and 17 m long. The mean flow was supplied by a high pressure source (6 atm compressor) and controlled by a precise pressure regulator. This arrangement insured that the mean flow rate was independent of the superimposed oscillations. The streamwise component of air velocity was measured by an array of nine hot wires distributed equidistantly along the radius at the exit cross section of the pipe. The distance between neighboring wires was equivalent to \( r/R = 0.12 \); thus by locating the first wire at the centerline of the pipe, the ninth wire was located at 0.5 mm from the wall, i.e., at \( r/R = 0.97 \). The hot wires were calibrated in a wind tunnel which provided a stable reference velocity between 30 cm/sec and 15 m/sec. Taking into account the low velocities which occur in the wall region of the pipe flow, the frequency of vortices shed downstream of a circular cylinder was used for calibration, instead of the generally accepted Pitot tube. The accuracy of velocity measurements was better than 1%.

The instantaneous value of the pressure drop along the pipe was measured by a pressure transducer. Pressure oscillations were introduced by a valveless piston pump, connected to the settling chamber. The period of pulsations could be changed in the range between 0.5 and 5 sec; the repeatability of the period was better than 0.3%. The amplitude of forcing could be varied by changing the displacement of the piston in 17 discrete steps. An optical switch provided a trigger signal for independent phase reference. A total of 11 channels was thus sampled: nine channels containing velocity information, one pressure information, and one phase information. All of the measured data were digitized by the PDP 11/60 minicomputer and recorded on magnetic tape for further processing.

Measurements were made at three Reynolds numbers: \( Re = 3600, Re = 5500, and Re = 9000 \) and at three periods of pulsations at each Reynolds number, namely \( T = 0.55 \) sec, \( T = 1.6 \) sec, and \( T = 5.5 \) sec. The chosen amplitude of bulk velocity pulsations was about 8%. In the previous investigation\(^2\) this amplitude was found to be high enough for accurate measurements of phase-averaged turbulent velocity fluctuations and sufficiently low to enable linearization and to assure independence of all normalized time-dependent quantities in the flow on the amplitude of pulsations. At each Re, measurements were also performed at two steady flows with flow rates differing by about 8%. In most measurements of an unsteady flow, 240 periods of pulsations were recorded, with the number of data points at each channel ranging from 1024 per period at the highest frequency of pulsations 8192 at the lowest frequency. In this way the sampling frequency was kept approximately constant at 1.5 kHz per channel. This sampling frequency was found quite adequate in the range of Reynolds numbers used\(^2\). The measurements in the steady turbulent pipe flow were made at the same sampling frequency.

III. RESULTS AND DISCUSSION

The radial distribution of the amplitudes of pulsations of the turbulent velocity fluctuations, normalized according to Eq. (6) are shown in Figs. 2(a)–2(c). The properly normalized time-mean values of \( \overline{u^2} \) and of the increment \( \Delta \overline{u^2} \) are also shown here for comparison. At low Reynolds number (\( Re = 3600 \)) the normalized value of \( \langle u'^2 \rangle \) at \( T = 5.5 \) sec is very close indeed to the radial distribution of \( \overline{u^2} \). At higher frequencies of pulsations, however, the values of \( \langle u'^2 \rangle \) are notably higher and the radial distribution is closer to that of the normalized time-mean value. At \( Re = 5500 \) the radial distribution of the amplitude of pulsations of \( \langle u'^2 \rangle \) at two lower frequencies (\( T = 5.5 \) and 1.6 sec) are closer to the distribution of \( \overline{u^2} \), while for shorter period of pulsations (\( T = 0.55 \) sec) the values of \( \langle u'^2 \rangle \) are notably higher. At the highest Reynolds number used (\( Re = 9000 \)) the amplitude distributions at all three frequencies are very similar to that...
of \( \Delta \bar{u}^2 \) and differ significantly from the distribution of the time-mean values. The behavior of \( \langle u'^2 \rangle \) thus corresponds to the expected quasisteady distribution at relatively low rates of change of flow rate. From Fig. 2 one can see that the radial distribution of \( \Delta \bar{u}^2 \) differs qualitatively from that of \( u'^2 \).

At low Reynolds numbers the difference occurs at all radial positions while at higher \( Re \) (= 9000) the region of the difference is more localized and adjacent to the wall. This behavior could be expected from the qualitative presentation given in Fig. 1. It seems reasonable to assume that at even higher Reynolds numbers the pattern will be essentially the same, but the proximity of the wall will make the detailed measurements more difficult.

It is worth noting that at higher frequencies the normalized distribution of \( \langle u'^2 \rangle \) resembles that of the time-mean values. While this similarity may be accidental, it should be stressed that the growth of \( \langle u'^2 \rangle \) with frequency, increasing from a quasisteady value, suggests that the dependence of the amplitude of \( u'^2 \) on frequency has a complex character and is not governed solely by a simple relaxation mechanism, which causes the vanishing of \( \langle u'^2 \rangle \) at very high frequencies.\(^4\) The radial distribution of the amplitude of \( u'^2 \) may serve for definition of the quasisteady behavior of turbulent pulsating flow. The flow is considered quasisteady, if Eq. (6) holds at all radial positions (with the exception of the viscous sublayer). The nondimensional parameter which defines whether the flow is quasisteady or not must depend on both the flow rate of the mean flow and on the frequency of pulsations and presumably only the local length scale. The relevant length scale is the thickness of Stokes' layer in a pulsating flow \( \delta_{st} = \sqrt{\nu/2\pi f} \). The nondimensional parameter may be constructed as a quotient of Strouhal and Reynolds numbers \( St/Re \). Taking the square root of this quantity allows us to redefine this parameter as Reynolds number based on \( \delta_{st} \):

\[
\sqrt{St/Re} = \sqrt{2\pi f \bar{U}} = (\bar{U} \delta_{st}/\nu)^{-1}.
\]

The analysis of the experimental data presented here as well as that available from Ref. 2 reveals that quasisteady flow behavior occurs for \( \sqrt{St/Re} < 1.8 \times 10^{-2} \).

The relaxational approach\(^5\) suggests that there must be a certain time lag in the response of the turbulent structure to a changing mean velocity. The experimentally measured difference between the phase angles of velocity \( \phi(u) \) and turbulent intensity \( \phi(u'^2) \) is presented in Figs. 3(a)–3(c). Generally speaking, \( \langle u'^2 \rangle \) lags behind \( \langle u \rangle \); this phase lag increases with increasing frequency (for constant \( Re \)). When frequency is kept constant, the phase lag of turbulent intensity behind the phase-averaged velocity decreases with increasing \( Re \). Such behavior is in full agreement with the concept of relaxation. It is worth noting, however, that at flow rates and frequencies at which the amplitude of turbulent intensity has the appearance of a quasisteady distribution, there is still a notable phase lag of \( \langle u'^2 \rangle \) behind \( \langle u \rangle \), especially in the central region of the pipe. The phase distribution thus seems to be more sensitive to deviations from quasisteadiness and shows that only the lowest frequency employed may be regarded as really quasisteady.
FIG. 3. The radial distributions of the phase difference between \( \langle u'^2 \rangle \) and \( \langle u \rangle \) at different Reynolds numbers and periods of pulsations.

The negative value of the phase difference \( \phi_{\langle u'^2 \rangle} - \phi_{\langle u \rangle} \) at some radial positions, however, seems to be incompatible with the relaxation concept. The range of radial locations where \( \phi_{\langle u'^2 \rangle} - \phi_{\langle u \rangle} < 0 \) decreases with increasing frequency of pulsations (at \( Re = \text{const} \)), and it is shifted towards the wall with increasing \( Re \). Such behavior of the phase relations in turbulent pulsating pipe flow may be explained by assuming nonmonotonic growth of \( u'^2 \) with \( Re \) in steady turbulent pipe flow, as suggested in Fig. 1. The analysis of Figs. 1 and 3 reveals that the oscillations in velocity lag behind those in turbulent intensity at the same radial locations where the nonmonotonic growth of \( u'^2 \) with \( Re \) can be expected. The assumption of nonmonotonicity leads to a conclusion that in a very slowly pulsating flow, at a certain part of the period, the increase in \( \langle u \rangle \) is accompanied by a decrease in \( \langle u'^2 \rangle \), thus causing an effective 180° “phase difference” between the two during the corresponding time intervals. In this case the time dependence of \( \langle u'^2 \rangle \) must be significantly nonharmonic, and the resulting phase angles obtained for the Fourier component at the fundamental frequency are apparently invalid.

Since the nonmonotonic growth of \( u'^2 \) with \( Re \) can only be noticed in steady turbulent flow at very small increases in the flow rate, its influence on the phase behavior of pulsations in \( u'^2 \) must decrease with increasing amplitude of the bulk velocity pulsations. The increase in the pulsation amplitude must also lead to more harmonic time behavior of \( u'^2 \). In order to verify this experimentally, measurements were made at constant \( Re = 4200 \) and period of pulsations \( T = 5.4 \text{ sec} \) (with the corresponding value of \( \sqrt{St/Re} = 0.0009 < 0.0018 \) and therefore well in the quasisteady region) and at amplitudes of bulk velocity pulsations ranging from 4.5% to 12%. The experimental results of Fig. 4 show both \( \phi_{\langle u \rangle} \) and \( \phi_{\langle u'^2 \rangle} \). One can notice that while, as expected, there is practically no influence of the amplitude on the radial distribution of the velocity angles \( \phi_{\langle u \rangle} \), the change in \( \langle (U')/U \rangle \) results in notable changes in the radial distribution of \( \phi_{\langle u'^2 \rangle} \). With the growth of the amplitude of pulsations, the scattering in data decreases, the locations of points with adverse phase difference between \( u'^2 \) and \( u \) move towards the wall, and, at even higher amplitude, this phenomenon practically disappears.

As a quantitative degree of harmonicity, the harmonic distortion ratio was chosen. In order to eliminate the influence of the meaningless oscillations at very high frequencies in the phase-averaged values of \( \langle u'^2 \rangle \), the “power spectrum” coefficients of the first ten harmonics in the Fourier decomposition of \( \langle u'^2 \rangle \) were calculated, and the ratio of the coefficient of the fundamental frequency of pulsation to the total energy of these ten frequencies was found. The results shown in Fig. 5 are in compliance with expectations: the pulsations of \( u'^2 \) become more harmonic with the increase in relative amplitude \[ \langle (U')/U \rangle \] (a small decrease in this ratio at the high-

FIG. 4. The dependence of the radial distribution of \( \phi_{\langle u \rangle} \) and \( \phi_{\langle u'^2 \rangle} \) on the amplitude of pulsation.

FIG. 5. The dependence of the radial distribution of the harmonic distortion ratio of \( u'^2 \) on the amplitude of pulsations.
est amplitude of pulsations of 11.9% may be attributed to either an experimental error, or to emerging nonlinear effects; and all curves have a pronounced minimum in the vicinity of the maximum in the radial distribution of $\bar{u}^2$, where the nonmonotonicity is expected, as shown in Fig. 1. The results of Fig. 5 on the whole justify the presentation of turbulent intensity by Eq. (3). The harmonic distortion ratio for pulsations of pressure and phase-averaged velocity was very close to unity in all cases.

The measurements performed in a slowly pulsating pipe flow thus can provide experimental data revealing some details of the structure of steady turbulent flow, as nonmonotonic growth of $\bar{u}^2$ with Re. Similar behavior can be expected at higher Reynolds numbers, but the region of nonmonotonic dependence of $\bar{u}^2$ on Re is then located closer to the wall.

As mentioned before, the flow investigated here was the simplest unsteady turbulent flow possible: it depends on a single spatial coordinate, the imposed pulsations are slow, harmonic, and of low amplitude; yet the response of turbulent structure to the changing mean flow is of rather complex character. In the case under consideration the basic steady flow is well documented so that the peculiarities in the behavior of the time-dependent part could be understood. However, in general, in different flow conditions, a straightforward extrapolation of the properties of a steady flow to a quasisteady time-dependent turbulent flow may be even more erroneous.

2. L. Shemer, I. Wygnanski, and E. Kit (submitted to J. Fluid Mech.).