A review study of instantaneous electric energy transport theories and their novel implementations

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A R T I C L E   I N F O

Article history:
Received 10 May 2015
Received in revised form 19 September 2015
Accepted 17 December 2015

Keywords:
Active power
Energy metering
Harmonic distortion
Power transport theory
Reactive power

A B S T R A C T

Electric energy transport theories are still a debatable subject regarding their validity and applicability in various aspects. This paper includes a comparative study of the group of theories which are categorized as instantaneous theories as opposed to periodic-averaged ones. As a preliminary review this paper presents first the three apparent power definitions in order to emphasize the difference between them. A discussion regarding their accuracy for energy metering of various conditions such as multiphase and non-sinusoidal environments is presented. Instantaneous theories are then reviewed and compared with periodic averaged theories. The review focuses mainly on p-q theory (PQT) and its derivative, instantaneous multi-phase theory. The legacy application of this theory for active filtering is briefly presented and the new idea of using the theory for multi-load and multi-source theory is presented as well. The comparison identifies the similarities and differences between the instantaneous and periodic averaging theories, and their inter-relations.

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1. Introduction

Electric energy transport theories are one of the most important theories in grid analysis as well as in the understanding of the transfer of energy in complex grids. The introduction of renewable energy and the development of smart grids have increased the interest in and the necessity of matching the appropriate theory with the appropriate application.

These theories are interesting by virtue of their suitability for: 1) grid monitoring and control, 2) grid energy management, 3) grid energy storage, 4) grid reactive power management, 5) grid multi-source management. The issue of suitability of each theory in terms of smart grid applications is an open debate. Recent publications show renewed interest in the subject as well as various applications of these theories [1–4].

Apparent power serves as a tool for grid generation and transmission design. Therefore, prior to dealing with the various power theories the understanding of the various apparent power definitions and their validity to multiphase and non-sinusoidal environments are essential. The paper starts with a short presentation of the three apparent power definitions which reflect recent changes in energy metering [5] and a discussion on the resemblance (not identity) of these various definitions is performed. These definitions yield different results, except for the case when they are applied on a purely sinusoidal balanced system. The known definitions are therefore initially comprehended through a survey of numerous sources of information and the following questions are answered: what is apparent power? What is it used for? What is the initial definition? and what “physical” explanation was later added to justify the previous definitions?. In this paper we try to evaluate to what extent a unified approach is possible for the various definitions of apparent power in non-sinusoidal environments. The understanding of the differences in apparent power definitions, especially in non-sinusoidal environments, facilitates a suitable choice of definition. This paper attempts to show that similarity relation exists at least between the Buchholz, geometric and vector apparent power definitions in the single phase case and that in the multi-phase case there is a bounded region of the difference.

There are two major groups of electric energy transport theories: 1) periodic averaged theories. These theories handle electric power parameters as averaged over an single network cycle and include the conservative frequency domain theory and the current physical components (CPC) theory, which are both discussed in length at [2, 6–8]; 2) instantaneous theories. These theories handle the electric power parameters in their instantaneous form in the time domain. This group includes the p–q theory (PQT), the conventional Poynting vector theory (PVT) and the conservative time domain theory (CPT time-domain). Instantaneous theories handle electric power parameters in their instantaneous form in time.

In a previous paper we reviewed and dealt with the various aspects of periodic averaged theories [8]. This paper is complementary to [5,8] and concludes the study of the power theories range by covering instantaneous power theories as well as comparison of the various theories with periodic averaged ones.

In Table 1, all the known active electric energy transport sub-theories are presented. It should be noted that there is a vast literature on each theory and therefore only a few major contributors are mentioned here. The highlighted rows in Table 1 are the instantaneous theories which are the scope and main focus of this paper. After clarification of the various definitions of each theory, a theoretical comparative study approach is taken. The current state is that many researchers today regard these theories as essentially different and present examples where different theories and different apparent power definitions yield different results.

Power theories are ongoing specialization fields [1–3,9]. However, if the theories are regarded as different perspectives of the same phenomena, then acceptance of a future unifying theory is an option [10]. There has been some debate regarding these instantaneous theories’ effectiveness compared with periodic averaged theories and some works claim that the theories are less useful for energy measurement and active/reactive load characterization [7–14]. There are also other comparative works such as [15,16] which deal with specific aspects of comparing between the theory groups. The work presented in this paper demonstrates that in-depth study of instantaneous theories’ apparent power definitions enables the development of new electric grid monitoring and energy management tools. Active filtering is a classical application in which instantaneous theory is applied [11,17–19]. Instantaneous theory is not limited to response time of more than half a cycle and does not necessarily require storage components [17], but rather only switches. Grid monitoring and control is an application which is addressed in the paper as well. The paper investigates the extent of applicability of instantaneous theories to power conditioning [3,20,21].

This paper shows in detail that instantaneous theory is different from periodic averaged theory with regard to physical active, reactive and distortion current/power physical components. Moreover, the paper studies whether multi-phase instantaneous theory is suitable for analysis of multi-load/multi-source computation for composition and decomposition of current/power in hierarchical grid transitions.

Because of the excessive size of a study covering all theories, this paper contains a comprehensive study of only instantaneous theories: p–q theory (PQT) and multi-phase instantaneous theory as a necessary addition to a previous study of periodic averaged theories and multi-vector Poynting vector theory (PVT) [7]. Multi-phase formalism brings in several merits which are discussed in this paper: (1) Physical comprehension. With it comes the understanding that the mathematical formalism of Clarke transform for PQT is not mandatory for instantaneous theories development [10,21]. (2) The generalized multi-phase theory is applicable to all the specific cases: single-phase, double-phase, three-phase balanced and unbalanced cases. (3) Multi-phase formalism is shown to be suitable for describing the single phase and two-phase networks as well as three phase systems. (4) Multi-phase vector formalism enables comparison with other multi-vector theories such as multi-harmonic, periodic averaging theories and instantaneous PVT. Differences and similarities are highlighted. (5) It is shown that instantaneous theory is not equal to periodic averaging in terms of optimization criteria for delivering active power through minimal losses to load. Explicit multi-phase formalism is investigated in this paper, because multi-phase formalism is potentially suitable for describing multi-loads and multi-sources, even if the network is a three-phase system.
2. Definitions of apparent power $S$

2.1. General

Apparent power is a figure intended to quantify the actual power required from the source in order to provide a specific active energy to the load [22]. This quantity is essential to transmission line design. An alternative definition is: the power measured at the terminals of a load under test. Apparent power is usually not considered as an actual physical power [2,9]. In the literature, three definitions of apparent power can be found [2,23]. Each definition yields a different apparent power value. In the following sections the various definitions are presented as well as a comparison between them. This, subject is still being debated by many researchers and the following sections are intended to lay-out the definitions and shed some light at some of these inequalities between the definitions.

2.2. The Buchholz definition

The Buchholz definition is based on the following equation [4, 2]:

$$S = \|V\| \cdot \|I\| = V_{\text{RMS}} \cdot I_{\text{RMS}} = \sqrt{\sum_{i=1}^{M} V_i^2} \cdot \sqrt{\sum_{i=1}^{M} I_i^2}$$

where: $i=1,2,3,\ldots, M$ is an integer and $L_i$ is the corresponding phase. The RMS value of the current and voltage of all phases is calculated in accordance with the chosen method. A single phase time-domain definition for the squared $L_2$ norm of the voltage is:

$$||V||^2 = V_{\text{RMS}}^2 = \langle v(t), v(t) \rangle = \int_{0}^{T} v^2(t)dt$$

where $v(t)$ is the instantaneous voltage. A frequency domain single-phase definition for the same voltage is:

$$||V||^2 = V_{\text{RMS}}^2 = \sum_{n=0}^{N} V_n^2$$

where $V_n$ is the harmonic voltage of order $n$. One possible meaning of the Buchholz definition formula for the single-phase case is that the load can be considered as a black-box with one port. At its terminals the instantaneous voltages and currents are measured. These parameters are the only observable and measurable parameters considered. In the three-phase case the RMS voltages and currents are defined as stated in [2,4]. The definition in (1) can be explained by applying the ‘Buchholz–Goodhue apparent power definition’. Namely, the RMS voltages and currents of each phase are the cause of the actual three-phase system line active power loss. This can be shown to be [2]:

$$\Delta P_V = \sum_{i=1}^{M} \frac{V_i^2}{R_i} \approx \sum_{i=1}^{M} R_i I_i^2 = R_{\text{eff}} \sum_{i=1}^{M} I_i^2$$

(4)

There is no physical meaning for the Buchholz apparent power as thought by major researchers [2,9]. There are however physical justifications such as (4). In [26] the Buchholz definition is derived by maximizing the active power $P$ according to Lagrange’s multiplier method in the case where: (1) phase voltages are given, (2) the phase currents are constant, (3) the net sum of the phases and neutral currents is zero. The Budeanu three-power type relation, active, reactive and distortion, requires the Buchholz definition. An alternative interpretation of the advantage of Buchholz power to reflect the system’s power is suggested next. In [2,20] the multi-phase apparent power is derived from phase voltages and currents (originally the operator in [20] was used to criticize $PVT$ for including useless non-active cross-phase product components). Next we show how it leads to preservation of power laws in a multi-phase system using the Buchholz definition:

$$S_{\text{star}}(t) = \sum_{i=1}^{M} V_i(t)I_i(t), \text{ or } S_A = \sum_{i=1}^{M} \int_{L_i} V_i(t)I_i(t)dt$$

$$\|S\|^2 = \int_{0}^{T} S(t)S^*(t)dt = \sum_{i=1}^{M} \sum_{j=1}^{M} V_i^2 V_j^2 R_i R_j = S_{\text{Buchholz}}^2$$

(5)

$$||S||_2 = S_{\text{Buchholz}}$$

The result in (5) is not trivial and is detailed in the Appendix. It is now shown that a simple product of $v(t)$ and $i(t)$ where $v(t)$ and $i(t)$ are the instantaneous voltage and current accordingly, represented as complex phasors, can be expanded as Fourier series and

<table>
<thead>
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<th>#</th>
<th>Theory</th>
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<tr>
<td>1</td>
<td>Conservative theory (CPT)</td>
<td>Buchholz</td>
<td>1922</td>
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<td>2</td>
<td>Time domain</td>
<td>Budeanu</td>
<td>1927</td>
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<td>3</td>
<td>Frequency domain</td>
<td>Fryze</td>
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<td>Fryze-Buchholz-Depenbrock (FBD) method</td>
<td>Quade, Rosenzweig</td>
<td>1934, 1939</td>
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<td>5</td>
<td>Tenti method (CPT)</td>
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<td>V. Tipsuwanporn, F. Cheevawit, W. Piyarat, P. Thepsatorn, Y. Parale</td>
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<td>J. Willems</td>
<td>J. Willems</td>
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then all known power types can be extracted:

\[ p = \text{Re}(\vec{V}(t) \cdot \vec{I}(t)) = \sum_{i} V_i(t)I_i(t) = \vec{V}_{ph} \cdot \vec{I}_{ph} = \vec{V}_{ph}^{T} \vec{I}_{ph} \]

\[ q_{\text{Budeanu}} = \text{Im}(\vec{V}_{ph} \cdot \vec{I}_{ph}), \]

\[ Q_{\text{D}} = \text{Re}(\vec{V}_{ph} \times \vec{I}_{ph}) = \sum_{p \neq q} (\vec{V}_p \times \vec{I}_q - \vec{V}_q \times \vec{I}_p) \]

The results in (6) are obtained in [10] for instantaneous multi-phase theory, and in [9,27] for single-phase multi-harmonic (which equals to the periodic averaged) model there, multi-vector Buchholz and is nonzero in otherwise.

\[ S = \sum_{i} (V_i \cdot I_i) \]

This definition implies that the total three-phase apparent power relates to the arithmetic (scalar) sum of the apparent power of all three phases. Because of energy conservation, that physical quantity is considered as the total load energy. This definition does not equivalent to the Buchholz definition. Next, a heuristic explanation of why this arithmetic definition is not necessarily physically correct is presented. It relies on proven theorems in the Poynting vector power theory [9]. If \( S \) is a vector it cannot be scalar. Most researchers agree that reactive power is a vector propagating with direction in the harmonics space (periodic averaged) or generator phase space (instantaneous theories) or both [9,23]. For example:

1) Reactive vector decomposition theory, which is a conservative power theory in which:

\[ \vec{Q} = \vec{V} \times \vec{I} \]

where: \( \vec{V} = [V_1, V_2, ..., V_n], \vec{I} = [I_1, I_2, ..., I_n] \) and therefore \( \vec{Q} \) is naturally a vector.

Then the total apparent power \( S \) is represented according to the relation:

\[ \vec{S} = P + j \vec{Q} + \vec{k} \vec{D} \Rightarrow \]

\[ S^2 = P^2 + Q^2 + Q_{\text{Budeanu}} + D_{\text{Budeanu}} \]

Where \( j, k \) are orthogonal unit vectors. It implies from (8) and (9) that \( S \) is also a vector. Regarding the distortion power \( \vec{D} \) in the Buchholz definition, this power is zero in the pure sine wave case and is nonzero in otherwise.

2) In Poynting vector power transport theory, as in [9]. The model there, multi-vector \( \text{PVT} \), uses the geometric apparent power definition. This also yields vector reactive power, as does (8). Extending the description to a multi-phase system means that the total apparent power should naturally be a vector:

\[ \vec{S}_{\text{Total}} = \left[ \vec{S}_{1a}, \vec{S}_{2a}, ..., \vec{S}_{na} \right] \]

the voltage and current vectors are temporally phase-angle shifted from one load phase to another. In general:

\[ \begin{align*}
D_{\text{Total}}(t) &= \sum_{i} V_i(t)I_i(t) + \sum_{i \neq p} (V_i(t)I_p(t) + V_p(t)I_i(t)) \\
|D_{\text{Total}}|^2 &= \sum_{i} V_i(t) \cdot I_i(t)
\end{align*} \tag{11} \]

as (5) indicates.

It is evident from (11) that the instantaneous distortion components are generated by cross-product phase voltages and currents. These components contribute net zero power to the periodic averaged distortion power. The arithmetic apparent power definition does not obey the orthogonal behavior of the current physical components as in (9). In [28,29], a definition that maintains orthogonality in the apparent power current components is demonstrated. An alternative argument that the arithmetic definition is to be less recommended is presented in [2], because of violation of parameters of the electric system line losses in the unbalanced case.

2.4. Geometric and vector definition of apparent power

The last definition of apparent power is the one in [29]:

\[ \vec{S} = P + j \vec{Q} \]

while: \[ j \vec{Q} = jQ_a + jQ_b \]

From (8), and (9) the following known result can be derived:

\[ S^2 = P^2 + Q^2 \]

This definition is also known as the geometric definition [24] because it obeys Pythagoras' theorem, and includes orthogonal power vectors \((P \pm Q)\). Eq. (13) implies orthogonally physical components, namely:

\[ \vec{P} \cdot \vec{Q} = 0 \quad \text{and} \quad \vec{P} \times \vec{Q} = |\vec{P}||\vec{Q}| \]

Orthogonal is meant not in the spatial sense (x, y, z) but with regard to inner products over harmonic components [11,9,23], or inner products over voltage vectors current phase vectors [15]. It should be mentioned that not all works use the above terminology for geometric apparent power beyond the single phase case. In [2], for example, the above definition is included as a fourth type named vector apparent power. In this case the geometrical apparent power for the three-phase case is: \( \vec{S}_1 + \vec{S}_2 + \vec{S}_3 \) and the vector notation is \( |S_1| + |S_2| + |S_3| \). The similarity of these mathematical extensions to the Buchholz, in the three phase case, is confined to limits as shown in the Appendix.

The completeness in terms of energy conservation of the active and reactive powers, as shown above, does not mean that in other theories and sub theories reactive power cannot be further decomposed into additional orthogonal physical components, as will be demonstrated.

Fig. 1(a), as in [2], shows decomposition of non-active power \( Q \) into Budeanu reactive \( Q_a \) and Budeanu distortion power \( D_b \). \( S_b \) \( S_c \) represents the phase apparent powers. \( S_{\text{vector/geom}} \) represents the total system load vector or geometric apparent power.

Fig. 1(b) demonstrates a comparison between the arithmetic apparent power and the vector apparent power. Each phase's apparent power is redrawn and the \( S_{\text{Arithmetic}} \) apparent power is shown along with the \( S_{\text{vector/geom}} \) apparent power.

There is no explicit mention of apparent power at \( \text{PQT} \). In [3] which is a book on \( \text{PQT} \) (2007), an equivalent variable is named as the complex instantaneous power vector \( S \). It is obtained by taking the total scalar power \( P \) and the total scalar amplitude of the reactive power \( Q \), and summing them as \( P + jQ \). The complex power vector is similar to the geometric apparent power definition.
2.5. Comparison of the apparent power definitions

The three apparent power definitions provide identical results only if the line currents are purely sinusoidal and symmetrical, namely they are a linear balanced system. Otherwise these definitions yield different results [2,7]. As long as Budeanu distortion power exists (D ≠ 0), there can be no equality between the Buchholz definition as derives from (9) and the geometrical apparent power definition of (13). It is shown in [23] that the distortion power is composed of the sum of cross-product harmonics \( V_n X_m \) (where \( n \neq n \)). Therefore it is evident that \( D = 0 \) only for a pure sine wave. When asymmetry is considered, the three-phase apparent power definitions should be treated carefully. Comparing the arithmetic apparent power with the geometric apparent power, we obtain the vector sum of the three phases:

\[
S_{\text{arithmetic}} = \sum_{i=1}^{4} V_i I_i \\
S_{\text{geometric}} = \sum_{i=1}^{4} V_i \times \sum_{i=1}^{4} I_i
\]

The phase vector product is defined in (8). Only if the voltage and current are orthogonal in phase, with no high-order harmonics and the phase reactive powers aligned, then the definitions yield equal results. That is only for pure sine and balanced load. However, the Buchholz apparent power and the geometrical apparent power are related by (9) and (13), and the distortion power has an explicit formula as shown in [2,23]. The theories mentioned in Table 1 are mapped by their apparent vector definitions. Conservative theories support the Buchholz definition. PVT as represented by the sample paper [9] uses the geometrical definition, but as explained it relates to the Buchholz definition. CPCT [7] is operable with all the apparent power definitions, but its author/developer prefers the Buchholz definition.

2.6. Concluding discussion on apparent power definitions

It is important to perform a shift from theoretical computations to real practice. It should be noted that geometrical apparent power (instantaneous) leads to values that may seem not to reflect active/reactive load characterization correctly [5,33]. It is known that even when restive load is considered the imbalance between the phases results with cross-phase voltages and currents. An analogous issue regarding the Poynting vector was discussed in [14,20] where the cross-voltage current product was found to display the same phenomena. Another similar issue concerns instantaneous power definitions of the active and reactive powers is discussed in [33]. As a result of [33], some of the scientific community do not use instantaneous theories [14] for identification of active/reactive load characteristics. Geometric multi-phase apparent power is not suitable for active/reactive load characterization and consequently not for energy measurement either. It is suitable for the single-phase case. One might therefore ask what it is good for. Apparent power is not a physical parameter, but rather some kind of mathematical vector sum of active and reactive powers, reflecting the power at load boundary.

Moreover, it can be shown using simple manipulation that in the single phase case, the geometrical instantaneous definition is equivalent to the Buchholz definition (see Appendix); Buchholz serves as the RMS of the geometric apparent power definition for the single phase but not in the multiphase case.

Researchers who wish to avoid fictitious active and reactive expressions can use Buchholz apparent power which is always positive. The Buchholz apparent power yields the RMS value of the geometric apparent power in the single-phase case but not in the multiphase case.

3. Modern instantaneous power theories

3.1. The p-q theory or instantaneous reactive power (IRP) p-q theory

The p-q theory (PQT) was published in 1984 and described in [17,15]. The theory has evolved over the last three decades. Initially formal theory foundations were established for the three-phase case [4,19] followed by implementation for active filter control. Enhancement to multi-phase theory and usage of modernized terminology is covered in [10,21,22]. A modern source on PQT reflecting its state of the art status is [3]. There, numerous enhancements of PQT are discussed, although the discussion is confined to three phases. The first in-depth theoretical explanation of the interpretation of reactive power during unbalanced load or star system with a neutral is given in [20]. Because of criticism [36] regarding its difference from other accepted theories, an in-depth analysis of PQT physics was offered by Willems [10]. There, he proposed a new interpretation of PQT enabling its generalization to a multi-phase system, with the special single-phase case which was not included in the original theory.

3.2. PQT: formal theory foundation for three phases

3.2.1. The Clarke transform

Let \( a, b, c \) be the three-phase interface between source and load. The Clarke transform [34] of phase voltages transfers the three-phase balanced vector \([a, b, c]\) to an \([\alpha, \beta, 0]\) reference frame. The instantaneous variables \( i(t), v(t), p(t), q(t) \) are denoted as \( i, v, p \), and \( q \). Applying the Clarke transformation to the vector of phase voltages yields:

\[
\begin{bmatrix}
    v_v \\
    v_\alpha
\end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix}
    \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
    1 & -\frac{1}{2} & -\frac{1}{2}
\end{bmatrix} \cdot \begin{bmatrix}
    v_a \\
    v_b
\end{bmatrix}
\]

The phase current vector transformation is identical:

\[
\begin{bmatrix}
    i_i \\
    i_\alpha
\end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix}
    \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
    1 & -\frac{1}{2} & -\frac{1}{2}
\end{bmatrix} \cdot \begin{bmatrix}
    i_a \\
    i_b
\end{bmatrix}
\]
The inverse voltage and current transforms are provided in [3].

### 3.2.2. Justification of usage of Clarke transformation

In two important cases the Clarke transformation degenerates the three-phase system into a two-phase system [35]:

1. In the three-phase four-wire system

   \[ v_a + v_b + v_c = 0 \]  \hspace{1cm} (18)

   The \( a,\beta,0 \) transformation separates the three-phase voltages into voltages lying on the zero sequence axes. Condition (18) occurs for balanced loads, or at a \( \Delta \) connection without leakages.

2. A three-wire star (Y) system without a neutral (three-phase three-wire) yields

   \[ i_a + i_b + i_c = 0, \quad \text{or} \quad i_{\text{neutral}} = 0 \]  \hspace{1cm} (19)

   This means that one current variable is eliminated. The initial theory is developed in favor of these two instances.

#### 3.2.3. Definition of powers in PQT and the geometrical interpretation of the electric parameters

The active power of the three-phase \([a, \beta, 0]\) system is defined as:

\[ p = v \cdot i = v_a i_a + v_b i_b + v_c i_c \]  \hspace{1cm} (20)

The reactive power is defined as a sum of vector products of the phase variables:

\[ q = v_a \times i_a + v_b \times i_b + v_c \times i_c \]  \hspace{1cm} (21)

Another result discussed in multi-phase instantaneous theory is reduced for the three-phase case as follows [22]:

\[ q = \tilde{v} \times \tilde{i}, \quad \tilde{v} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}, \quad \tilde{i} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \]  \hspace{1cm} (22)

The resulting power is formulated in matrix form as follows:

\[
\begin{bmatrix}
  p \\
  q
\end{bmatrix} =
\begin{bmatrix}
  v_a & v_b & v_c \\
  -v_b & v_a & v_c \\
  -v_c & v_b & v_a
\end{bmatrix}
\begin{bmatrix}
  i_a \\
  i_b \\
  i_c
\end{bmatrix}
\]  \hspace{1cm} (23)

where \( p \) is a scalar and \( q \) is the amplitude of the reactive power. It should be noted that:

1. Instantaneous active power is a scalar product of the phase voltage and current vectors. At \([a, \beta, 0]\) space, the voltages and the currents are parallel to each other and to the \( a \) and \( \beta \) axes.
2. The instantaneous reactive power is a vector product of the three-phase voltage and current vectors at the \([a, \beta, 0]\) space.

Fig. 2 and Fig. 3 are inspired by [3,17]. In Fig. 2, there is a 120° difference between successive phases, whereas in the \([a, \beta, 0]\) system the phases \( a, \beta \) are orthogonal. In Fig. 3, the power diagram is presented in the \([a, \beta, 0]\) system. The active power is on a real plane consisting of voltages and currents which are in the same phase.

The illustration is of a balanced resistive load, i.e. no phase shift between voltage and current.

The reactive power is on an orthogonal vector, consisting only of the cross-products of the voltages and currents.

### 3.3. Generalization of PQT to include zero-phase sequence components for load imbalance

In the theory presented above, the third-phase element was zero in the \([a, \beta, 0]\) space, since the voltage was balanced and did not include a zero phase sequence voltage [36]. The generalization for the inclusion of a zero phase sequence component is handled in [3,15,35,36]. The enhancement of an unbalanced load system or a star (Y) system with a neutral line is owed to Aeredes et.al. in [15]. Suitable infrastructure was already introduced by maintaining a 3 \( \times \) 3 matrix formulation in (16), (17) and (20). An additional instantaneous power is defined on top of the scalar active power, \( p \), and the vector reactive power \( q \). this power is named instantaneous zero-phase sequence power, \( p_0 \). When the load is balanced only a positive sequence phase vector exists, similarly to CPCT [7].

\[ p_0 = v \cdot i = v_0 \cdot i_0 \]  \hspace{1cm} (24)

Eq. (24) reminds us that every general three-phase vector of voltage/current waveforms is describable by the sum of three symmetrical balanced sequences the positive, negative, and zero sequences.

The generalized matrix formulation relating the voltage and current as inputs and the power as output [18,3,35] is:

\[
\begin{bmatrix}
  p_0 \\
  q
\end{bmatrix} =
\begin{bmatrix}
  v_0 & 0 & 0 \\
  0 & v_a & v_b \\
  0 & -v_b & v_a
\end{bmatrix}
\begin{bmatrix}
  i_0 \\
  i_a \\
  i_b
\end{bmatrix}
\]  \hspace{1cm} (25)

In (25) the zero phase sequence voltage is an incremental factor of the previous system described by (23). It provides further interpretation of Fig. 1, where the 2 \( \times \) 2 sub-matrix in the
right-hand corner represents both positive and negative components. Next the currents are calculated from the power and the voltages as follows:

\[
\begin{bmatrix}
  i_0 \\
  i_a \\
  i_b \\
  i_c
\end{bmatrix}
= \begin{bmatrix}
  0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\
  -\frac{1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} \\
  -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0
\end{bmatrix}^{-1}
\begin{bmatrix}
  p_0 \\
  p_a \\
  p_b \\
  p_c
\end{bmatrix}
\]

(26)

Therefore:

\[
\begin{bmatrix}
  i_0 \\
  i_a \\
  i_b \\
  i_c
\end{bmatrix}
= \begin{bmatrix}
  0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\
  -\frac{1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} \\
  -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0
\end{bmatrix}
\begin{bmatrix}
  i_0 \\
  i_a \\
  i_b \\
  i_c
\end{bmatrix}
\]

(27)

Eqs. (26) and (27) separate the zero-phase sequence component from the rest of the system (positive sequence phase vector). When transferring from the \([a, b, 0]\) plane to the \([a, b, c]\) plane, the phase currents are introduced as follows:

\[
\begin{bmatrix}
  i_0 \\
  i_a \\
  i_b \\
  i_c
\end{bmatrix}
= \begin{bmatrix}
  0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\
  -\frac{1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} \\
  -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0
\end{bmatrix}
\begin{bmatrix}
  l_0 \\
  l_a \\
  l_b \\
  l_c
\end{bmatrix}
\]

(28)

Next, the current components where the zero-phase sequence is separated can be presented as follows:

\[
\begin{bmatrix}
  i_0 \\
  l_a \\
  l_b \\
  l_c
\end{bmatrix}
= \begin{bmatrix}
  l_0 \\
  l_a \\
  l_b \\
  l_c
\end{bmatrix}
\begin{bmatrix}
  0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\
  -\frac{1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} \\
  -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0
\end{bmatrix}
\begin{bmatrix}
  0 \\
  l_a \\
  l_b \\
  l_c
\end{bmatrix}
\]

(29)

The same formulation can be derived with the instantaneous phase power:

\[
\begin{bmatrix}
  p_a \\
  p_b \\
  p_c
\end{bmatrix}
= \begin{bmatrix}
  v_a \cdot i_a \\
  v_b \cdot i_b \\
  v_c \cdot i_c
\end{bmatrix}
\]

(30)

where:

\[
\begin{bmatrix}
  p_a \\
  p_b \\
  p_c
\end{bmatrix}
= \begin{bmatrix}
  0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\
  -\frac{1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} \\
  -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0
\end{bmatrix}
\begin{bmatrix}
  p_a \\
  p_b \\
  p_c
\end{bmatrix}
\]

(31)

3.4. Generalization of PQT to include multi-phase systems

3.4.1. General

A multi-phase generalization to PQT was initially presented in [21] and [37]. Later on, in [10] a significant comparative work on the difference between instantaneous and periodic averaged theories is performed. The state-of-the-art generalization of multi-phase instantaneous power theory is reflected in [35,38]. Willems in [21] was the only work of the time dealing with generalizing PQT and has become a foundation of modern multi-phase PQT. Akagi et al. [3] provide an extensive study of PQT. The development of multi-phase PQT was gradual: initially developed for multi-phase, but still not generalized to cases such as unbalanced load and neutral line and very brief formulations were developed [21], then a separate branch of three-phase unbalanced load was developed [38], followed by three-phase four-wire explicit formulation [39]. Finally, a detailed formulation of modern multi-phase PQT [22] appeared, and then a critical comparison of multi-phase instantaneous with other theories [10]. The motivation for multi-phase theory development is that the single-phase case cannot be derived from the three-phase case, and also a single formulation of all three-phase variants. The multi-phase (\(m=1,2,3,\ldots,M\)) system cannot be derived as well. This is because the generalization requires comprehension of the instantaneous electrical parameters. The zero-phase sequence treatment is different from the early work performed in [24,40] and will be described ahead. The comparative work of Willems is described in [10,21,37,41].

3.4.2. Outline of multi-phase generalization for PQT

The generalization is summarized next, with emphasis on more elaborated proofs which are only briefly mentioned in an original paper [21]. In [21,42] it is shown that the Clarke transformation is not required, for manifesting PQT generalization ideas. In [3] Akagi et al. accepts this idea. Willems also relates to the difference between instantaneous power theories and power averaged theories. The original PQT development [34,15,17] presents Clark’s transformation as a key element of PQT. The knowledge developed by Willems is a new interpretation of PQT. The extension performed in [21,42] is acceptable to the scientific community as shown in [3]. In [10,21] a modern terminology of current and power decomposition of components is used. This terminology resembles CPCT [7] and conservative reactive vector space decomposition theory [23]. This terminology was used rarely in the eighties when PQT was initially developed from the FBD method [2,32,42].

The initial definitions of multi-phase voltage vector and phase currents are [3]:

\[
v(t) = [v_a(t), v_b(t), \ldots, v_M(t)]
\]

and

\[
l(t) = [l_a(t), l_b(t), \ldots, l_M(t)]
\]

(32)

where: \(L_i = 1, 2, \ldots, M\) are the phase indices. [43] defines active power current:

\[
p(t) = \left[\frac{\sqrt{3}}{2} v(t) \right] \cdot \left[\frac{1}{\sqrt{3}} l(t)\right] = v(t)^T l(t)
\]

(33)

\[
\Rightarrow l_p(t) = \frac{v(t)^T l(t)}{|v(t)|^2} v(t);
\]

where: \(v(t) = \frac{\sum_{i=1}^{M} v_i(t)}{|v(t)|} v(t)\)

The active power function is defined as the sum of phase active powers. It is similar to periodic averaged definition in the sense that it has the same arithmetical operation but it is not equivalent. The instantaneous reactive current is defined as:

\[
i_q(t) = i(t) - i_p(t)
\]

(34)

The significance of (34) [21], is repeated here. The reactive power is defined as:

\[
q(t) = |v(t)| \cdot i_q(t)
\]

(35)

In the three-phase case, the definition in \([a,b,0]\) space is of a vector product as in (21), but that is not \(i_q(t)\) as in (35). The majority of the scientific community regards reactive power as a vector. There is no violation of that assumption here. The following relations hold:

\[
i(t) = i_p(t) + i_q(t)
\]

(36)

Orthogonality is also obtained:

\[
|i_q(t)|^2 = \frac{|v(t)|^2}{|v(t)|^2}
\]

(37)
The proof is given by analogy to a proof from conservative theories [25] on the orthogonality of active and reactive powers. In the periodic averaging theory:

\[
\begin{align*}
S^2 &= \left( V^T I \right)^T \left( V^T I \right) \\
P^2 &= \left( V^T I \right) I \\
Q^2 &= S^2 - P^2
\end{align*}
\]

(38)

From (33) and (36), for the M-phase instantaneous theory we can derive:

\[
i_p = \frac{V_i^T I}{|v_i|^2}, \quad |v_i|^2 = v_i^* v_i, \text{ analogous to: } P^2 = \left( V^T I \right) I
\]

(39)

and:

\[
i_q = i - i_p = -\frac{V_i^T I}{|v_i|^2} = \frac{(v_i^* v_i - (V_i^T I) v_i)}{|v_i|^2}, \text{ analogous to: } (S^2 - P^2) I^{-1}
\]

(40)

Therefore, by analogy with (38), where it implies that Q and P are orthogonal (\(i_q \perp i_p\)).

Watanabe et al. [43] demonstrate orthogonality with phase vectors and not with harmonic vector components. In the specific case of a three-phase system the total current can be calculated from the orthogonality of phase voltages:

\[
|i(t)|^2 = |i_d(t)|^2 + |i_q(t)|^2 + |i_k(t)|^2
\]

(41)

Eq. (41) is acceptable for special cases: balanced load, from De-Moivre’s theorem, or star (3) connection for multi-phase developed here [21] converge to the single-phase classical definition. In the three-phase case, they converge to the original PQT development [15]. They converge to the two-phase definition of phases \(\alpha, \beta\) as defined by Clarke transformation. In [21] there is no explicit multi-phase mathematical formalism per phase M, it does provide explicit vector formalism and also reduced it to single phase, two-phase and three-phase theories. In [22] explicit variables appear, but for the three-phase case only. Next a complementary development is presented. From (33):

\[
i_{p_n}(t) = \frac{V_i^T I}{|v_i|^2} V_n(t) = \frac{v_i^* v_i}{\sum_{n} v_i^* v_i} P_n(t) = \frac{v_i^* v_i}{\sum_{n} v_i^* v_i} V_n(t)
\]

(42)

Since reactive power was defined in (35), it can be shown, based on analogy of (40), to (38) that distortion power is a sum of cross-product phase voltage pairs. The harmonic vectors are replaced with instantaneous phase vectors. According to (34), (36) and (40):

\[
i_q(t) = i(t) - \frac{V_i^T I}{|v_i|^2} V_n(t) = \frac{v_i^* v_i - (V_i^T I) v_i}{|v_i|^2}
\]

(43)

Relating the reactive current to the reactive power:

\[
\begin{align*}
S^2 / i &= vv_i^* \\
P^2 / i &= v_i^* v_i \\
I_q(t) i(t) &= \frac{v_i^* v_i - (V_i^T I) v_i}{|v_i|^2} = \frac{\varphi_i}{|v_i|^2}
\end{align*}
\]

(44)

In order to comprehend the first equation of (44), it is necessary either to compare physical units or follow (38), originally developed for periodic averaged theories. We have used an instantaneous apparent power definition and equivalence to (38). Then, by using the identity to the last expression in (48) we obtain:

\[
\begin{align*}
I_q(t) i(t) &= \sum_{n=1}^{N} \sum_{j=1}^{N} \frac{v_i^* v_j - (V_i^T I) v_j}{|v_i|^2} \\
I_{q,n,k}(t) &= \frac{1}{|v_i|^2} \frac{v_i^* v_j - (V_i^T I) v_j}{|v_i|^2}
\end{align*}
\]

(45)

The result of (45) is now here and show the relation between the total system reactive power and the cross-phase decomposed power elements:

\[
\begin{align*}
I_q(t) &= \sum_{n=1}^{N} \sum_{k=1}^{N} I_{q,n,k}(t) - 2 \sqrt{I_{q,n,k}(t) I_{q,k,n}(t)} \\
I_{q,n,k}(t) &= \frac{1}{|v_i|^2} \frac{v_i^* v_j - (V_i^T I) v_j}{|v_i|^2} \sum_{n=1}^{N} \sum_{j=1}^{N} I_{q,n,k}(t) - I_{q,k,n}(t)
\end{align*}
\]

(46)

Results (43) for the n-th phase active current component and results (45) for the nth, kth cross-phase reactive current component, and the nth phase reactive current component established equations similar to those of conservative theory, except that they were instantaneous expressions. The analogy step that was performed is to the harmonic vector, while here it is a phase vector. This makes them energetically un-equivalent. The results are not surprising, since it is known that the theory was developed on the basic representation in (22), but in the time domain instead of the frequency domain. Eqs. (42) and (45) are multi-phase explicit equations for the nth and kth cross-phase current component. A very close result for multi-phase PQT is found in [22]. More information on three-phase three and four wires can be found in [38,39]. All these papers use the generalized result (43) to extract three-phase, three-wire and four-wire special cases. The reactive \((n, k)\) component represents a flow of energy between non-cross-product phases of voltage and current. Emanuel [44] shows, in a paper on \(PVT\), that this power flow is owed to the interaction of the radiation of the electro-magnetic field from one phase with the current and voltage of another phase. It’s net contribution over one period is zero. Eqs. (43) and (45) set PQT in a mathematical framework which enables comparison with the periodic averaged theories and with \(PVT\). This result when reduced to the two-phase case resembles (26):

\[
\begin{align*}
I_q(t) &= I_{q,p}(t) + I_{q,q}(t) \\
I_{q,q}(t) &= I_{q,p}(t) + I_{q,q}(t)
\end{align*}
\]

(47)

The decomposition is identical to PQT active current (26):

\[
\begin{align*}
I_{q,p}(t) &= \frac{v_i(t)}{|v_i|} p(t), \\
I_{q,q}(t) &= \frac{v_i(t)}{|v_i|} q(t)
\end{align*}
\]

(48)

and the original PQT reactive current and power from:

\[
\begin{align*}
I_q(t) &= \frac{(v_i(t) \bar{q}_i(t) - q_i(t) \bar{v}_i(t))}{|v_i|} = \frac{\varphi_i}{v_i} q(t) \\
q(t) &= v_i(t) \bar{q}_i(t) - q_i(t) \bar{v}_i(t)
\end{align*}
\]

(49)

The work presented in [22] is similar and is summarized here:

\[
\begin{align*}
p = \frac{\bar{v}_i \cdot \bar{u}}{|u|} = \bar{u}^T i, \quad q = \bar{u} \times \bar{i} = \bar{u} \times \left( \bar{I}_p + \bar{I}_q \right), \quad s = \frac{|u|}{|\bar{u}|} \bar{I}_p \\
l_p = \frac{\bar{u} \cdot \bar{v}}{|u|} = \bar{v} \bar{u} = \frac{\bar{u}}{|u|} \frac{\bar{v}}{|v|} = \frac{\bar{u}}{|u|} \frac{\bar{v}}{|v|} = \frac{\bar{u}}{|u|} \frac{\bar{v}}{|v|}
\end{align*}
\]

(50)
Applying (50) to multi-phase vectors, for any phase, \( M \), yields identical results to (42) and (46) which are driven here from algebraic manipulation of (32)–(37), and were developed in [3]. In [22] the explicit development is implemented for three phases. Dai et al. [22] suggest that its novelty compared with [21] is explicit reactive power definition, whereas Willems [10] provides an identical reactive current definition. The two papers provide identical results. The next discussion is whether the multi-phase enhancement of instantaneous reactive power as a \( \vec{u} \times \vec{I} \) vector product is physically meaningful. Initially, physical meaningfulness is explained as an analogous harmonic vector cross-product in (50) which is beyond the requirement to design, a formalism where active and reactive power and current components are orthogonal. That requirement is the first justification for the usage of orthogonal power components: \( P \) and \( Q \). The second reasoning is as follows: if a simple product operator \( v(t) \cdot i(t) \) is performed, expanding the variables to their Fourier series as in [23] indicates for (38), and then taking the real and imaginary parts of only the \((n,m)\) product pairs, the active \( (P = \text{Re}(\sum_n V_n I_n)) \) and reactive \( (Q = \text{Im}(\sum_n V_n I_n)) \) power components arise. The reactive distortion component emerges as the harmonic vector of cross-harmonic product pairs \((n,k)\) where \( n \neq k \) \( (P = \text{Re}(\sum_n V_n I_n)) \) which is a vector product \([V_n] \times [I_n]\) of the voltage and current vectors of harmonics. A simple multiplication yields active, reactive and distortion power.

According to [9] in the instantaneous reactive power, is the phase cross-product of non-active power fluctuations through the electric and magnetic field radiation from one phase to the another phase. Fig. 4 shows a vector scheme of a multi-phase system. These fluctuations contribute over a single period a zero net power, and do not reflect oscillations between the source and the load. Thus, usage of multi-phase instantaneous vector product formalism is correct for phases. It is not physically meaningful for load to source oscillations, but it is correct for phase to phase radiation oscillations.

Furthermore (50) appears as a legitimate multi-phase description. This description is no worse than describing a multi-phase system only as a vector of independent phase behaviors \([\vec{Q}_1, \vec{Q}_2, \vec{Q}_3]\).

3.5. Difference between PQT and periodic averaged time-dependent theories

In [23] it is demonstrated that there is a time-dependent reactive power operation equivalent to the periodic averaged reactive power in terms of energy balance. That operation is applicable in multi-vector PVT, for example.

\[
q(t) = V(t) \frac{dI(t)}{dt} - i(t) \frac{dV(t)}{dt}
\]  

Equation (51) is valid for \( PVT \) by itself an instantaneous theory. Multi-vector \( PVT \) is periodic averaged since Fourier transform is applied. The equivalent periodic averaged reactive power operation is, [25]:

\[
\vec{Q}_{\text{total}} = [V_n] \times [I_n]
\]  

Equation (52) is derived from the Buchholz apparent power definition in (8) as shown in [26]. The instantaneous reactive power definition (21) is not equivalent to periodic averaged theories. PQT is thus not equivalent to the periodic averaged theories. This difference does not rule it out and it is successful in active filtering. Buchholz apparent power definition is a formal definition without a relation to a physical parameter. However, the definitions of (21) and (52) have the same formalism. The first in the time domain for the multiphase case and the second relates to the frequency domain with referring to each phase separately.

Opinions in the literature regarding the usage of instantaneous reactive power and instantaneous power theory are split. Some say that power theory does not reflect well the active and reactive load characteristics [21]. As a result, for applications requiring true reflection of load active and reactive characteristics, some researchers do not use instantaneous theory [6]. Counter-opinion states that the modern electronics with switching circuits implemented in converters require real-time active compensation, which instantaneous reactive power enables [3]. Overall, \( PVT \) does compensate for non-linear waveforms successfully. It appears that both periodic and instantaneous theories groups are relevant.

4. PQT and multi-phase theory: areas of strength compared with other theories

There are several reasons why PQT is considered a dominant implementation theory:

1. **Cheaper and simpler active filter implementation**: PQT provides the option of implementing active filters, with energy storing devices [3] such as capacitors, inductors, and current sources. PQT also enables implementation of active filters without energy storage devices [15], using only switching circuits. Periodic averaged theories enable implementation only with energy storage devices. Implementation without energy storage is cheaper and easier to maintain. That is a major factor in PQT’s success.

2. **Ability of instantaneous response to transient power conditioning**: current switches in regulators and inverters insert transients to the grid. It is hard to handle non-periodic waveforms with periodic averaged theories for compensation with response time smaller than a single cycle. Periodic averaged theories are therefore referred to as quasi-periodic [41].

3. **Black box load knowledge**: there is no need to know load structure in order to break it down to the current/power physical components. That property is shared by periodic averaged theories such as CPVT, and multi-vector PVT [9].

4. **Offers theoretical simplicity and sophistication**: early PQT was a simple theory. Modern multi-phase PQT [3,22] and modern three-phase unbalanced theory [3,39], offer a sophistication comparable to the rest of modern theories such as multi-vector PVT, CPVT, and reactive vector space decomposition. Simplicity is an advantage in terms of both comprehension and implementation. Sophistication is important for modern-day applications.

5. **Computation complexity**: PQT is a time domain. It requires \( N \) digital samples per cycle per phase, and \( O(N) \) computation complexity. A periodic averaged theory operating at the frequency domain requires \( N \log N \) operations to compute FFT, times four wire phases, times voltage and current separately. Czarnecki [7] suggests a look-up table to reduce computation complexity.
complexity to $O(N)$. With current computation technology, performing FFT is not such an issue any more [45]. The computational issue was a key factor historically in terms of accelerating PQT development as compared with CPCT [46].

For all these reasons PQT has been more dominant than other theories over the past three decades. However, this review of past papers suggests that the last word has not yet been spoken.

5. Conclusion

In this paper a review of apparent power definitions and instantaneous electric energy transport group of theories are presented. The review is believed to cover most of today's instantaneous theories while other review papers on the subject focus on a partial group or different aspects of the theories. In the first part of the paper several different apparent power definitions were presented: Buchholz, arithmetic, geometric and vector power (which is similar to geometric). The theories are first shown to yield identical results in the linear balanced three phase system. It is shown that each definition results in different terms when nonlinear and unbalanced loads are concerned. In order to contribute to the knowledge in this field, this paper attempts to point out similarities of terms as well as total inequalities between the various definitions. Some insights into a possible physical notation were presented, although there is no proven physical meaning to any of the definitions. The Appendix shows expressions which support that a single-phase geometric apparent power is equivalent to the Buchholz apparent power in the sense that the latter is simply the RMS value of the former. For a three-phase instantaneous reactive power vector, it is shown that Buchholz apparent power is bounded in a finite region of magnitude in relation to the geometric apparent power and therefore for design purposes it can be used though the terms are not equal. The problem of load characteristic active/reactive identification was discussed in terms of vector power for three phases, and its physical source is explained. The question of which definition to use was also handled by emphasizing the physical point of view of each definition, although apparent power is generally accepted as not being a physical parameter.

The basic instantaneous theory PQT is presented. Then, enhancement to include zero phase sequence in order to generalize the theory in the cases of three-phase four-wire, three-phase three-wire, is also described. Then, a multi-phase instantaneous theory was studied. By using explicit formalism from existing papers, power and current physical components were written for the nth and $k^{th}$ cross-phase. Usually multi-phase formalism is developed for three-phase systems, but here the $n^{th}$ phase formulation was provided to enable cases such as multi-load and multi-source usage. This presentation is not new, but the formalism in this paper offers a new perspective on multi-phase usage, and also sheds some light on how orthogonality of current components and non-active power are obtained. A comparative study is presented, showing two distinct differences between instantaneous theories and all other theories: (1) there is un-equivalence of calculating conduction losses of active power. (2) un-equivalence of energy formulas. It is shown that the study of instantaneous theory enables novel grid implementations such as: (a) Active grid filtering and power conditioning, (b) Grid power composition and decomposition of sub-branches into a main branch and vice versa (as shown in (46)). The use of instantaneous reactive current as grid indicator for inter-phase energy flow is demonstrated as well. A comparative of instantaneous and periodic averaged theories as they were presented here is finally summarized in Table 2.

### Acknowledgment

This research was supported in part by the ISG (Israeli Smart Grid) Consortium, administrated by the Office of the Chief Scientist of the Israeli Ministry of Industry, Trade and Labor.

### Appendix. Relations between Buchholz and geometric power

In the Single phase case the Buchholz apparent power definition and the geometric apparent power definition or vector definitions are equivalent. [9] discusses multi-vector PVT and demonstrates periodic averaging instantaneous variables and executes a Fourier transform. The distortion power operator obtained is:

$$ D(t) = \sum_{j \neq k, \text{linear}} (V_j I_k e^{j \phi_{jk}} - V_j I_k e^{j \phi_{jk}}) e^{(\eta_{jk} - \eta_{jk}) e^{j \alpha_{jk}}} $$

$$ + \sum_{j \neq k, \text{non-linear}} V_j I_k e^{(\eta_{jk} - \eta_{jk}) e^{j \alpha_{jk}}} $$

(53)

Taking the RMS value of (53) yields:

$$ D^2 = \int D \cdot D' dt = \left\{ \sum_{j \neq k} (V_j I_k e^{j \phi_{jk}} - V_j I_k e^{j \phi_{jk}})^2 \right\}^{1/2} $$

(54)

Eq. (54) is the single-phase Buchholz apparent power definition.

In the multi-phase case the suggested multi-phase apparent power $S$ based on geometric apparent power orientation yields a
Buchholz apparent power as order of magnitude:
\[
\Delta S_{app}(t) = \sum_{i} v_i(t) j_i(t), \quad \text{or} \quad S_a = \sum_{i} v_i L_i j_i L_i(t)
\]

Putting the apparent power operator further yields:
\[
\Delta S_a^2 = \sum_{i} \int_0^T |v_i(t) j_i(t)|^2 dt + \sum_{i \neq j} \int_0^T (v_i(t) j_i(t))^* (v_j(t) j_j(t)) dt
\]

Developing the apparent power operator further yields:
\[
\Delta S_a^2 = \sum_{i} \int_0^T |v_i(t) j_i(t)|^2 dt + \sum_{i \neq j} \int_0^T (v_i(t) j_i(t))^* (v_j(t) j_j(t)) dt
\]

There is no constraint regarding the magnitude relationships between \(V_L^2\), \(V_i^2\), \(I_L^2\), and \(I_i^2\) and therefore only order of magnitude equivalence is obtained:
\[
\Delta S_a^2 = \sum_{i} \int_0^T |v_i(t) j_i(t)|^2 dt + \sum_{i \neq j} \int_0^T (v_i(t) j_i(t))^* (v_j(t) j_j(t)) dt
\]

The upper and lower bounds of the difference between the geometric and the Buchholz apparent powers are as follows: The Buchholz power which is the upper bound, because “it assumes” all harmonics are simultaneously maximized, takes values according to the following equation:
\[
S = V_{RMS} I_{RMS} \sqrt{1 + y^2 + z^2} \sqrt{1 + u^2 + w^2}
\]

where \(y, z\) are the normalized RMS \(S, T\) phase voltages as compared with the \(R\) phase with a value of 1 p.u. Also, \(u\) and \(w\) are the normalized \(S, T\) phase currents. The region of existence is bounded in a product of two balls and is bounded by:
\[
1 \leq \sqrt{1 + y^2 + z^2} \sqrt{1 + u^2 + w^2} \leq 3
\]

A ratio of 3 is not undermined but it is bounded.

References


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