On the Definition of Total Harmonic Distortion and Its Effect on Measurement Interpretation

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***Abstract*—The existence of two different definitions for total harmonic distortion (one in comparison to the fundamental and one in comparison to the signal's root mean square) might cause ambiguity and misinterpretation of measured data. The difference between those definitions is stressed out in this letter. It is suggested that total harmonic distortion measurements in the context of power systems should always adopt the first definition and never the second.**

***Index Terms*—Harmonic distortion, nonlinear load, nonsinusoidal waveforms, power measurements, power meters.**

I. Introduction

TOTAL harmonic distortion (THD) is an important figure of merit used to quantify the level of harmonics in voltage or current waveforms. Two different definitions for THD may be found in the literature. According to one definition, the harmonic content of a waveform is compared to its fundamental [1], [2]. By the second definition, the harmonic content of a waveform is compared to the waveform's rms value [3]. In order to distinguish between the two, the former is occasionally denoted by $\mathrm{THD_F}$ and the second by $\mathrm{THD_R}$. For instance, current THDs are defined as

$$\mathrm{THD_F} = \frac{\sqrt{\sum_{n=2}^{\infty} I_n^2}}{I_1}; \quad \mathrm{THD_R} = \sqrt{\frac{\sum_{n=2}^{\infty} I_n^2}{\sum_{n=1}^{\infty} I_n^2}} \tag{1}$$

where I_n are either the rms values or the amplitudes of the harmonics. At low values of THD, there is not much difference between the two. However, the two definitions may cause ambiguity, confusion, and misinterpretation when measuring waveforms of high harmonic content.

The relation and difference between the two definitions of THD are stressed in this letter. Usage of $\mathrm{THD_F}$ rather than $\mathrm{THD_R}$ is advocated.

II. Relation Between $\mathrm{THD_F}$ and $\mathrm{THD_R}$

Actually there is a consensus as to the basic definition of THD in the context of power measurements, by which it is defined with respect to the fundamental [1], [2], [4].

It seems that the second definition $\mathrm{THD_R}$ was inherited from the area of audio amplifiers, where the THD serves as a measure of the systems linearity and its numerical value is always much less than 1 (it practically ranges from 0.1–0.3% in Hi-Fi systems up to a few percent in conventional audio systems). Thus, for this range of THD values, the error caused by mixing up the two definitions of THD was acceptable. For instance, if the actual THD $(\mathrm{THD_F})$ is 10%, $\mathrm{THD_R}$ will have the value of 9.95% (less than a 0.5% difference). Moreover, with the older type, analog distortion analyzers incorporated for amplifier testing, it is easier to measure $\mathrm{THD_R}$; the nominator in (1) is obtained by filtering out the fundamental with a notch filter, and the denominator is simply the signals' rms value. These two quantities are related by (2) and plotted in Fig. 1

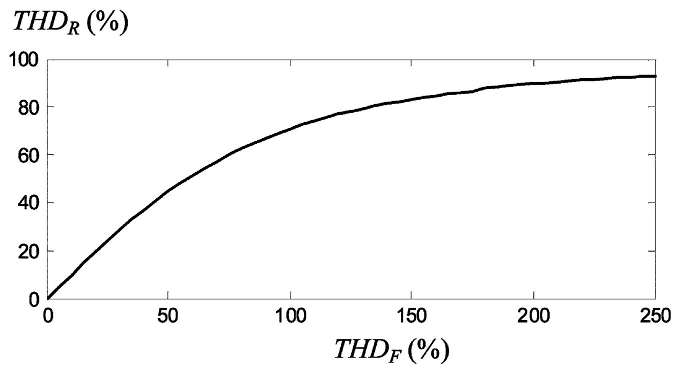

Fig. 1. Total harmonic distortion in percent of the signals rms versus its basic definition (in percentage of the fundamental) yields (4).

$$\mathrm{THD_R} = \frac{\mathrm{THD_F}}{\sqrt{1+\mathrm{THD_F^2}}}. \tag{2}$$

Evidently, at high values of THD, the difference becomes essential. $\mathrm{THD_R}$ cannot exceed 100% whereas $\mathrm{THD_F}$ may reach higher values when the spectral energy of the harmonics exceeds that of the fundamental (mathematically, it may reach infinity if a waveform contains no fundamental).

III. Interpretation and Accuracy Issues

High current THDs are quite common in electronic loads [5], [6]. For instance, values of 140–170% are typical for currents drawn by peak detection rectifiers (Fig. 2). Fig. 1 shows that at high THDs, large variations in $\mathrm{THD_F}$ manifest in very little variation in $\mathrm{THD_R}$, and vice versa, small differences of $\mathrm{THD_R}$ are, in fact, large differences in the THD $(\mathrm{THD_F})$ content. This can be quantified by the sensitivity of the THD with respect to variations in $\mathrm{THD_R}$, defined by

$$S_{\mathrm{THD_R}}^{\mathrm{THD_F}} = \frac{\partial \mathrm{THD_F}}{\partial \mathrm{THD_R}} \cdot \left(\frac{\mathrm{THD_R}}{\mathrm{THD_F}}\right) \tag{3}$$

$$S_{\mathrm{THD_R}}^{\mathrm{THD_F}} = \frac{\sqrt{1+\mathrm{THD_F^2}}}{\sqrt{1+\mathrm{THD_F^2}} - \mathrm{THD_F}}. \tag{4}$$

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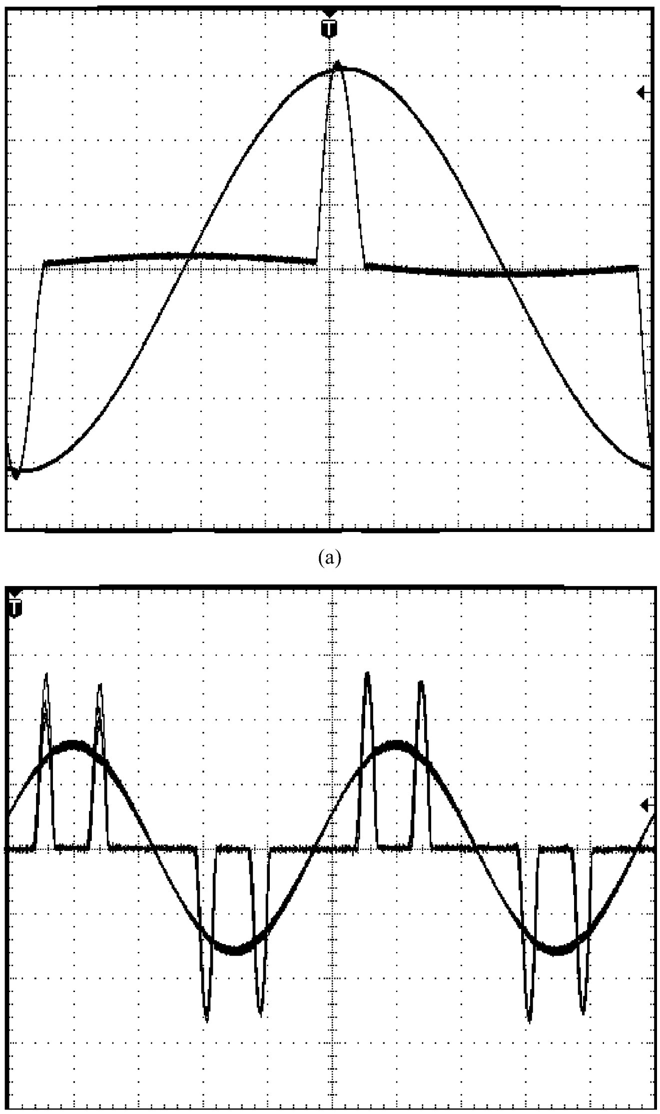

(a)

(b)

Fig. 2. Peak detection rectifiers' typical wave forms. (a) Single phase (94 W, pc load), 100 V/div, 1 A/div, THD $\cong 178\%$. (b) Three phase, low load (880 W, ASD), 200 V/div, 5 A/div, $\mathrm{THD} \cong 172\%$.

This sensitivity is quite high at high THDs, for instance, for $\mathrm{THD_F}$ equal to 170%, the sensitivity is above 7 and inaccuracy may be caused in two ways:

A. Human Perception

Values of 87% or 89% may be thought to be pretty much the same. However, should the above numbers represent $\mathrm{THD_R}$, this actually represents a difference of 19%, between 176% and 195%, respectively, in terms of $\mathrm{THD_F}$. Thus, what seems to be a 2% difference (in terms of $\mathrm{THD_R}$) is, in fact, a 10% difference in $\mathrm{THD_F}$. A 10% difference will be caused in the distortion factor, DF as well ($\mathrm{DF}(\mathrm{THD_F} = 176\%) = 45\%$ and $\mathrm{DF}(\mathrm{THD_F} = 195\%) = 49\%$), where DF is defined as

$$\mathrm{DF} = \frac{1}{\sqrt{1+\mathrm{THD_F^2}}} = \sqrt{1-\mathrm{THD_R^2}} \quad (5)$$

TABLE I
SPECIFIED ACCURACIES OF THD MEASUREMENT FOR DIFFERENT POWER ANALYZER MODELS

POWER QUALITY ANALYZER MODEL	THD MEASUREMENT ACCURACY
FLUKE 43B	3%+8COUNTS
FLUKE 41B	0.03*READING+2%
LEM LH1060	3%
AGILENT 6813B	0.05*READING+0.1%

TABLE II
EXAMPLE OF MEASUREMENT ERROR

	$\mathrm{THD_F}$	$\Delta\mathrm{THD_F}$(%)	$\mathrm{THD_R}$	$\Delta\mathrm{THD_R}$(%)
FLUKE 43B	183%	2.73%	84%	3.7%
FLUKE 41B	173%	2.73	82%	6%
LEM LH1060	181%	1.69%	89.2%	2.3%
Agilent 6813B	175%	1.70%	93.6%	7.3%

B. Instrumentation Accuracy

The effect of instrumentation finite accuracy is similar to the one above. The specified THD measurement accuracies of four power-quality analyzer models are listed in Table I. As can be seen, all of the accuracies contain terms, which are independent of the reading. That implies reduced accuracy if $\mathrm{THD_R}$ is used.

IV. EXPERIMENTAL RESULTS

The power-quality analyzers listed in Table I have been compared when measuring highly distorted signals. The measurement accuracy was always higher when $\mathrm{THD_F}$ was used. Measurements results for the single-phase rectifier [Fig. 2(a)] are summarized in Table II. The average measured values are referred to as accurate ones. The measurement error $\Delta\mathrm{THD}$ is calculated as the difference of the measured value from the average, in percent, of the average value

$$\Delta\mathrm{THD_R}(\%) = \frac{|\mathrm{THD_R} - \langle\mathrm{THD_R}\rangle|}{\langle\mathrm{THD_R}\rangle}\cdot 100\%$$

$$\Delta\mathrm{THD_F}(\%) = \frac{|\mathrm{THD_F} - \langle\mathrm{THD_F}\rangle|}{\langle\mathrm{THD_F}\rangle}\cdot 100\%. \quad (6)$$

In this test case, the average $\mathrm{THD_R}(\langle\mathrm{THD_R}\rangle)$ was 87.2% and the average $\mathrm{THD_F}(\langle\mathrm{THD_F}\rangle)$ was 178% [which does not comply with (2)]. Not only are the measurement errors higher when using $\mathrm{THD_R}$, but also the error generated when computing $\mathrm{THD_F}$ from $\mathrm{THD_R}$ is enormous. For instance, $\mathrm{THD_R} = 93.6\%$ (Table II) yields $\mathrm{THD_F} = 266\%$, an error of 49%.

V. CONCLUSION

It has been shown that $\mathrm{THD_F}$ is a much better measure of harmonics content. Employment of $\mathrm{THD_R}$ in measurements may yield high errors in significant quantities such as power factor and distortion factor, derived from THD measurement [7]. Modern power analyzers incorporate discrete Fourier transform (DFT)-based algorithms (as opposed to older, analog analyzers). Thus, there is no reason to include $\mathrm{THD_R}$ even as an optional measurement as it may cause errors and misinterpretation.

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