Characteristics of modern nonlinear loads and their influence on systems with distributed generation

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Abstract: Nonlinear electronic loads may demonstrate unexpected reactions to variations in power system voltage. This effect of the electrical load may be very significant in distributed generation power systems containing relatively small generators. This paper reviews modern nonlinear electronic load current consumption mechanisms and provides typical current waveforms. Based on extensive measurements and field surveys, modified ZIP models have been developed to produce descriptive behaviour for a large class of nonlinear loads that account for cutoff voltages. Finally, an independent PC program was developed to analyse individual load equipment under varying voltage conditions based on the active and reactive powers at the rated voltage. The program also enables the study of combined loads (such as in a single residential or commercial building).

Keywords: harmonics; load estimation; ZIP model; varying voltage; reactive power; power quality; nonlinear load.

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1 Introduction

Global load composition in power systems has changed significantly during the past two decades (Arrillaga et al., 2000; 1985; Emanuel, 1990; IEEE Working Group in Nonsinusoidal Situations, 1996; IEEE Task Force on Load Representation for Dynamic Performance, 1995; Subjak Jr. and McQuilkin, 1990). The share of nonlinear loads nowadays is over 50% due to the modern electronic interface in the majority of loads. The distortion of sinusoidal voltage and current waveforms caused by harmonics is one of the major power quality concerns in the electric power industry (Subjak Jr. and McQuilkin, 1990; Hajagos and Danai, 1998; Duan et al., 2003; 2004; Dugan et al., 1996).

In the traditional secondary-network power distribution load-flow program, customer demand is represented as a constant kVA load. Although this model has been very useful in the past, substantial changes in the nature of the electrical loads supplied by utility companies, especially in the past ten years, have made it increasingly clear that a more accurate load representation is needed. Computer loads, the use of electronic ballasts for fluorescent lighting and the extensive use of power semiconductor switches, such as in solid-state drives, all have greatly increased the nonlinear load on power systems. One way for utility companies to reduce power consumption during the summer months is by decreasing voltage. However, with the large increase in the number of nonlinear loads in recent years, the distribution network response may not be so straightforward. That is, it is no longer possible to predict the network response with the same level of confidence as in the past. For example, voltage reduction reduces power consumption; but at the same time, it may cause significant increases in feeder and transformer currents. Furthermore, some electronic loads with regulated output voltages, such as active power factor-corrected rectifiers, exhibit constant power demand against voltage variation. In response to a voltage decrease, these loads will respond by increasing their current consumption. Considerable effort has been made in recent years attempting to model the load and predict it (Hajagos and Danai, 1998; Duan et al., 2003; 2004; Dugan et al., 1996; Aggarwal and Song, 1997; Leeb et al., 1995; Dillon and Niebur, 1996; Park et al., 1991; Wagner, 1995; Chen et al., 1992; Bakirtzis et al., 1995; Peng et al., 1993; Kandil et al., 2001; Yan et al., 1999).

A more accurate load model (or customer load profile function) is suggested which estimates the actual customer demand ($P$ and $Q$) from the nominal demand ($P_0$ and $Q_0$) and the actual voltage level over the range 60–130 V. This model would be different for commercial and residential loads. In addition, the load model will tabulate the error between this single customer load file function and commercial or residential functions that are developed. The load model will also tabulate the error for the constant kVA representation.

A modified load model was developed in this work. The basic ideas behind this model and the laboratory tests used to develop and validate the model are described. The code simulates the effects that a combination of linear and nonlinear loads will have on the utility network at the distribution level. A separate, independent PC program was created to allow the study of individual pieces of load equipment (e.g., pumps, variable-speed drives, fluorescent lights) as well as combined loads (such as in a single residential or commercial building).
2 Current waveforms and power variation of modern loads

The measured waveforms of the current consumed by several office loads (e.g., a personal computer, a copying machine and a laser printer) and by some lighting loads are discussed in this section. The variation in these loads’ power versus voltage variation within a narrowly defined range is also shown. In order to differentiate between the nonlinear effect of the loads, they were energised by a line emulator (programmable ac power supply, HP6138) that generates a nearly pure sinusoidal voltage with total harmonic distortion (THD) <0.3%. The reactive power, $Q$, was calculated according to Equation (1):

$$ Q = \sqrt{\frac{1}{1+THD^2}} S^2 - P^2 $$

where THD is the total harmonic distortion, $S$ the apparent power and $P$ the active power.

The current waveform of a PC (with no power factor correction) is a waveform characteristic for appliances with a diode bridge rectifier at the front end and is not shown here. It should be noted that while this load consumes only 94 watts on average, it momentarily draws over 900 watts. In an industrial plant containing many loads of this type, the combined momentary power demand of all the loads adds up in the vicinity of the peak line voltage.

Another interesting phenomenon produced by the PC load (or any other peak detection rectifier load) is its response to a sudden change in voltage. Figure 1 shows the normalised power level and how it varies in response to a 9% decrease in voltage.

The normalised quantities $S_n$, $P_n$ and $Q_n$ are defined in Equation 2:

$$ S_n = \frac{S}{S_{\text{nominal}}}; \quad P_n = \frac{P}{S_{\text{nominal}}}; \quad Q_n = \frac{Q}{S_{\text{nominal}}} $$

with $S_{\text{nominal}}$ being the apparent power measured at nominal voltage (prior to the voltage reduction). As can be seen, the load drops to zero for 1–2 line periods. This may be explained by the large capacitance in the peak detection rectifier which, prior to the voltage step down, was charged to nearly peak voltage (311 V). Immediately after the line voltage decreases there is no conduction in the diode bridge as the capacitors are charged to a voltage higher than the line. After 1–2 line periods, the capacitor discharges and conduction resumes when the capacitor voltage drops below the peak line voltage.
The copying machine is a time-varying load. Most of the time (while in standby), it consumes a triangularly shaped current with medium harmonic content (THD = 30%), (see Figure 2). However, while it is in full operation, sinusoidal in-phase currents add to the triangular current component resulting in a nearly sinusoidal total current with a THD equal to 4% (see Figure 2(b)).

Other measured values for the copying machine load while in standby mode include the rms current, active power, apparent power, reactive power, power factor and displacement factor:

\[ I = 0.42 \text{ A}; P = 89 \text{ W}; S = 93 \text{ VA}; Q = 22 \text{ VAR}; PF = 0.96; DPF = 0.981. \]

Laser printers exhibit quite similar behaviour. Measurements of the waveforms produced by a HP2100 printer are shown herein. In standby mode, it consumes a narrow, 1.6 A current pulse with high harmonic content (THD = 166%) (see Figure 3(a)). This current waveform is typical to diode bridge rectifiers like the one employed to energise the printer’s electronic circuits. The power consumption during standby mode was 39.7 W. Other measurements of the laser printer in standby mode were the following: \[ V = 220 \text{ V}; I = 0.36 \text{ A}; P = 39.7 \text{ W}; S = 80 \text{ VA}; Q = 11 \text{ VAR}; PF = 0.497; DPF = 0.983; THD = 166\%. \] These numbers are quite similar to those obtained for the PC load since a similar peak detection rectifier supplies the electronic circuits within the printer.
Figure 2  Current and voltage waveforms of a copying machine

(a) in standby, 100 V/div, 2 A/div

(b) while in operation, 100 V/div, 2 A/div

Every once in a while a sinusoidal current component appears (which powers the printer's heater). The total current consumed during heating, which is the sum of the pulsating and the sinusoidal currents, is clearly seen in Figure 3(b). During heating, the THD decreases to 14% and the power consumption increases to 416 W.
Figure 3  Current and voltage waveforms of a laser printer

(a) while the heater is not energised, 100 V/div, 1 A/div

(b) while the heater is energised, 100 V/div, 2 A/div
The current waveform of a high intensity 125 W mercury-vapour light bulb with magnetic ballast is shown in Figure 4. Measured data are the following: \(V = 220\, \text{V}\), \(I = 0.983\, \text{A}\); \(S = 214\, \text{VA}\); \(Q = 175\, \text{VAR}\) (inductive); \(P = 123\, \text{W}\); \(PF = 0.57\); \(DPF = 0.58\); \(THD = 10.1\%\). As can be seen, this load is highly linear due to the large magnetic ballast, which is also the reason for the low displacement factor.

**Figure 4**  Current and voltage waveforms of a high intensity mercury-vapor light bulb with magnetic ballast, 100 V/div, 0.5 A/div

The current waveform of a magnetic ballast that energises four 18 W fluorescent lamps is shown in Figure 5. Measured data are the following: \(V = 220\, \text{V}\); \(I = 0.397\, \text{A}\); \(P = 83\, \text{W}\); \(S = 86\, \text{VA}\); \(Q = 15\, \text{VAR}\) (inductive); \(PF = 0.97\); \(DPF = 1\); \(THD = 19.3\%\).

**Figure 5**  Current and voltage waveforms of a fluorescent bulb’s magnetic ballast, 100 V/div, 0.5 A/div
The current waveform of a high frequency, switched mode electronic ballast with active power factor correction that energises four 18 W fluorescent lamps is shown to exhibit a nearly sinusoidal shape. Measured data for this type of load are the following: $V = 220 \, V$; $I = 0.352 \, A$; $P = 76.6 \, W$; $S = 76.9 \, VA$; $Q = 7.5 \, VAR$ (capacitive); $PF = 5$; $DPF = 0.99$; $THD = 5\%$.

In addition to the fact that the loads exhibit different current waveforms with different levels of distortion and that some consume reactive power while others supply capacitive power, the loads also respond differently to line voltage variation.

Reactive power of the PC load hardly varies, and the active power is nearly constant. This may be explained by the PC’s voltage-regulated DC power supply. The slight variation in reactive (and apparent) power is accounted for by reactive elements such as inductors and cooling fans.

A very similar relationship applies to the laser printer while in standby mode.

The power variation of a high-intensity mercury-vapour light bulb with a magnetic ballast is shown in Figure 6. As the large inductor dominates the nature of this load, the powers are proportional to the voltage square. A quite similar dependence was observed for the fluorescent magnetic ballast. Normalised voltage is employed $V_n = \frac{V}{V_{\text{nominal}}}$ in the figure as well as the normalised power definitions given in Equation (2).

Figure 6  Normalised power of a of a high intensity mercury-vapor light bulb with a magnetic ballast
The powers of the fluorescent electronic ballast do not change with the voltage. The electronic ballast has internal controls so as to maintain a constant voltage across its output DC-link capacitor, which produces a constant illumination intensity and constant power consumption, \( P \). It is also designed to draw sinusoidal current in phase with the voltage, resulting in almost no reactive power and, therefore, a constant apparent power.

For the purposes of this study, we have selected a large variety of loads, each having a distinctive mechanism of current consumption. The various loads’ current distortion, apparent power, active power and reactive power characteristics as well as their responses to line voltage variations are also different from one another. In the following sections these loads have been categorised into 17 groups, and a load model suggested to predict the responses of these loads to line voltage variations.

3 Load estimation problem

3.1 Problem formulation

The initial problem of load distribution estimation is to be able to accurately determine the composition of a customer’s load from the measured bus current waveforms (Hajagos and Danai, 1998; Duan et al., 2004). The measured bus current waveform \( I(t) \) can be approximated by a composite waveform which consists of the sum of a set of connected load current waveforms that are weighted appropriately. The connected loads are characterised by the frequency spectrum of their current waveforms. The loads are assigned to groups based on their harmonic spectrum. For an individual load:

\[
I_i(t) = \sum_{k=0}^{N} [c_{ik} \cos(k\omega t + \phi_k)] = \sum_{k=0}^{N} [a_k \sin(k\omega t) + b_k \cos(k\omega t)]
\]

(3)

where \( N \) is the highest harmonic order used for the load composition and \( i = 1, 2, \ldots M \) is the number of the load. The total composite load will be the following:

\[
L(t, w) = \sum_{i=0}^{M} w_i I_i(t)
\]

(4)

where \( w \) is the vector of weighting factors \( w_i \) of each load.

To match the shape of the composite waveform \( L(t, w) \) to that of the actual bus waveform \( I(t) \), the root mean square error can be used. This results in the minimisation criterion for the quantity \( J \) determined in Equation (5).

\[
J(t, w) = \sqrt{\frac{1}{T} \int_0^T [I(t) - L(t, w)]^2 dt}
\]

(5)

where \( T = 2\pi/\omega \) is the period of the fundamental component of the power system and the physical constraint for \( w \) is that \( w_i \geq 0 \) for all \( i \)'s. Expanding \( I(t) \) into a Fourier series, we have:
Characteristics of modern nonlinear loads

\[ I(t) = \sum_{k=0}^{N} \left[ A_k \sin(k \omega t) + B_k \cos(k \omega t) \right] = \ldots \]
\[ = \sum_{k=0}^{N} \left[ A_k \sin(k \omega t) + B_k \cos(k \omega t) \right] + \ldots \]
\[ + \sum_{k=N+1}^{\infty} \left[ A_k \sin(k \omega t) + B_k \cos(k \omega t) \right] \]

and substituting it into Equation (5) results in \( J(t, w) \) becoming independent of the time variable \( t \) due to the orthogonality property of sine and cosine functions:

\[ J(w) = \left\{ \frac{1}{2} \left[ \sum_{i=1}^{n} \left( A_i - \sum_{j=1}^{m} w_i a_{ij} \right)^2 + \sum_{i=1}^{n} \left( B_i - \sum_{j=1}^{m} w_i b_{ij} \right)^2 + \sum_{k=N+1}^{\infty} (A_k^2 + B_k^2) \right] \right\}^{\frac{1}{2}} \] (7)

Hence, the requirement that \( J(t, w) \) of Equation (5) be minimal has been transformed into the requirement that \( J(w) \) of Equation (7) be minimal. The calculation of load composition is then formulated as a minimisation of \( J(w) \) which becomes a quadratic programming problem. Thus, it is theoretically possible to estimate percentage shares of various types of loads connected to a service point using the frequency spectrum of the current at that point. In practice, the goal function \( J(w) \) is a complex nonlinear function of load harmonic characteristics, current waveform at the service point and uncertain external factors like harmonic deviation and voltage disturbance from the outside. With traditional methods such as linear or multiple regression methods and general exponential smoothing, the load models would be very complicated and would not be able to ensure the accuracy of the estimation due to the local minima problem. Thus, an artificial neural network approach is considered to be a promising one due to its adaptability, general applicability and nonlinear character.

4 Load model under varying voltage conditions

4.1 General approach

Field surveys were conducted to determine what types of equipment and appliances are currently in use at commercial and residential sites, the power consumed by each type of equipment and the times of the day each equipment and/or appliance is used. The equipment was then grouped into 17 separate categories (see Table 1). To obtain the load characteristics under varying voltage conditions, laboratory tests of individual electric loads were conducted for voltages in the range of 50%–110% of the nominal voltage. \( P, Q \) and \( VA \) data were obtained in this manner. Low-voltage functional-cutoff voltages were also recorded. Where such testing was not feasible, published data in the literature was used as the source (Hajagos and Danai, 1998). For each equipment type, the \( P, Q \) data were quadratic-curve-fitted to generate functions for \( P(V) \) and \( Q(V) \). To include the low-voltage functional-cutoff data (i.e., voltages of \( V_{min} \)), these functions were multiplied by a factor \( Y_v \) which has a value of 0 below \( V_{min} \) and 1 above \( V_{min} \).
From the results of surveys, a list of 17 basic loads was constructed, each basic load having its individual electrical characteristics. This list is provided in Table 1, along with the number of such devices tested in the laboratory and the voltage range (percentage of rated voltage) of the tests.

### Table 1  Typical electric loads of modern buildings

<table>
<thead>
<tr>
<th>Load</th>
<th>Number tested</th>
<th>Percentage in voltage range (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pump 1 – Constant speed</td>
<td>4</td>
<td>63–110</td>
</tr>
<tr>
<td>Pump 2 – Variable speed</td>
<td>1</td>
<td>91–110</td>
</tr>
<tr>
<td>Fan 1 – Constant speed</td>
<td>2</td>
<td>50–110</td>
</tr>
<tr>
<td>Fan 2 – Variable speed</td>
<td>1</td>
<td>65–110</td>
</tr>
<tr>
<td><em>Elevator – Variable speed</em></td>
<td>1</td>
<td>85–110</td>
</tr>
<tr>
<td>#Elevator/escalator – motor generator set</td>
<td>1</td>
<td>75–110</td>
</tr>
<tr>
<td>Fluorescent lights – magnetic</td>
<td>3</td>
<td>76–110</td>
</tr>
<tr>
<td>Fluorescent lights – electronic</td>
<td>2</td>
<td>50–110</td>
</tr>
<tr>
<td>Fluorescent lights – u-shaped</td>
<td>2</td>
<td>50–110</td>
</tr>
<tr>
<td>Fluorescent lights – spotlight</td>
<td>2</td>
<td>83–110</td>
</tr>
<tr>
<td>Halogen</td>
<td>1</td>
<td>83–110</td>
</tr>
<tr>
<td>Incandescent</td>
<td>4</td>
<td>87–110</td>
</tr>
<tr>
<td>Resistive load</td>
<td>2</td>
<td>75–110</td>
</tr>
<tr>
<td>TV; printers; fax</td>
<td>10</td>
<td>83–110</td>
</tr>
<tr>
<td>Computers</td>
<td>4</td>
<td>50–110</td>
</tr>
<tr>
<td>Microwave oven</td>
<td>2</td>
<td>75–110</td>
</tr>
<tr>
<td>UPS (what is this?)</td>
<td>1</td>
<td>77–110</td>
</tr>
</tbody>
</table>

*Note:  
* Elevator – Variable Speed was simulated with AC/DC Converter and DC motor set.  
# Elevator/escalator – M/G set was simulated with DC-DC M/G set in our lab.

#### 4.2 Load profile function formulation

The goal here was to formulate ‘load profile functions’ $P(V)$ and $Q(V)$. These are continuous functions that could be used to model each of the device categories of Table 1. Then, by combining them with appropriate weighting functions, they could be used to model a composite load. This provided the basic tool for modelling representative loads in secondary network distribution buses. These $P(V)$, $Q(V)$ functions, which are quadratic expressions in $V$, were based on studies by Hajagos and Danai (1998), and on the recommendations of the IEEE Task Force (IEEE Task Force on Load Representation for Dynamic Performance, 1995). This type of representation is called the ‘ZIP’ model. To incorporate functional-cutoff voltages ($V_{min}$), the ZIP model was modified using a special multiplier function $Y_v$, and the new model was termed the ‘Modified ZIP Model’. From $P(V)$ and $Q(V)$, one can calculate $I$, $Z$, $VA$ and $PF$.

The active and reactive power versus voltage data for the steady state were fitted to the following two ZIP models:
Characteristics of modern nonlinear loads

\[ P = P_0 \left[ Z_r \left( \frac{V}{V_0} \right)^2 + I_r \left( \frac{V}{V_0} \right) + P_r \right] \]  

\[ Q = Q_0 \left[ Z_r \left( \frac{V}{V_0} \right)^2 + I_r \left( \frac{V}{V_0} \right) + P_r \right] \]  

where \( P_0 \) and \( Q_0 \) are the rated real and reactive powers; \( Z, I \) and \( P \) are the constant ‘impedance’, ‘current’ and ‘power’ coefficients; \( V_0 \) is the rated voltage; and \( V \) is the actual load voltage. It is noted that the sum of the coefficients \( -Z + I + P \) is unity.

The following multiplier function was used in modifying the load model to accommodate low-voltage functional cutoff:

\[ Y_v = 0.5 \left[ 1 + \tanh \left( \frac{V - V_{min}}{V_0} \right) \right] \]  

and results in Equations (11) and (12).

\[ P = P_0 \left[ Z_r \left( \frac{V}{V_0} \right)^2 + I_r \left( \frac{V}{V_0} \right) + P_r \right] Y_v \]  

\[ Q = Q_0 \left[ Z_r \left( \frac{V}{V_0} \right)^2 + I_r \left( \frac{V}{V_0} \right) + P_r \right] Y_v. \]

\( Y_v \) is shown graphically in Figure 7 for the case \( V_0 = 120 \text{ V} \) and \( V_{min} = 70 \text{ V} \). It has the special property that it is nearly 0 when \( V < V_{min} \) and nearly 1 when \( V > V_{min} \). It is seen that the modified ZIP model describes the practical load situation better than the traditional load model.

**Figure 7** Behavior of Multiplier \( Y_v \) as a function of load voltage
As an example, a single ‘adjustable frequency drive’ load-unit was chosen. \( P \) and \( Q \) are given by quadratic equations that use an impedance coefficient, \( Z \), a current coefficient, \( I \), and a power coefficient, \( P \). The data given for that load are the following: \( V_0 = 120 \) V; \( VA = 1780 \) VA; \( PF = 0.79 \); \( V_{min} = 90 \) V; for active power: \( Z = 3.19, I = -3.84, P = 1.65 \) (the sum \( Z + I + P = 1 \)); for reactive power: \( Z = 1.09, I = -0.18, P = 0.09 \) (again, the sum \( Z + I + P = 1 \)). The modified equations are as follows:

\[
P(V) = 1780 \times 0.79 \left[ 3.19 \left( \frac{V}{120} \right)^2 - 3.84 \left( \frac{V}{120} \right) + 1.65 \right] \times 0.5 \left[ 1 + \tanh \left( \frac{V}{120} - 0.75 \right) \right]
\]

\[
Q(V) = 1780 \sqrt{1 - 0.79^2} \left[ 1.09 \left( \frac{V}{120} \right)^2 - 0.18 \left( \frac{V}{120} \right) + 0.09 \right] \times 0.5 \left[ 1 + \tanh \left( \frac{V}{120} - 0.75 \right) \right]
\]

Figure 8 below illustrates the behaviour of the above equations. Below the minimum voltage, \( 120 \times 0.75 = 90 \) V, \( P(V) \) and \( Q(V) \) become zero. The dashed lines show how the curves would look if Hajagos’ original equations (Hajagos and Danai, 1998) were used, that is, if our suggested multiplier, the hyperbolic function \( 0.5 \left[ 1 + \tanh \left( \frac{V}{120} - 0.75 \right) \right] \), were absent from Equations (13) and (14).

**Figure 8** Power curves for a single appliance as a function of voltage
Using the following relations:

\[
I_i = \frac{\sqrt{P_i^2 + Q_i^2}}{V_i}
\]  

(15)

\[ [P.F.] = \frac{P_i}{\sqrt{P_i^2 + Q_i^2}} \]  

(16)

\[ |Z_i| = \frac{V_i^2}{\sqrt{P_i^2 + Q_i^2}} \]  

(17)

one can also determine how the load current, power factor and impedance vary as a function of voltage.

5 Experimental tests

Each load was subjected to a range of voltages, varying stepwise in a slow ramp from 50% below to 10% above its rated value. For small loads, several devices could be tested simultaneously in parallel. The step tests were performed in accordance with the IEEE recommendations (IEEE Working Group in Nonsinusoidal Situations, 1996; IEEE Task Force on Load Representation for Dynamic Performance, 1995). Thirty loads in 17 categories were tested. Emphasis was placed on modern equipment loads such as heat pumps, computers, laser printers and microwave ovens. The laboratory tests were carried out for at least ten voltage levels. For a 120 V device, the voltage was stepped through the following: 60 V; 70 V; 80 V; 90 V; 100 V; 105 V; 110 V; 115 V; 120 V; 125 V and 130 V. For 208 V devices, the voltage was stepped through the following: 160 V; 170 V; 180 V; 190 V; 200 V; 205 V; 210 V; 215 V; 220 V; 225 V and 230 V. At each voltage level, the real power, reactive power, current and power factor were measured. Figure 9 below shows a typical steady-state data acquisition. Point 3, referred as to as \( V_{min} \), may be noticed where the loads began to drop out. The voltage was reduced further to Point 4, where all loads dropped out.

Figure 9   Typical steady-state data acquisition
6 Load model parameters development

A MATLAB program was used for curve fitting with the least square optimisation employing measured data points. The output of the curve fitting is a set of 3 parameters for each piece of equipment to simulate it by the modified ZIP load model. In this way, a ZIP model library was built for all 17 types of devices. Two sets of ZIP parameters were derived: ‘constrained’ and ‘accurate’. The ‘constrained’ model parameters were obtained as a fit to the measured data with the constraint that the sum $Z+I+P=1$. This produced load models most familiar to system planners. The ‘accurate’ parameters were fitted to the measured data with no constraints. Finally, the ‘constrained’ ZIP model was selected since it describes the situation in the field better. One example showing the difference between the ‘constrained’ and ‘accurate’ models is shown in Figure 10 together with the measured results.

Figure 10 Constrained and accurate ZIP models

![ZIP Models Curve](image)

Table 2 shows the load ZIP model coefficients derived from the curve fitting of measurements.
### Table 2
Load ZIP model coefficients derived from curve fitting

<table>
<thead>
<tr>
<th>Number</th>
<th>Devices</th>
<th>$S_0$(VA)</th>
<th>PF</th>
<th>$V_{min}$</th>
<th>Active power</th>
<th>Reactive power</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
<td>$Z$</td>
<td>$I$</td>
</tr>
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<td>1</td>
<td>Pump 1</td>
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<td>0.99</td>
<td>0.63</td>
<td>5.51</td>
<td>-11.3</td>
</tr>
<tr>
<td>2</td>
<td>Pump 2 (V.S.D.)</td>
<td>2153.7</td>
<td>1.00</td>
<td>0.91</td>
<td>-35.5</td>
<td>75.71</td>
</tr>
<tr>
<td>3</td>
<td>Fan 1</td>
<td>210.45</td>
<td>0.69</td>
<td>0.61</td>
<td>0.42</td>
<td>-0.04</td>
</tr>
<tr>
<td>4</td>
<td>Fan 2 (V.S.D.)</td>
<td>965.4</td>
<td>0.98</td>
<td>0.65</td>
<td>-0.96</td>
<td>3.05</td>
</tr>
<tr>
<td>5</td>
<td>Elevator</td>
<td>1976.4</td>
<td>0.51</td>
<td>0.85</td>
<td>-2.33</td>
<td>4.95</td>
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<tr>
<td>6</td>
<td>Escalators</td>
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<td>0.75</td>
<td>12.58</td>
<td>-26.3</td>
</tr>
<tr>
<td>7</td>
<td>Flu. (Magnetic)</td>
<td>94.8</td>
<td>0.99</td>
<td>0.76</td>
<td>-5.24</td>
<td>10.71</td>
</tr>
<tr>
<td>8</td>
<td>Flu. (Electronic)</td>
<td>77.7</td>
<td>0.99</td>
<td>0.50</td>
<td>-7.42</td>
<td>13.97</td>
</tr>
<tr>
<td>9</td>
<td>Flu. (U-shape)</td>
<td>84.1</td>
<td>0.97</td>
<td>0.50</td>
<td>-0.30</td>
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<tr>
<td>10</td>
<td>Flu. (Spot)</td>
<td>14.4</td>
<td>0.82</td>
<td>0.83</td>
<td>-0.64</td>
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<td>11</td>
<td>Halogen</td>
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<td>1.00</td>
<td>0.87</td>
<td>0.43</td>
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<td>13</td>
<td>Resistive Load</td>
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<td>1.00</td>
<td>0.75</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>14</td>
<td>TV; Printers; Fax</td>
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<td>1.00</td>
<td>0.83</td>
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<td>0.00</td>
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<tr>
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<td>17</td>
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<td>0.77</td>
<td>0.13</td>
<td>-0.14</td>
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</table>
7 Computer program for analysis of voltage reduction contingencies

In general, a load-flow program is quite large and complex. It is designed for making thorough studies of transmission and distribution networks of large systems. Using the load-flow program to examine one individual load, or the behaviour of a small commercial load, is possible but is very inefficient. Based on the modified ZIP load model suggested in Section 4, a separate, independent PC program was developed allowing for the study of individual load equipment (e.g., pumps, variable speed drives, fluorescent lighting) under varying voltage conditions, given the active power and reactive power at the rated voltage. It also allows combined loads (such as a single residential or commercial building) to be studied. Outputs from this program provide the following quantities as a function of the input voltage: real power, reactive power, apparent power, load impedance and feeder current. To represent a composite load consisting of several types of equipment, a program was written that combines these functions with proper weighting factors. Typical ‘commercial load’, or a typical ‘residential load’, could be represented. The program flowchart is shown in Figure 11.

Figure 11 Flowchart of computer aided program for load analysis
Figure 12 is the first user input form to receive input of power distribution in four areas including both a large and small commercial area, and a large and small residential area. The input data is transferred to the data bank, and the user has the option to input power percentage or kW distribution in these four areas. The program calculates the current, the power factor and the load impedance as a function of voltage. The program users can create different composite loads and obtain their characteristics. The output can be provided as set of curves either on screen or in hard copy. The program was written in Visual Basic.

**Figure 12** Input form of different area load distribution

![UserInputForm_AreaRatio](image)

The second user form of load distribution is shown in Figure 13. The program has the option of changing the load weighting factors for a combined load. For instance, a 100% weighting factor for electronic fluorescent lights will generate behaviour curves for that load only. Any other combination of weighting factors will generate a composite load that has its own characteristic behaviour.
Figure 13  Input form of 17 electric loads for load analysis

8 Summary

The electric load estimation problem was mathematically formulated and the existence of optimal solutions of the load distribution problem was proved. Field surveys were conducted to determine the typical commercial and residential electric loads. A representation of the load under low-voltage contingency conditions that is more accurate than the traditional ZIP load model was suggested. This was done by developing customer load versus voltage characteristics (i.e., for $P$, $Q$, $I$, $Z$, $PF$, $VA$ vs. $V$) as seen from the service point, for commercial and for residential loads. A function that will permit the above characteristics to be introduced into the secondary-network power-distribution load-flow computer program is presented.

A separate stand-alone computer program was developed that permits individual appliances to be modelled, as well as small combined loads. It allows an individual building, or a complex, or a load-centre to be modelled, so that its load-voltage characteristics (i.e., $P$, $Q$, $I$, $Z$, $PF$, $VA$ vs. $V$) can be studied without the need to use the main load-flow program.

Acknowledgement

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References


