Transmission Line–Based Loss-Free Resistor

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Abstract—The loss-free resistor (LFR) is a two-port network with a resistive input characteristic that transfers all the energy from the input to the output terminals. A novel LFR realization based on a transmission line, which we denoted as TLFR, is presented. The TLFR is suitable for the recycling of the energy of narrow, high-power, high-voltage pulses. Such pulses appear in some high-power lasers, such as the copper-vapor laser.

I. INTRODUCTION

The loss-free resistor (LFR) is a two-port network with a resistive characteristic at the input terminals that transfers all the energy from the input to the output. The first LFR realization has been achieved by the continuous control of circuits, characterized by a time-variable transformer or gyrator matrices [1], in such a way that a resistive characteristic was created at the input terminals [2]. It has been found lately that there is a group of switched mode circuits that "naturally" (without controller) exhibit LFR behavior [3]. Actually, both of the preceding realizations switched mode circuits are applied, with the merits and the disadvantages implied by the properties of the switching elements. Those circuits offer a lot of flexibility. They can be operated with various types of load characteristics at a wide range of input resistance. On the other hand, these circuits have limited voltage current and power ratings, slower dynamics, and good (but not excellent) efficiency, so they are not suitable for the processing of waveforms characterized by high voltage and power levels and high frequency.

In some cases there is a need to process such types of pulses. For example, in copper-vapor lasers (CVL) a train of narrow (0.4 μs) parasitic pulses [4] at high voltage and power levels (up to 20KV, 5MW) are generated. These pulses consume up to 40% of the CVL power, so their recycling by means of an LFR is convenient. For this, a new, more robust type of LFR was required, which was the motivation for the transmission-line–based LFR (TLFR) development.

The properties of the transmission line (TL) led us to the TLFR realization. Like the LFR, the uniform ideal TL is a loss-free two-port network with a resistive input characteristic. The TLFR is formed by terminating the TL using a suitably matched storage element with a voltage-source characteristic that recycles the energy of the pulses.

II. THE BASIC TLFR STRUCTURE

The basic TL-based LFR (TLFR) consists of a uniform TL terminated by a voltage source $E$ in series with a blocking diode (Fig. 1). A pulse applied to the input terminals travels along the TL. Part of its energy is absorbed by $E$, which recycles it. By suitable matching, it is possible to recycle all the energy of the pulse.

It should be noted that the matching conditions, in the case of voltage source termination, are different from those that are implied by a resistive termination. It can be shown that the matching should be done between the voltage value of the pulse (applied to the input) and the voltage source at the output of the TL. Actually, the voltage source, which powers the total system (whose parasitic pulses should be recycled), can be applied as the source element that recycles the pulses.

III. THE POWER FLOW IN THE TRANSMISSION-LINE–BASED LFR

Let us look at a uniform, loss-free transmission line (TL), terminated by an arbitrary one-port element. The TL equations are given as follows:

\[ L \frac{\partial I}{\partial t} = - \frac{\partial V}{\partial z} \frac{\partial^2 V}{\partial t^2} = \frac{1}{LC} \frac{\partial^2 V}{\partial z^2} \]

\[ C \frac{\partial V}{\partial t} = - \frac{\partial I}{\partial z} \frac{\partial^2 I}{\partial t^2} = \frac{1}{LC} \frac{\partial^2 I}{\partial z^2} \]

(1)

where $L$, $C$ are the inductance and the capacitance per a unit of length $z$, respectively, substituting

\[ Z_0 = \sqrt{\frac{L}{C}} \]

\[ v = \frac{1}{\sqrt{LC}} \]

(2)

The solutions of (1) are given by:

\[ V^+(t-z/v) = Z_0 I^+(t-z/v) \]

\[ V^-(t+z/v) = Z_0 I^-(t+z/v) \]

(3)

where $V^+$ is the voltage wave, which propagates in the positive direction (i.e. to the right) with a velocity $v$; $V^-$ is the reflected voltage wave, which propagates to the left; $I^+$ is the current wave, which propagates to the right; and $I^-$ is the reflected current wave, which propagates to the left.

Equation (3) can also be written in the following form:

\[ \begin{pmatrix} I^+ \\ I^- \end{pmatrix} = \begin{pmatrix} 1/Z_0 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} V^+ \\ V^- \end{pmatrix} \]  

(4)
For \( z = 1 \) (at the termination) we have
\[
V^+ + V^- = v_A
\]
\[
I^+ + I^- = i_A
\]  
(5)
where \( v_A, i_A \) are the voltage and current on \( A \) terminals, respectively, but
\[
I^+ + I^- = \frac{1}{Z_0} (V^+ - V^-)
\]  
(6)
which implies
\[
i_A = \frac{2V^+ - v_A}{Z_0}.
\]  
(7)
Equation (7) can be written as
\[
i_A = \frac{e(t) - v_A}{Z_0}
\]
\[
e(t) = 2V^+|_{z=1}
\]  
(8)
which means that the termination of the TL is viewed by \( A \) as a voltage source \( e(t) \) with an internal resistance \( Z_0 \), as shown in Fig. 3.

Let \( v_s(t) \) be a pulse, applied to the input terminals of the TL, i.e.
\[
V^+|_{z=0} = v_s(t)
\]  
(9)
At the TL termination, the propagating wave \( v^+ \) is given by
\[
V^+|_{z=1} = V^+|_{z=0}(t - \tau)
\]  
(10)
where \( \tau = 1/v = 1/\sqrt{LC} \), so
\[
e(t) = 2v_s(t - \tau)
\]  
(11)
By terminating the TL by a voltage source \( E \) and diode combination, it is possible to recycle the energy of the pulse. The diode prevents the TL from being changed by the source. The conduction mode of the diode is determined by the difference between \( e(t) \) and \( E \). It is convenient to apply the same source that powers the total system (which produces the parasitic pulses) as a recycling source \( E \). Defining "conduction function" of the diode \( D(t) \) as follows:
\[
e(t) > E \Rightarrow D(t) = 1
\]
\[
e(t) \leq E \Rightarrow D(t) = 0.
\]  
(12)
The current-through-the-voltage source \( E \) is given by
\[
i_E = D(t) \cdot \frac{e(t) - E}{Z_0} = D(t) \frac{2v_s(t - \tau) - E}{Z_0}.
\]  
(13)
The amount of the energy that is absorbed by the source \( E \) (the recycled energy) is given by
\[
W_a = \int_0^t E i_E \, dt.
\]  
(14)
Substituting (13) into (14) is
\[
W_a = \frac{1}{Z_0} \int_0^t E D(t) [2v_s(t - \tau) - E] \, dt
\]  
(15)
and the energy received by the input terminals of the TL is given by
\[
W_s = \frac{1}{Z_0} \int_0^t v_s^2(t) \, dt.
\]  
(16)
The recycling efficiency is given by
\[
\mu = \frac{W_a}{W_s}.
\]  
(17)
Among all the waveforms, the rectangular pulse whose amplitude equals \( E \) [defined by (18)], has a unique property; this pulse is totally absorbed, and no reflections occur
\[
0 < t < d \Rightarrow v_s = E
\]
\[
d < t < \tau \Rightarrow v_s = 0
\]
\[
d \leq \tau; \quad \tau = 1/\sqrt{LC}
\]  
(18)
where \( d \) is the pulse width.
Substitution of (18) into (5) implies that the reflected voltage \( V^- \) is zero, and substitution of (18) into (15) implies
\[
W_a = W_s = d \frac{E^2}{Z_0}
\]
\[
\mu = \frac{W_a}{W_s} = 1
\]  
(19)
In cases in which the voltage of the pulse differs from \( E \), reflections are caused due to the mismatch of the voltages.
IV. OPTIMAL OUTPUT MATCHING

In case of pulses with a nonrectangular waveform, given in a general form by \( v_s(t) \), it is possible to achieve optimal matching by applying a time-variable matching network. The simplest networks of this type are those that can be modeled by a time-variable transformer (TVT), whose transfer ratio \( n(t) \), is controlled in such a way that the following equation is satisfied:

\[
\alpha(t) = \frac{v_s(t - \tau)}{E}
\]

(20)

By doing so, continuous matching between the pulse (delayed by \( \tau \)) and the voltage source is achieved, so that all the energy of the pulse is absorbed and no reflections appear.

TVT realization is possible by means of switched mode circuits, such as buck, boost, flyback, and so forth, but that implies power, voltage level, and pulse-width limitations, so we will try to obtain results that are good enough by applying a conventional transformer with a fixed transfer ratio value. In the next section, it will be shown that it is possible to operate the circuit at a nonmatched mode and still recycle almost all the energy of the pulse by the absorption of the reflections.

V. THE MISMATCH IMPLIED REFLECTIONS

As mentioned above, the mismatch between the voltage of the absorbed pulse, \( v_s(t) \), which propagates along the transmission line, and the voltage of the recycling storage element \( E \), implies reflections. Defining the parameter \( \alpha(t) \) as follows:

\[
\alpha(t) \equiv \frac{v_s(t - \tau)}{E}
\]

i.e., \( \alpha(t) = n(t) \)

(21)

Let us analyze those reflections as a function of \( \alpha \). We will also assume that the pulses source has a pure voltage source characteristic, which means that at zero input voltage, a short circuit appears at the input terminals.

5.1 \( 0 \leq \alpha \leq 0.5 \)

At this range of \( \alpha \), (13) indicates that the current flowing through \( E \) is zero, so an open circuit appears at the TL termination. The reflected pulse \( V^- \) travels along the TL; at the input its polarity is reversed by the short circuit, and it continues to travel forward and backward along the TL. Since the pulse (and its reflections) can't be absorbed by the storage element, its energy remains "trapped" in the TL.

5.2 \( 0.5 < \alpha < 1 \)

In this case, the diode is in the conducting mode (at the first propagation of the pulse only), and part of the energy is absorbed. The power applied to the recycling element is given by

\[
p_a(t) = \frac{E^2}{2z_0} [2\alpha(t - \tau) - 1]
\]

(22)

And the recycled energy is

\[
W_a = \int_{0}^{r+d} p_a(t) \, dt
\]

(23a)

where \( d \) is the pulse width.

The amplitude of the reflected pulse is given by

\[
V^- = (1 - \alpha)E
\]

(23b)

And since the amplitude is smaller than \( E/2 \), the reflection remains trapped on the TL.

5.3 \( 1 < \alpha < \infty \)

In cases in which \( \alpha \) has a large value, part of the pulse is absorbed, and the reflection has a negative value with an amplitude that is given by the amplitude of the pulse, reduced by the value of \( E \). At the input terminals, the pulse polarity is reversed, and it propagates once again toward the source \( E \). Each traveling part of the pulse is absorbed, and the amplitude of the reflection is "chopped" by the value of \( E \) until it goes below the value of \( E/2 \), and then remains trapped. Denoting the number of the "travels" of the pulse or its reflections by \( j \), the amplitude of the \( j \)th reflection is calculated as follows:

\[
V^-_j = (V^+_j - E)
\]

(24)

Equation (24) implies

\[
V^+_j = V^+_j + E
\]

(25)

Let us split \( \alpha \) into the integer part \( a \) and the remainder \( b \)

\[
\alpha = a + b
\]

(26)

Equations (24–26) imply

\[
V^-_j = -(\alpha - j)E
\]

(27)

The power that is absorbed by \( E \) at the \( j \)th collision is given by

\[
p_{aj}(t) = \frac{E^2}{2z_0} [2\alpha(t - \tau_j) - 2j + 1]
\]

\[
\tau_j = (1 + 2j)
\]

(28)

And the absorbed energy by

\[
W_{aj} = \int_{\tau_j}^{\tau_j + d} p_{aj}(t) \, dt
\]

(29)

where \( d \) is the pulse width.

And the total recycled energy is given by

\[
W_a = \sum_{j=1}^{a} W_{aj}
\]

(30)
VI. THE RECYCLING EFFICIENCY
The recycling efficiency is given by

$$\mu = \frac{W_a}{W_s}$$

(31)

where $W_s$ is the energy of the pulse

$$W_s = \int_0^d v^2_a \, t \, dt = \frac{E^2}{Z_0} \int_0^d \alpha^2(t) \, dt$$

(32)

The efficiency for each waveshape can be calculated applying (28–32), but in order to see clearly how the efficiency depends on $\alpha$, let us calculate $\mu$ for constant values of $\alpha$ (rectangular pulses)

For constant value of $\alpha$, $W_s$ is given by

$$W_s = \frac{d\alpha^2E^2}{Z_0}.$$  

(33)

Equations (22), (23), (28)–(30) imply

$$0 < \alpha < 0.5 \Rightarrow \mu = 0$$

$$0.5 \leq \alpha \leq 1 \Rightarrow \mu = \frac{2\alpha - 1}{\alpha^2}$$

$$1 \leq \alpha < \infty \Rightarrow \mu = \frac{1}{\alpha^2} \sum_{j=1}^\infty 2(\alpha - j) + 1.$$  

(34)

Substituting $\alpha = a + b$, and making the summation we have

$$1 < \alpha < \infty \Rightarrow \mu = \frac{\alpha^2 - b^2}{\alpha^2}.$$  

(35)

The efficiency as a function of $\alpha$ is shown in Fig. 4. Looking at this figure, we observe that the efficiency tends to increase as $\alpha$ increases, so by operating the circuit at high values of $\alpha$ it is possible, in principle, to recycle almost all the energy of the pulse by the absorption of the reflections. Practically, operation of real elements implies some losses; an increase in $\alpha$ implies more reflections, which results in an increase in the conduction losses, so there is a tradeoff in the choice of the value of $\alpha$.

The values of $v_s(t)$ and $E$ are both determined by the circuit whose pulses should be recycled. $E$ is actually the voltage source that powers the total system, so no additional source is required for the TLFR realization. The suitable value of $\alpha$ can be achieved by applying a pulse transformer, which couples the TL termination to the recycling source.

VII. THE TLFR MODEL
It has been found that an LFR, which is based on a controlled transformer can be modeled by a two-port network, consisting of a resistive element at the input terminals and a controlled power source at the output as, shown in Fig. 5 [3].

The power source is defined as a one-port element that supplies constant power. Like the voltage and current source, the power source can also be dependent; in the case of LFR the output power value is controlled by the input power. This type of LFR is a power conservative two-port circuit, which belongs to the family of POPI (Pout = Pin) networks [3]. Also, the TLFR is a loss-free two-port network. It is energy conservative, but not instantaneously power conservative. The instantaneous power imbalance is a result of the transmission-line delay.
In case of the optimally matched TLFR, (applying a controlled, time-variable transformer) the TLFR is described by a two-port network with a resistive element at the input and controlled power source at the output, whose power value $P_{0}(t)$ is determined by the power (delayed by $\tau$) that is absorbed at the input terminals.

$$
P_{0}(t) = P_{i}(t - \tau)
$$

$$
\tau = 1/\sqrt{LC}
$$

$$
P_{i} = v_{s}^{2}/Z_{0}
$$

(36)

All practical circuits that realize the TLFR do not include a controlled, time-variable matching network, so the recycling is achieved by the absorption of the pulse and its reflections. As mentioned previously, it is possible to apply the voltage source, which powers the total system as a storage element $E$, which recycles the pulses. No additional voltage source is required for the TLFR realization, since the source element is already embodied in each system that produces a parasitic pulse that should be recycled. Since the values of $E$ and $v_{s}(t)$ are determined by the system (which generates the pulses), the suitable value of $\alpha$ can be obtained by applying a pulse transformer whose transfer ratio is $n$.

$$
\alpha(t) = v_{s}(t - \tau)/nE
$$

(37)

As mentioned above, the integer part of $\alpha$, denoted by $a$, determines the number of the reflections that can be recycled.

The power applied to the output terminals by the $j$th reflection, $P_{0j}$, given in (28), can be written as follows:

$$
P_{0j}(t) = A_{j}P_{i}(t - \tau_{j})
$$

$$
\tau_{j} = \tau(1 + 2j)
$$

$$
P_{i}(t) = v_{s}^{2}(t)/Z_{0}
$$

$$
A_{j} = \frac{2(\alpha - j) + 1}{\alpha^{2}}.
$$

(38)

The group of the power sources [Fig. 8(a)] can be described by a single power source [Fig. 8(b)] whose power value is given by

$$
P_{0}(t) = \sum_{j=1}^{a} P_{0j}(t) = \sum_{j=1}^{a} A_{j}P_{i}(t - \tau_{j})
$$

(39)

where $a$ is the integer part of $\alpha$.

VIII. SIMULATION AND EXPERIMENTAL RESULTS

The process of absorption of various pulses by the resistive input of TLFR, and the recycling of their energy has been simulated applying Microcap III software. The waveforms of the pulses, the reflections, the input and output currents, and the absorbed and recycled energy are presented in Figs. 9–19. As expected, it has been shown that by utilizing ideal (loss-free) elements, it is possible to achieve recycling efficiencies that tend to one: in the case of rectangular waveshapes the efficiency reaches one. Also nonideal elements, with some
losses, have been simulated, and it has been shown that by using high quality (but still realistic) elements, it is still possible to achieve very high recycling efficiency. The results have been experimentally verified by applying rectangular pulses to a TLFR consisting of a coax cable (RG 58) whose characteristic impedance is 50 Ω and whose length is 90 m, diode CGOSAN814, and matched voltage source of 15V.

The recycling efficiency in this case was about 85%; similar type of TLFR, in which the transmission line was replaced by 12 LC units in cascade connection, was operated at a recycling efficiency of over 90%.

IX. DISCUSSION

Realization of transmission-line–based loss-free resistor, TLFR, has been described, and the realization concept has been verified by simulation and experimentally.

The TLFR is a totally new type of LFR, its structure is very simple, and it can be operated at high efficiency at high voltage and power levels.

The development of the TLFR has been motivated by the need for recycling a “train” of parasitic pulses that arises...
X. CONCLUSIONS

A new type of LFR realization has been described. The transmission-line-based LFR (TLFR) is suitable for application at high power, high voltage conditions as well as at low power and voltage levels. The TLFR is suitable for the recycling of the energy of narrow pulses and can find applications in high-power laser systems, snubbers, and waveshaping circuits.

REFERENCES


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