Defining the Unique Signatures of Loads using the Currents’ Physical Components Theory and Z-Transform

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Abstract— A method to extract unique features from measured waveforms of non-linear loads is presented. These features can be used for the identification of loads connected to the grid. The method is based on the currents’ physical components (CPC) electric power transport theory combined with the Z-transform (for implementation using digital signal processing (DSP)). A set of admittance-based Z-transform functions that reflect the current physical components suggested by the CPC is constructed. The resulting transfer functions are shown to reflect the electric physical significance and are used for electric load and machine identification through its electric characteristics. In this paper, the Z-transform analytic expressions are developed and extended along with their physical comprehension. Moreover, the strength of the theory over more traditional spectral analysis is explained. The method presented in this paper is demonstrated and analyzed using real-world measured waveforms (measured by power quality monitors).

Index Terms— Load Identification, Power transfer theory, Smart Grid.

I. INTRODUCTION

The great interest in smart grids in recent years has given rise to new research areas and to a renewed interest in the fields of power systems and energy transport theories. Smart grids and increased penetration of renewable power sources [1], require increased power grid monitoring capabilities as well as the diagnostics of the various loads connected to the grid. In residential as well as in commercial and industrial environments, there are numerous loads with different imprints on the grid and on microgrids [1][2], [3]. It is not

only the loads’ nature that influences these imprints but also the characteristics of the power converters that interface it to the grid [4], [5]. Power monitoring an load identification bear the potential to also improve power quality (complementing PFC and harmonics cancelation methods [6]) There are a few existing methods for load identification in power grids. There are several studies on load identification: some studies focus on fault events and power quality events identification, partially with machine learning [7]-[10]. Other studies on load identification were performed using non-intrusive load monitoring and identified electric loads by learning the energy consumption and pattern identification of the load profile [11]-[14]. There have been several studies of power quality events identification using wavelets and fuzzy logic [15]-[17] as well as of load identification by measuring the harmonic impedances and the current and voltage harmonic parameters [18], [19]. Furthermore, there has been work on the system identification of specific electric machines using active filters and data summarization [20]-[23]. Recent technological advances have augmented the performance of power quality monitors and some smart meters beyond just monitoring power quality events[24]-[26]. In this paper, a new generic method for electric load identification is presented. The method is used for identifying loads during their regular operation as well as during periodical faults. The method is based on the currents’ physical components (CPC) theory as well as an enhancement via the Z-transform. The CPC theory is an advanced electric power transport theory that was initially presented in 1988 [27]-[30]. In this paper, the theory is expanded by first using the Z-transform for system identification and also by using digital signal processing (DSP) tools for enhancing the applicability of the theory. The Z-transform has been widely used in other disciplines, for example, in radar algorithms [31], [32] and speech recognition algorithms [33], but it has not been projected into electric power. Other works use the Z-transform in the context of electric machinery and electro-magnetic phenomena identification [34], [35], control theory and filter design [36].

Combining the CPC theory and Z-transform has several merits that make the presented theory successful for load identification in comparison to other theories: a) CPC reflects the currents’ physical components, which are components that bare a physical meaning. Although the validity of these components and their physical meaning were questioned because they are not entirely separable [30], it can be shown...
that the decomposition maps can be used uniquely to characterize the waveforms. Therefore, the transfer functions produced by the CPC theory either have a physical meaning or are mathematically equivalent to the original waveforms, making them useful for the identification of the characteristic signatures of electric machines and other loads. b) The Z-transform displays the overall spectral picture. Therefore, a transfer function that presents the entire spectrum at each point Z has an improved likelihood to observe fine system characteristics. c) The merging of the digital signal processing (DSP) enables the application of powerful tools to the CPC theory in particular and to power theories in general. d) The orthonormality of the current physical components base maintains the benefits of the Fourier base. The current physical components maintain the same two rules: (1) the total load current is the sum of the physical currents, and (2) the sum of squares of the currents is equal to the total current square. These rules imply that the current components are independent. As a result, the theory proposed herein has advantages over the conventional spectral treatment or even the conventional total admittance Z-transform. e) the merging of Z-transform and CPC enhances meaningful grid indicators such as load unbalance and backward current (which will not be detected by Z-transform only). Moreover, it may enhance the unique signature in the case the signature is weak and contained only in one of the proposed components. g) The theorems that are presented and developed in this paper and the use of the DSP tools enable the deduction of unique load features.

The massive use of power electronics in modern loads requires us to address their impact [38]. Non-linear loads generate distorted currents, which in turn insert voltage distortions. Nonetheless, these distortions are periodic, i.e., they are harmonic distortions [39]. One objective of this work is to express the CPC power theory in terms of the Z-transform. Then, building on this representation facilitates the accomplishment of the main objective: identification of the load power properties. Moreover, we developed a generic theory and procedure for the unique identification of these harmonic loads.

In Section II, the CPC theory is briefly presented. In Section III, the enhancement of the CPC theory by the addition and the new formulation via the Z-transform is described. In Section IV, the insights for load identification are explained. Finally, in Section V, the experimental results are shown for the implementation of the theory.

The computations proposed herein for load identification are possible for implementation as a module in the Metering Data Management (MDM) central software, or in an additional module let it be firmware of hardware + firmware inside the meters themselves.

II. CURRENTS’ PHYSICAL COMPONENTS OUTLINE

The current’s physical components (CPC) theory is a well-known power theory; see for a comprehensive presentation of this theory.

A general way to express the voltage and current waveforms in their Fourier series form (in a linear time invariant (LTI) harmonic system) is given in (1) and (2) respectively.

\[ v(t) = U_o + \sqrt{2} R e \left\{ \sum_{m=0}^{N} V_m e^{-2\pi imt/T} \right\} \]

\[ i(t) = I_o + \sqrt{2} R e \left\{ \sum_{m=0}^{N} I_m e^{-2\pi imt/T} \right\} \]

\[ = \frac{1}{Y_o} U_o + \sqrt{2} R e \left\{ \sum_{m=0}^{N} \frac{1}{Y_m} \cdot V_m e^{-2\pi imt/T} \right\} \]

where \( V_m \) and \( I_m \) are the voltage and current phasors. \( m \) are all the harmonic indices in which there are current components in the frequency domain, and \( Y_m \) is the admittance for the \( m \)-th harmonic, which can be expressed as a real \( G_m \) part and imaginary \( jB_m \) part as follows:

\[ Y_m = G_m + jB_m = \frac{I_m}{U_m} \]

Eq. (3) suggests a practical method of extraction of admittances while recording voltage and current waveforms of an unknown load.

The theory is valid for LTI systems as well as for harmonic generating loads (HGL) and quasi-periodic signals with a slowly varying envelope in comparison to the network period. It is important to note that for HGL (namely nonlinear systems), there might be various harmonics existing in the current; however, they are non-existent in the voltage. In the CPC theory, the total current is decomposed of several physical components that are independent and orthogonal:

\[ i(t) = i_a(t) + i_r(t) + i_s(t) + i_q(t) + i_b(t) \]

where \( i_a \) is the active current, \( i_r \) is the reactive current, \( i_s \) is the scattered (active) current, \( i_q \) is the unbalanced current (in three-phase systems), and \( i_b \) is the backward load generated current.

The assembly of the current by its physical components yields a possibility to enhance the identification of different loads by means of these currents, their respective values and their various transfer functions, as will be demonstrated next.

III. ENHANCEMENT OF THE CPC THEORY USING THE Z-TRANSFORM

First, the current expression (2) is written by the Z-transform (neglecting the DC component \( I_0 \)) as follows:

\[ I(Z) = \sum_{m=0}^{N} I_m Z^{-m} \]

This expression includes all the harmonics simultaneously. Each harmonic has its own weight at the Z polynomial.

Now, by applying \( Z \) as: \( Z = \frac{1}{Y_0} e^{-2\pi imT} \), expressions (1) and (2) can be written as the actual Z-transform of the digitally sampled signals as:

\[ V(Z) = U_o + \sqrt{2} R e \left\{ \sum_{m=0}^{N} V_m Z^{-m} \right\} \]
In the case when the load is LTI, then \( M \) in (6) equals \( N \) in (7).

Next, let us consider the problem of any electric load as a problem of the transfer function, where the voltage is the input signal, while the current is considered as the system's output, as seen in Fig. 1.

\[
\begin{align*}
V(Z) & \rightarrow Y(Z) \rightarrow I(Z)
\end{align*}
\]

Fig. 1. Visualization of a load as a system with an input voltage and output current.

The transfer function of a general load can be written as:

\[
Y(Z) = \frac{I(Z)}{V(Z)} = \frac{I_o + \sqrt{2} \text{Re} \left\{ \sum_{n \in N} I_n Z^{-n} \right\}}{U_o + \sqrt{2} \text{Re} \left\{ \sum_{n \in M} V_n Z^{-n} \right\}} = \frac{A_N(Z)}{D_M(Z)}
\]

where \( A_N(Z) \) is the numerator polynomial of rank \( N \) (depending on the current harmonic content) and \( D_M(Z) \) is the denominator polynomial of \( M \) rank (depending on the voltage harmonic content). In the \( Z \)-transform terminology, the CPC leads to a rational polynomial. The \( Z \)-transform preserves several of the CPC original properties as in the Fourier representation. The current orthonormality and decomposition ability are maintained. The following discrete Fourier transform (DFT) relations hold for CPC as well as for most power transport theories:

\[
S^2 = \sum_j P_j^2 + \sum_k Q_k^2
\]

\[
i = \sum_j i_j + \sum_k i_k
\]

\[
\|i\|^2 = \sum_j \|i_j\|^2 + \sum_k \|i_k\|^2
\]

where \( j \) relates to all the active harmonic components of the power and current and \( k \) relates to all non-active harmonic components of the power and currents. At the \( Z \) domain, the following CPC presentation of the current can be written as:

\[
i(Z) = i_a(Z) + i_s(Z) + i_r(Z) + i_d(Z) + i_b(Z)
\]

and because the components are orthogonal, the following expression applies:

\[
\|i(Z)\|^2 = \|i_a(Z)\|^2 + \|i_s(Z)\|^2 + \|i_r(Z)\|^2 + \|i_d(Z)\|^2 + \|i_b(Z)\|^2
\]

Dividing the current expression by the total voltage yields:

\[
Y(Z) = Y_a(Z) + Y_s(Z) + Y_r(Z) + Y_d(Z) + Y_b(Z)
\]

Expressions (11) and (12) imply the orthogonality of the various components of the admittance transfer functions, namely:

\[
\|Y(Z)\|^2 = \|Y_a(Z)\|^2 + \|Y_s(Z)\|^2 + \|Y_r(Z)\|^2 + \|Y_d(Z)\|^2 + \|Y_b(Z)\|^2
\]

where:

\[
Y_k(Z) = \frac{i_k(Z)}{V(Z)}
\]

and \( k \) represents each of the physical components in the CPC theory. Expression (13) is not mentioned in the original CPC theory and is introduced as an extension here. It is important to understand that only the current as in (4) is physically maintained, as indicated by Fig. 2. The admittance in (12) is not physically maintained. It can be viewed as a normalization of the currents by the total voltage \( V(Z) \) or as seen in Fig. 3, as parallel-connected branches, which are connected to a common voltage \( V_{\text{Total}} \). The various transfer functions (admittances) in (12) are independent of one another and enable the separation of a multi-problem into separate admittance physical components. Moreover, these admittances (transfer functions) enable separate handling of the various transfer characteristics per simultaneous phenomena. It is very important to note that the CPC by means of DSP (CPC-DSP) is not simply a \( Z \)-transform of the CPC for the transfer function \( Y(Z) \) but rather is an extension.

To summarize the information above, it can be observed that in the original theory of the CPC, the current \( Z \)-transform of (7) is preserving the CPC, but the transfer function at the CPC is maintaining single harmonic relations:

\[
Y_n = I_n / V_n (n \in N)
\]

Equation (8) suggests that the total current is related to the total voltage \( V(Z) \) as an input, and (14) implies that the current physical component \( i_k(Z) \) is related to the total voltage \( V(Z) \) as an input. For the conventional CPC, the \( Z \)-transform is preserved at the current physical components level \( i_k(Z) \). It differs from conventional CPC
because the total voltage $V(Z)$ is considered. For the total current, (8) is valid if the load is viewed as a transfer function from the input voltage to the current, as the CPC implies.

IV. ANALYTICAL EXTRACTION OF INFORMATION FOR ELECTRIC LOAD CLASSIFICATION

A. General

The work presented in [33], [35] may explain in more detail the theory of how the zero-pole pairs affect the spectra. However, complementary additions are presented in this section. A series of equations and notifications are presented in the next subsections, and the following is demonstrated: a) The electrical characteristics of an electric load (or machine) is comprehended from the transfer function zeroes and poles. b) All of the characteristic spectral features of a machine load’s transfer function are calculable from the zero-pole diagram. Analytical expressions for the magnitude response, phase response, group response, phase delay and transfer function are developed.

The analysis enables an investigation of several electric properties of loads. These results are significant from many aspects, especially in the case of classifying and identifying loads. The results are known from other disciplines, such as speech recognition [33]. The theorems are presented next in a format suitable for electric machine identification. The result of the zero-pole theorems is that at the spectra of DSP, features of the load transfer function are: amplitude response, phase response, group delay, phase delay - the zeroes and poles usually appear in pairs. The upper bound on the number of zeroes/poles is the systems sample rate $N$. The actual count does not exceed the number of harmonics at the voltage (denominator) and current (enumerator) waveforms. That count is substantially less than the sampling rate. Next, the analysis is shown to be a general qualitative analysis for demonstrating the theory of uniquely defining the signature of the loads.

B. Zero-pole unit-like circle

De Moivre's theorem states that the $n$ complex roots of 1 are located on the unit circle and are equally spaced in terms of the phase angle. In the Z-transform presentation of various loads, as presented in Fig. 4, the zeroes are marked with an ‘o’ and represent the roots of the current polynomial. The poles are marked with an ‘x’ and represent the roots of the voltage polynomial. When applying the Z-transform over a pure resistive load, the voltage polynomial has the exact same roots as the current polynomial, and all roots are equally spaced on a unit-like circle.

It should be mentioned here that the “unit like” circle in this paper does not refer to the range of Convergence (ROC) but rather to the loci of zeros and poles of the admittance transfer function.

The circle radius is determined by the admittance of the load, as seen in Fig. 4(a). When inductive or capacitive components are introduced, the roots of the current polynomial are phase shifted from the voltage polynomial and are still on the unit like circle as shown in Fig. 4(b). This is because of the change in the reactive admittance at each harmonic. From Fig. 4, it is evident that the matching zero-pole pairs represent the RLC linear behavior of the load. Note that the number of zeroes (current) and number of poles (voltage) are equal. The escaped zeroes, as in Fig. 4(c), represent the nonlinear behavior (HGL) of the load.

In this case, the number of zeroes is larger than the number of poles because there are additional current harmonics.

When introducing slight non-linearity to the load, some zeroes (current roots) “escape” away from the unit-like circle, as seen in Fig. 4(c). As the load tends to higher non-linearity, more zero-pole pairs will escape from one another and from the unit-like circle, and the distance in terms of the radius and phase angle will increase. This characteristic of the zero-pole map is important for the unique identification of various loads. It is not obvious at all, at a first glance, as to why a general polynomial of the denominator has on a regular basis the De-Moivre's roots distribution:

![Fig. 4: (a) Zero-pole diagram of a resistive load. (b) Zero-pole diagram of a resistive-inductive load. (c) Zero-pole diagram of a non-linear load in which the current harmonics are different from the voltage harmonics.](image-url)
The resolution of the theorem is simple when one observes that the zero poles are spread according to angle \( \theta \), where it is linear with harmonic \( \omega = \alpha \).

C. Amplitude-response zero-pole analysis

The amplitude response spectra can be written in a closed form as follows:

\[
\log|Y(Z)| = \sum_{n=1}^{N_I} \log|Z-Z_n| - \sum_{n=1}^{N_V} \log|Z-Z'_n| = \log \left\{ \prod_{n=1}^{N_I} |Z-Z_n| \right\} - \log \left\{ \prod_{n=1}^{N_V} |Z-Z'_n| \right\}
\]

(16)

The expression in (16) is well known [33]. The expression separating \( N_I \) and \( N_V \) is a general one. It emphasizes the fact that there is a different number of poles and zeroes.

D. Analytic analysis of the group-delay

In [33]-[36], the group delay feature is used for speech recognition and radar pattern analysis. In these papers, the analytical results are limited to up to two pole–zero pairs or are used as an empiric result for the general case. Next, the theory is presented in the general case.

Let \( Y(Z) \) be a transfer function as in (8). The zero-pole representation of this expression is as follows:

\[
Y(Z) = \prod_{n=1}^{N_I} (Z-Z_n) \prod_{n=1}^{N_V} (Z-Z'_n)
\]

(17)

where \( N_I, N_V \) are the current and voltage polynomial order accordingly. Then, the expression for the spectra of the group delay can be written as:

\[
\frac{\partial \theta(Z)}{\partial \omega} = \sum_{n=1}^{N_I} \frac{-1}{1+\varphi_n(\omega)} \frac{\partial Z}{\partial \omega} - \sum_{n=1}^{N_V} \frac{-1}{1+\varphi_n'(\omega)} \frac{\partial Z}{\partial \omega}
\]

(18)

where:

\[
\varphi_n(\omega) = \frac{\text{Im}((Z-Z_n)(\omega))}{\text{Re}((Z-Z_n)(\omega))},
\]

\[
Z = e^{-j\theta} = e^{-j\omega}
\]

(19)

The proof for (18) is based on: (a) a closed form expression for the derivative of: \( \tan^{-1}(X) = 1/(1+X^2), [\tan^{-1}(X)]' = 1/(1+X^2) \) and (b) the \( \tan^{-1} \) of a product of variables is the sum of the \( \tan^{-1} \) of each variable, and the \( \tan^{-1} \) of a division of two variables is the subtraction of the \( \tan^{-1} \) of each variable. Equation (18) states that there is a direct link between the zero-pole diagram and the group-delay spectra, and it may be computed. Furthermore, the physical implication is that most zeroes and poles are coupled into identical zero-pole pairs in the form: \( (Z-Z_n)/(Z-Z'_n) \thicksim 1 \). When the zeroes and poles deviate from one another, they generate a local minimum and local maximum, as seen in Fig. 5. The interpretation of this characteristic will be demonstrated in the experimental results section later.

\[
\theta(Z) = \sum_{n=0}^{N_I} \left\{ \tan^{-1}(Z-Z_n) - \tan^{-1}(Z-Z'_n) \right\}
\]

(20)

The phase delay is defined as:

\[
\text{phase-delay} = -\frac{\theta(Z)}{\omega}
\]

(21)

F. Phase-response zero-pole analysis

The generalized formulation for the phase response is presented in [33].

\[
Y(Z) = \prod_{n=1}^{N_I} (Z-Z_n) e^{j\theta(Z-Z_n)} \prod_{n=1}^{N_V} (Z-Z'_n) e^{j\theta(Z-Z'_n)}
\]

(22)

Equation (22) can be now separated into the amplitude and phase spectra as follows:

\[
|Y(Z)| = \frac{\prod_{n=1}^{N_I} |Z-Z_n|}{\prod_{n=1}^{N_V} |Z-Z'_n|}
\]

(23)
\[
\theta(Z) = \sum_{n=0}^{N} \left\{ \tan^{-1}(Z - Z_n) - \tan^{-1}(Z - Z_{n+1}) \right\}
\]  

(24)

G. Procedure summary

Next, the entire procedure of extracting unique features to the measured time domain voltage and current waves by means of the above-mentioned theory is summarized. The process is shown in Fig. 6, as a block diagram, which illustrates the flow of data and the analytical and numerical procedures in each stage. The time domain voltage and current waves \(v(t)\) and \(i(t)\) are first transformed to the frequency domain by the FFT module and are normalized. Note that FFT is a technical algorithm for fast computation of Discrete Fourier Transform (DFT) and its inverse. Therefore we applied FFT coefficients. The spectra of these waves \(V_n\) and \(I_n\) are treated in the next module with the CPC and then proceed to the Z domain by the Z-transform in accordance with the theory presented in (1)-(7) and (10).

![Feature extraction block diagram](image)

The resulting current components along with the voltage \(V(Z)\) yield the admittance transfer function components as in (12) and (13). For each individual admittance transfer function, five features are generated (as seen on the right hand side of Fig. 6). These features are the zero-pole diagram, amplitude, phase, phase delay and group delay as shown in sections B-E.

Using the outcome of CPC and Z-transform suggests a wealth of uniquely identifiable features. Looking at (19), (20), (21), (24), shows that phase features of Z-transform are usually more informative than amplitude features. The phase features bear a linear relation to zero-pole pairs, while the amplitude features, as shown in (16), carry a logarithmic relation to zero-pole pairs, making them less sensitive to unique signatures. It has also been shown that the active component spectrum is uniform, therefore carrying little signature information, while active scattered spectrum is carrying variable spectrum. Finally, in a case where several components are unique to the load, it is not mandatory to use all of them.

We have implemented the computations according to diagram in Fig. (6) in a remote server at the Metering Data Management (MDM) system.

V. EXPERIMENTAL RESULTS

Next, a series of electric machines are analyzed using the above-mentioned theory. The current and voltage waveforms were measured using SATEC EM720 power quality monitors. These devices are certified for power quality monitoring and energy metering international standards with a 256 samples/cycle sampling rate.

A. First test case: computer center

The measured waveforms of a computer center are shown in Fig. 7. The waveforms are analyzed with a Matlab simulation of CPC-DSP code, which creates the transfer functions of the various current physical components. Then, each transfer function is analyzed by the MATLAB FDA-tool, which produces the various Z-transform spectral features. Observing the waveforms in Fig. 8, it is evident that the voltage is pseudo-linear with additional harmonics added to the pure sine wave. However, the current is pulsating and highly non-linear because of the converters’ characteristics.

![Phase voltage and current waveforms of a computer center](image)

Fig. 7: Phase voltage and current waveforms of a computer center. The three upper waveforms are the phase voltages, and the three lower waveforms are the phase currents.

![Zero-pole diagram of the active and scattered transfer function of the computer center waveforms](image)

Fig. 8: Zero-pole diagram of the active and scattered transfer function of the computer center waveforms.
We first present the DSP spectral features developed for the: $G_{\text{active-conservative}}(Z) = G_a(Z) + G_s(Z)$.

This is performed because the active component by itself has insufficient information (because it is a constant number $G_e$). It is easy to show that the sum of $G_a$ and $G_s$ is equivalent to the conservative active power [40]. The zero-pole diagram in this case is depicted in Fig. 8 where it can be seen that the poles ($x'$) are equally distant along a circle. The highlighted "escaped" zeroes appear in pairs and are directly represented by the peaks at the phase spectrum according to the theory above. Moreover, the remaining zeroes (current), which are slightly phase shifted relative to the poles (voltage), represent a linear behavior of the load, which includes reactive components.

The various DSP spectra of the non-linear load are presented in Fig. 9.

In each graph, black dashed lines are added to emphasize the location of the changes and interest in the various spectra. It is clear from Fig. 9 that in all spectra, the location of the peaks or graph change is at the same location regarding the frequency axis. In addition to noting the above-mentioned theory, there are other observations: a) the group delay presented in Fig. 9(a) for the non-linear load of Fig. 7 is characterized by sharp peaks. The sharpness of the peaks is directly related to the strong non-linearity of the load. The further the zero escapes, the sharper the peak is (as shown in Fig. 5). These sharp peaks will be compared to the results of a relatively linear load later.

b) Observing Fig. 9(a)-(d) and from the developed theory, it is evident that the more interesting and uniquely defined information of the load is located at the phase spectrum functions. The magnitude according to (24) is also influenced by the zero-pole pairs, but it is much less observable because $\log|G(Z)|\log|G(Z)|$ is a highly smoothing and descaled function. A weak effect in the extreme case barely causes an amplitude response but is noticeable in the phase spectrum.

**B. Second test case: electric AC motor.**

Next, we consider the voltage and current waveforms measured by the same equipment. The measurements were taken in a site containing a large AC motor with a motor drive. Because the motor drive consists of power electronics, nonlinearity is introduced into the motor. The zero-pole diagram of this load is presented in Fig. 11.

![Fig. 10: Voltage and current waveforms of an AC motor that is a linear load with an additional HGL.](image-url)
At first glance, it seems that the zero-pole diagram does not include "escaped" zeroes and that the poles highly match the zeroes.

Examining the results further, it is noticeable from Fig. 11(b), which is a zoom-in view of the same diagram in Fig. 11(a), that there are weak escapes of zeroes from the unit-like circle at the marked locations in Fig. 11(b). This means that this load includes a small amount of non-linearity.

Next in Fig. 12 (as in Fig. 9), the various DSP spectra of this load are presented for each graph. The black dashed lines are also added to emphasize the location of the changes and interest in the various spectra.

The spectrum in Fig. 12(a), being proportional to the derivative of the spectrum in Fig. 12(b), yields a phase shift similar to the $\pi/2$ phase shift between sine and cosine.

The results of Fig. 9 and Fig. 12, as proposed by the theory, imply the uniqueness of the waveforms for each load. Moreover, an inspection of the waveform peaks of each load in Fig. 12(a) reveals that they are thicker and less definite compared to the peaks presented in Fig. 9(a). These results indicate that the linear load can be distinctively identified from a nonlinear load. Furthermore, we can conclude that the sharper the peaks are, the higher the nonlinearity is.

C. Third test case: a dimmer with variable excitation angle.

The first and second test cases demonstrated the difference and uniqueness of the features that are produced for two significantly different loads to show the general theorems in action. The next test case, shown in Fig. 13, demonstrates the unique feature generation of a dimmer circuit with two only slightly different excitation angles: $85^\circ$ and $90^\circ$. There is hardly any noticeable difference in the current waveforms, as shown Fig. 13(a) and (d). Nevertheless, the analysis of $G_r(Z)$, $G_a(Z)+G_s(Z)$ of the CPC-DSP theory reveals noticeable changes in the various features. The zero-pole diagram reveals different spacing between the zeroes and poles. Moreover, the phase delay of the reactive transfer function $G_r(Z)$ for $85^\circ$ and $90^\circ$ have different characteristics, as can be seen, for example, in Fig. 13(b) and (e) for the group delay and Fig. 13(c) and (f) for the phase delay. Note the dashed arrows which connect the same graph change in both graphs of the same feature in the group delay and the circled area in the phase delay features of each state of operation. These results show noticeable differences for a slight angle change in the operation of the
device. Same results are achieved for the active+scattered transfer functions \( G_d(Z) + G_s(Z) \). Another noticeable difference between the various operation modes of the dimmer can be seen in the phase delay. It also shows that the CPC-DSP offers a wealth of features, and it suffices if only one/some of these features show a significant load signature.

VI. PRACTICAL IMPLEMENTATION FOR SMART GRID

Next, the motivation and practical implementation of this work is discussed.

Power measurements use waveform analysis mainly for power quality analyses, which require fewer resources than the analyses described here. Therefore, the implementation of this method will most likely require larger computational resources in comparison to the resources existing in today's smart meters and power monitors. Nevertheless, many currently available power monitors are designed for modularity with the ability for extended capabilities by adding modules for the existing firmware. Moreover, in several load identification applications, the analysis does not require real-time results, and therefore off-line analysis on remote servers is quite satisfactory. We can present numerous examples by which this costly method (these days) will be worthwhile. There are many very expensive processes in which adding a system, such as the method implementing this theory as explained in this paper, will be worthwhile in the aspect of preventing maintenance. For example, one application for smart grids might be the detection of electric loads by waveform measurements recorded by power quality monitors and also by detecting irregularities in these waveforms. Another application could be the installation of the algorithm based on this theory for a production line of wafers in the silicon industry or any expensive production line in which this system can predict a malfunction prior to it becoming destructive. Typically, a pause in such a system means loss of millions of dollars, and therefore this method, applied with additional software for a waveform monitor that will only cost a few thousands of dollars, would be very attractive. At this stage, this technology might be too expensive for implementation in every power monitor, but the future of smart grids predicts substantially more measurements in the grid, which will imply that this theory and similar theories can be implemented in other hierarchies of the grid.

The above-mentioned spectral features presented in Fig. 9, Fig. 12 and Fig. 13 for the group- and phase-delay spectra as well as the zero-pole diagram in Fig. 8, Fig. 11 and Fig. 13 are shown to be unique for each electric load, as was demonstrated theoretically and through examples. The spectrum is characterized with unique peaks with a specific width and height, which characterize specific loads and other harmonic phenomena. These newly proposed signatures are significantly more unique than the current's physical components spectra without the DSP enhancement because 1) they include the load spectral response function; 2) they include the phase information; and 3) the Z-transform is a continuous function.

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**Fig. 13:** Dimmer circuit: (a) current waveform with 85° excitation angle and the associated FFT distribution (b) The group delay of analysis of the Reactive transfer function at 85° angle (c) The phase delay of analysis of the Reactive transfer function at 85° angle (d) current waveform with 90° excitation angle and the associated FFT distribution (e) The group delay of analysis of the Reactive transfer function at 90° angle (f) The phase delay of analysis of the Reactive transfer function at 90° angle.
VII. DISCUSSION AND CONCLUSIONS

The theory presented in this paper is used for generating features which define a load, electric machine or any phenomenon that can be represented by its current and voltage waveforms. The method combines the CPC power transport theory with the Z-transform to generate unique features for the measured waveforms of a specific load. The motivation for the development of this theory poses advantages compared to more conventional machine identification procedures because a) the admittances originate from the currents with physical component significance; b) the Z-transform presents the entire spectral signature of the specific admittance with distinct weighting for each individual harmonic; c) the DSP spectral techniques are used to investigate power problems and anomalies; and d) orthonormality introduces separateness for the current physical components.

In this paper, analytic expressions are given and developed, which demonstrate that all the spectra are analytically comprehended through the knowledge of the zero-pole diagram. To uniquely describe a load, machine, or electrical phenomena, five transfer functions are introduced (according to the extension of the CPC). These transfer functions are $Y_a$, $Y_c$, $Y_r$, $Y_y$, and $Y_b$. While $Y_a$ is constant, it leaves four transfer functions for which five DSP features, such as amplitude response, phase response, phase delay, group delay and zero-pole diagram, can be produced. This means that there are potentially twenty different spectra for uniquely identifying the electric load. It should be mentioned here that not all features exist for every load, and therefore, in addition to the qualitative analysis, the quantity of features is added to the identification of the load.

The spectrum analysis presented in the paper shows first that for linear loads, such as $R_1$, $L$, and $C$-like, yields at the most a shift of the roots of the voltage (by a phase angle). The zeroes that are the current roots are therefore not distant from their pair of matching poles. With non-linear loads, there are escaped zeroes, which are distant from the unit-like circle, which in turn cause the sharp peaks of the spectrum. It was shown that the further the zeroes migrate, the sharper the peaks and the higher the derivatives for the correlated group delay spectrum in the phase, which indicates a higher nonlinearity of the load. It should be stressed here that these spectra are saved in a database, and therefore a difference in the spectra is sufficient for uniquely defining a load or a phenomenon. Test cases A and B, which include different loads, demonstrate this idea clearly. The various spectrum analyses show that the phase spectrum provides more profound information than the amplitude spectrum. The results are demonstrated here on real measured waveforms, demonstrating the proposed technique for uniquely identifying a load.

A worthy discussion is concerning the added value of CPC Z-transform over simply spectral identification. The added value can be summarized in the next few points: 1) comparing to only Z-transform, the proposed merger of CPC and Z-transform preserve physical properties of currents and power. For example test case 3 deals with highly similar loads as shown in Fig. 13. Simple DTFT transform results in pretty much equal load signature as shown In Fig. 13 (a) and (d). However, the proposed method reveals notable difference between the two as shown in Fig. 13 (b), (c), (e), and (f). 2) the use of CPC enhances meaningful indicators such as load unbalance and backward current (which wouldn’t have been identified through Z-transform alone) and 3) the proposed Z-transform combined with CPC may enhance the unique signature in case in which the signature is weak and contains only in one of the proposed components. For example: the CPC active admittance manifests as a constant variable. Thus it is advantageous to focus on the scattered components solely (without the constant) to enhances phenomena related to active power.

REFERENCES


