Finite-difference time-domain study of modulated and disordered coupled resonator optical waveguide rotation sensors

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Abstract: We present a full-wave finite difference time domain (FDTD) study of a coupled resonator optical waveguide (CROW) rotation sensor consisting of 8 doubly degenerate ring resonators. First we demonstrate the formation of rotation-induced gap in the spectral pass-band of the CROW and show the existence of a dead-zone at low rotation rates which is mainly due to its finite size and partly because of the individual cavities losses. In order to overcome this deficiency, we modulate periodically the refractive indices of the resonators to effectively move CROW's operating point away from this dead-zone. Finally, we analyze the performance of a structurally disordered CROW to model the unavoidable fabrication errors and inaccuracies. We show that in some cases structural disorder can increase the sensitivity to rotation by breaking the degeneracy of the resonators, thus making such CROW even more sensitive to rotation than its unperturbed ideal counterpart.

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1. Introduction

In recent years there has been an increased interest in the coupled resonator optical waveguides (CROWs) as potential candidate for an ultra-small optical rotating sensor [1-7]. Due to rotation each cavity experiences the Sagnac effect which manifests itself in the resonance frequency splitting of the resonators [8, 9]. As the direction of light propagation in every pair of successive resonators is in opposite direction, a rotation-induced periodic modulation of the resonance frequencies is effectively present, resulting in the formation of the rotation-induced gap in the CROW's pass-band [10]. The most widely utilized technique for analyzing such devices is the transfer-matrix method [11]. This technique is relatively simple and easy to implement with quickly obtainable results, but it requires the *a priori* knowledge of the impact of Sagnac effect, loss, inter-cavity coupling, etc. in the device. All these parameters highly affect the performance of the device, and may be mutually dependent; therefore a full-wave simulation based directly on Maxwell's equations is necessary during the design stage of such a device. Recently, a few finite-difference-time-domain (FDTD) methods for modeling the Sagnac effect were proposed in the literature [12,13], but their ability to model relatively large optical structures, such as CROW, where the computational widow can have an area of hundreds or more of square microns, was not demonstrated and the issue of numerical stability was not addressed. The development of a stable FDTD scheme, and its successful utilization for modeling such devices was only briefly introduced in our previous work for a CROW consisting of 4 racetrack ring resonators [14].

In this paper we present a comprehensive study of a CROW-based rotation sensor comprising 8 coupled doubly degenerate micro rings resonators using full-wave simulations. We study its performance under periodic modulation of the structure parameters and under structural disorder. We employ the implicit RCN-4 (Rotating Crank-Nicolson) FDTD method developed in [14]. This method is a modification of the Crank-Nicolson scheme holding for rotating frames of reference, having 4th order accuracy in space and 2nd order accuracy in time.

In this work we show that a finite CROW exhibits a rotation-insensitive region-the deadzone-that extends from zero to a minimum detectable rotation rate. Although this range can be narrowed by adding more resonators, it still does not allow for measuring practical angular velocities. To overcome this problem, we apply a periodic modulation of the refractive indices of the rings, thereby shifting the operating point of the CROW out of the dead-zone and improving the sensitivity at low rotation rates. We also investigate the impact of disorder on the CROW performances by creating random local changes to the inner and outer boundaries of the waveguides comprising each resonator. The effect of such disorder is threefold: First, the noise modifies the CROW by introducing random fluctuations in the positions of the boundaries of the waveguides, thereby azimuthally modifying the effective radius and waveguide width of each ring, consequently increasing their surface roughness and scattering losses. Second, it breaks the rotational symmetry of the rings thus lifting the degeneracy of the individual resonators, creating two new *rotation eigenmodes* [9]. In other words, the clockwise (CW) and counter clockwise (CCW) modes of every ring have different resonance frequencies and these frequencies differ from one ring to another. Last but not least, the noise randomly modifies the inter-ring coupling between the resonators. The consequences of the two last factors can be quite surprising as the sensitivity to rotation of a disordered CROW can sometime surpass that of an ideal device.

The structure of the rest of this paper is as follows. In Sec. 2 we study the ideal finite-size CROW, its dead-zone, and the effect of periodic modulation. In Sec. 3 we analyze a randomly disordered CROW. Concluding remarks are provided in Section 4.

2. Ideal finite length CROW design and simulation

2.1 CROW without index modulation

A CROW model of 8 ring resonators, together with its parameters used in our simulations, is shown in Fig. 1. We assume that it lies in a two-dimensional slowly rotating medium at the xy plane with a rotation rate Ω . Our choice of gaps between the straight waveguides and the rings ensures that the output of the drop ('D') (through ('T')) port is high (low) and stable over the frequency pass band and does not exhibit strong Fabry-Perot oscillations due to impedance mismatch [15]. The FDTD simulations employed throughout this paper utilize a Cartesian grid with a spatial resolution $\Delta x = \Delta y = 20$ nm, a time step $\Delta t = 5/3 \times 10^{-17}$ sec. TM polarization (Ez,Hx,Hy) is assumed. The vacuum wavelength of the excitation pulse is 1.42µm, corresponding to an exact resonance of the individual ring resonators, with angular modal number of M = 15. M is chosen to be odd in order to ensure that both modes (CW and CCW) do not have a spurious resonance frequency splitting due to the grid discretization [16]. Prior to running the simulation of the entire CROW it is necessary to verify that the resonance frequencies of all resonators are identical. As the first and the last resonators have a different dielectric structure on one of their sides (straight waveguide with a gap of 200nm) as opposed to resonators 2-7, their resonance frequency differs from that of the inner cavities due to the stronger coupling-induced resonance frequency shift (CIFS) [16]. Using FDTD simulations we found the CIFS to be approximately + 550GHz for ring 1 and 8, and + 50GHz for rings 2 to 7, all relative to the resonance frequency of 210.95THz of an isolated resonator with the refractive index n = 2. Therefore, there is a + 500GHz frequency mismatch between the resonance frequency of resonators 1, 8 and the resonance frequency of resonators 2-7. Tuning can be performed by either reducing the refractive indices of the rings 2-7 or by increasing the indices of the rings 1 and 8. By choosing the latter option we get a resonance frequency of 210.96THz for the rings 1 and 8 using n = 2.00464; comparing it to the resonance frequency of 211THz for the rings 2-7 there is still a small mismatch of 40GHz. Additional iterations of index tuning can be done to reduce the resonance frequency mismatch further, but as we show below it has no major consequences on the performance of the CROW structure studied here for rotation sensing. We note that the above fine tuning of the resonators refraction index can be practically realized through, e.g., the electro-optic or thermo-optic effects [17, 18].



Fig. 1. CROW consisting of 8 ring resonators and rotating at the rate Ω . The resonance frequency shift in each resonator due to rotation is $\pm \delta \omega(\Omega)$. Alternatively, the effect of rotation can be mimicked in a static CROW by modulating the indices of the resonators by $\pm \Delta n$, thus shifting the operating point (or bias) from null rotation rate. The first and the last rings have higher indices (different color) in order to reduce CIFS to allow more symmetric frequency response.

Figures 2(a) and 2(b) show the spectral responses of the drop and through ports, respectively, vs. the optical frequency for an ideal CROW, i.e. without index modulation ($\Delta n = 0$). Each rotation rate is a multiple of $\Omega_0 = 10^{10}$ [rad/s].



Fig. 2. Spectrum of an ideal unmodulated CROW ($\Delta n = 0$) for different rotation rates; (a) drop (b) through.

The maximal value of the drop transmission is approximately 0.26 which is substantially lower than 1 due to bending losses caused by the fact that each resonator has a small radius. Another loss mechanism is the computational grid discretization resulting in radiation scattering. Using FDTD simulation the intrinsic quality factor of an isolated ring resonator was found to be $Q_{int} \approx 8000$. The change in transmission of both ports is very small for rotation rates lower than $\Omega = 2 \times \Omega_0$, especially at the through port. This rotation insensitive region, which we designate as the dead-zone, is formed by two major mechanisms: (1) losses in the

CROW as described in [19] and (2) the finite number of the ring resonators in the device. In fact, the second factor is generally the dominant one when dealing with relatively small number of rings, which is definitely the case for our CROW. In Appendix A. we demonstrate this observation by solving Eq. (4.8) in [19], derived by the tight-binding approach, for the amplitude coefficients of the modes of the resonators in the CROW, while taking into account the finite number of resonators. Another important dead-zone mechanism which generally dominates in conventional RLGs is frequency locking due to back-scattering of radiation in the ring cavity. Such effect is caused by imperfections of the waveguides and, in conventional RLGs, by imperfections of the mirrors. In our FDTD simulations, non-negligible backscattering is generated by the staircase approximation used for the realization of the resonators in the computational grid. As this artificial roughness (~20nm) is generally larger than that of waveguides fabricated using state-of-the-art techniques it is reasonable to assume that the numerical backscattering in our simulations is actually larger than the real values. Nonetheless, as we show in appendix A, the dead-zone formed in the CROW is dominated by its finiteness and the impact of the backscattering is, therefore, negligible.

In order to obtain the drop response to rotation from the spectral data plotted in Fig. 2, for every rotation rate we integrate the power over the range 210.5-211.5 [THz], yielding an estimation of the output power. We then normalize it to the value obtained for a static CROW ($\Omega = 0$). The results are shown by the blue curves ($\Delta n = 0$) in Fig. 3 for both ports. Clearly, there is a dead-zone for $\Omega < 1 \times \Omega_0$, which renders the device practically insensitive to rotation.



Fig. 3. Response to rotation of an ideal CROW without modulation (blue, $\Delta n = 0$), and with modulation (green, $\Delta n = 7.74 \times 10^{-4}$). All values normalized to the value with $\Delta n = 0$ and $\Omega = 0$; (a) drop (b) through.

2.2 Modulated CROW

In order to overcome the dead-zone problem one can either bias the device at an extremely high rotation rate in the order of 10^{11} [rad/s], a solution which is not practical, or modulate the refractive indices of the resonators, thus effectively attaining the same result [19]. The latter can be achieved in practice by utilizing, e.g., the thermo-optic effect for modifying the resonance frequencies of the individual microrings. In our case a periodic change of the index of refraction by a value of $\Delta n = 7.74e-4$ mimics a CROW rotating at the rate $\Omega = 10 \times \Omega_0 =$ 10^{11} [rad/s]. Figures 4(a)-4(b) show the transmission at the drop and through ports, respectively, with the imposed periodic modulation of the indices. We note that the dead-zone no longer exists. In order to evaluate the power available from the ports of the device for each

rotation rate, we again integrate the spectrum in the range 210.5-211.5 [THz] and normalize it to the value obtained for a static CROW without index modulation.



Fig. 4. Spectrum of an ideal modulated CROW using $\Delta n = 7.74 \times 10^{-4}$ for different rotation rates; (a) drop (b) through.

The corresponding curves (green curves, $\Delta n = 7.74 \times 10^{-4}$) for drop and through ports are shown in Fig. 3. Again, once the index modulation is introduced, the dead-zone vanishes and the device retains its sensitivity even in the region of zero rotation rates. The index modulation effectively shifts the blue curves ($\Delta n = 0$) of Fig. 3 leftwards; it artificially "adds rotation" without any actual rotation. Hence, the dead-zone disappears independently of the rotation rate, and the device sensitivity (curve slope) is at its best value right at $\Omega = 0$. Clearly, this improvement comes at the expense of reduced output power at the drop port (-2.75 dB), but it is compensated by an appropriate increase in power at the through port (+ 2 dB). These values are identical because the drop and through plots of Fig. 3 are normalized to different values; therefore in practice there is no actual power loss as a result of modulation. We emphasize that once the modulation is applied, the sensitivity is no longer limited by the deadzone and instead it is limited by the minimum detectable power of the receiver at the output of the CROW.

In order to demonstrate more clearly that index modulation mimics the impact of rotation, we compare in Fig. 5 the spectrum of an unmodulated CROW rotating at $\Omega = 10 \times \Omega_0$ and that of a static CROW with index modulation of $\Delta n = 7.74 \times 10^{-4}$. There is an excellent agreement between the spectra. Thus we conclude that one can choose a desired "out of the dead-zone" operating point using this technique.

Thermo-optic tuning of the refractive indices has been demonstrated in recent years for a number of CROW platforms such as SOI and polymers [20]. The thermo-optic coefficient for polymer materials is of the order of -10^{-4} [K⁻¹], and 10^{-4} [K⁻¹] for silicon [17, 21]. Therefore, a modulation of $\Delta n = 7.74 \times 10-4$ would require a temperature difference of $\Delta T \approx 16$ [K] between two consecutive rings, which is not difficult to achieve. In addition, employing larger resonators than those used in the simulations would further reduce the required index change, thus rendering the resonance modulation quite straight-forward.



Fig. 5. Spectrum of an unmodulated ($\Delta n = 0$) CROW rotating at $\Omega = 10 \times \Omega_0$ (blue), and a modulated static CROW with $\Omega = 0$ and $\Delta n = 7.74 \times 10^{-4}$ (green); (a) drop (b) through.

3. Disordered CROW simulation

In this section we investigate the influence of structural disorder in CROW on its response to rotation. We consider a disorder model which introduces random surface roughness on the inner and outer walls of the rings waveguides. Such disorder can be generated by the errors and tolerances of conventional fabrication methods such as E-beam and photo lithography. The smallest roughness that can be introduced is the computational grid resolution of $\pm \Delta x =$ $\pm \Delta y = \pm 20$ nm. Thus we generate a disordered CROW by randomly erasing, adding (or leaving as is) a single cell from/to the dielectric structure at the wall boundaries of every ring in the ideal CROW. This procedure is illustrated in Fig. 6, which shows the relative permittivity of one of the rings in the 1st realization of the randomly generated CROW. The introduction of such noise into the device has the following impacts: (1) increased surface roughness, resulting in greater scattering losses; (2) The rotational symmetry of each ring is broken, thus lifting mode degeneracy and creating two new modes with different resonance frequencies;; (3) the effective separation between adjacent rings is not constant, resulting in modification of the inter-ring couplings. As the structural noise presented into the structure in our simulations (\pm 20nm) is in the order of what can be attained using conventional photolithographic techniques [22], we believe that the simulated effects reported here can be observed in a realistic device.



Fig. 6. Relative permittivity of a section of a ring constituting the CROW; (a) ideal CROW (b) instance #1 of a randomly disordered CROW.

We randomly generated 4 realizations of a disordered CROW and, in the same manner as for the ideal CROW, obtained their responses to rotation. Figure 7 shows the drop and through responses for all 4 realizations. Each curve was normalized to the value obtained from the simulation of the corresponding static CROW. Therefore all curves intersect at the point of 0 [dB] at $\Omega = 0$. The first interesting feature of the plots is that they are no longer symmetric around $\Omega = 0$, as opposed to the ideal CROW. We note that the response curves of realizations #1 and #4 are blue-shifted and those of #2-3 are red-shifted. This result actually indicates that there is no dead-zone around zero rotation rates for any of the CROWs. Thus, the randomly generated structural noise effectively imposes modulation of the effective refractive indices of the ring modes thereby causing a similar effect to that presented in Section 2.2 for the modulated ideal CROW. In fact, here the effect may be even stronger due to the changes of the inter-ring couplings, modulation that can also mimic the effect of rotation. The dead-zones of the disordered CROWs are shifted and their flat regions do not become wider, but rather the opposite. A closer look reveals another interesting feature of the drop response, that is, the slope of all 4 curves is actually steeper than that of the ideal CROW, rendering the disordered CROW even more sensitive to rotation. Table 1 shows the absolute value of the maximal slope for each CROW realization, as well as the ratio between the maximal powers at the drop port of the disordered CROWs normalized to that of the ideal CROW. Clearly, the ideal CROW exhibits inferior performance in terms of sensitivity. On the other hand, the disordered CROWs exhibit strong attenuation due to radiation scattering, requiring larger input powers (up to 5.75 times more) to retain the same output signal level as the ideal CROW. Therefore, since the minimum measurable rotation rate is tied to the signal-to-noise ratio, which changes with power, the ideal CROW may still have better performances in that regard.

It is also worth noting that the results in Fig. 7 were obtained using the same frequency range of 210.5-211.5 [THz] as for the ideal CROW. This underlines the advantage of the band-gap power measurement of the CROW rotation sensors, as opposed to the conventional measurement of a frequency shift such as in a RFOG, for example, where there is a need to stabilize the resonance frequency of the device in order to extract the rotation rate in a reliable manner.



Fig. 7. Response to rotation of 4 different random realizations of a disordered CROW; (a) drop (b) through.

Table 1. Maximal absolute slopes and ratio max(D)/max(D _{ideal}) for drop response ide	eal	
and disordered CROWs		

CROW	Max. Slope [dB/rad/s]	$max(D)/max(D_{ideal}) [dB]$
Ideal	38.1×10^{-11}	0
#1	60.4×10^{-11}	-6.9
#2	70.1×10^{-11}	-7.6
#3	58.7×10^{-11}	-3.5
#4	46.8×10^{-11}	-7

4. Conclusions

We performed a full-wave FDTD study of ideal and disordered CROWs consisting of 8 ring resonators. We showed the formation of rotation induced-gap and demonstrated the existence of a dead-zone at low rotation rates. For such small devices the primary cause for the dead-zone was found to be the finite length of the CROWs and not the losses of the individual cavities. Modulating the refractive indices of the rings effectively biases the device at the desired operating point and thus resolves the dead-zone obstacle. Finally, we studied 4 instances of randomly generated CROWs with surface roughness noise. In terms of sensitivity all 4 devices are superior to the ideal CROW, but require more power to obtain a similar level of output signal because of the higher radiation scattering losses. We believe that these are encouraging results as it might suggest that the use of CROWs as rotation sensors is a tangible option.

Appendix A. Dead-zone formation: Finite-size vs. Loss comparison

The results shown in this section are obtained by solving Eq. (4.8) in [19]. That equation was derived by the tight-binding approach, for the amplitude coefficients of the modes of the resonators in the CROW. This approach is valid only if the inter-cavity power coupling is weak (i.e. a few percents), but within this limit it provides very accurate results, very close to those obtained by the FDTD method. We note, however, that the tight-binding approach cannot model directly structural disorder effects. Here, we first consider a lossless CROW comprising N=8, 16, 32 and 64 cavities; the inter-cavity power coupling is 1%, the mode order is M=15, and ε_r =4, ε_i =0 (real and imaginary parts of relative permittivity). Figure 8(a) shows the drop response as a function of the rotation rate for the various CROWs. Figure 8(b)

shows the corresponding sensitivity defined as the ratio of the drop response in [dB] to the rotation rate Ω in [rad/s].



Fig. 8. (a) Drop response calculated by the tight-binding approach [19] for a lossless CROW with different number of rings. (b) The corresponding sensitivity.



Fig. 9. (a) Drop response calculated by the tight-binding approach [19] for a CROW of 8 rings with different round-trip power loss rates in a single ring. (b) The corresponding sensitivity.

For high rotation rates the response is exponential as expected, while at slow rotation rates the response becomes flat, thus generating a dead-zone. Note that adding more resonators only narrows that region but does not eliminate it completely. In order to ensure that losses play no major role in dead-zone creation in our FDTD simulations, we set N=8 and varied the imaginary part of the refractive index as $\varepsilon_i=0$, 106e-6, 212e-6, 424e-6 corresponding to a single ring round trip power loss of 0%, 0.5%, 1% and 2% respectively. Figure 9(a) shows the calculated drop response and Fig. 9(b) shows the corresponding sensitivity. At very low rotation rates the response decreases with losses, but the slope of each curve is similar, thus indicating similar (and small) sensitivity. The sensitivity grows significantly and achieves its maximum value around $\Omega=20\times\Omega_0$ for all losses. Thus we conclude that the dead-zone

formation in the CROWs which are studied in this paper is primarily due to the finite number of resonators in the device.

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