HIERARCHY BASED SCALING IN COMPLEX SYSTEMS

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**ABSTRACT**

In the present paper the method of scaling proposed and applied in our previous works is developed and generalized. This method is based on the sets' hierarchy defined on the grounds of the information types treated by different parts of the considered system. In the present paper this method of the scaling is developed to be fit for problems arising in different fields, in systems of quite different nature, as for example, scaling in complex systems such as living organisms, systems considered in economics, micro- and macro evolution, human society and its parts etc.. In the general case of complex systems the hierarchy is stopped to be a number, but becomes to be a set of numbers corresponding to different ways of the information transfer.

In the case of living organisms an interesting task is to study how the molecular genetic information transferred to macroscopic levels of organization influences the behavior and thinking, for example, of human being. For this purpose it is necessary to establish the scaling and to find its connection with different levels of the organism organization.

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1. INTRODUCTION

The idea of scaling based on the hierarchy has turned out to be useful for a basic problem of the high-energy elementary particle physics by the following reason. The advancement of the high-energy physics (HEP) experiments toward higher-and-higher energies allows one to penetrate to smaller-and-smaller spatial (or space-time) regions of the micro-World, in other terms, to smaller-and-smaller micro-World scales. However, the use of terms like "smaller-and-smaller spatial (or space-time) regions of the micro-World" demands that the concept of space or space-time could be defined for the unlimitedly small region, which is very doubtful. So it would be reasonable to get rid of the dependence of micro-World scaling on the possibility/impossibility to define concept of the space (or space-time), and find another, alternative way for it, not based on space-time regions' dimensions.

The method proposed, developed and applied in [1, 2] is based on the use of the hierarchy among well-ordered sets forming a well-ordered set instead some "natural" factors as, for example, dimensions of spatial and time interval. The meaning and structure of these well-ordered sets cannot be defined as pure mathematical ones, but only together with taking into account the considered concrete studied object properties and natural meaning, such as, for example, the economical, physical, chemical, biological, of evolutionary biology [7], ones etc.. Each of well-ordered sets forming the abovementioned well-ordered set represents a scale. In the present paper the method of scaling proposed and developed in [1] is generalized to be valid for different kinds of systems and of scaling, but not only for the elementary particle physics. Notice that it is not supposed that only well-ordered sets (see below) could be used. In [1, 2] as well as in the present paper
the hierarchy of subsets is established on the grounds of their information processing level and properties, using, in particular, Bertrand Russell's theory of types [6].

In [3] (see also the same author web pages linked with [3]) the scaling is considered for different kinds of measurements. In Sec. "Purpose of Scaling" of [3] its author writes: "Why do we do scaling? Why not just create text statements or questions and use response formats to collect the answers? First, sometimes we do scaling to test a hypothesis. We might want to know whether the construct or concept is a single dimensional or multidimensional one (more about dimensionality later). Sometimes, we do scaling as part of exploratory research. We want to know what dimensions underlie a set of ratings. For instance, if you create a set of questions, you can use scaling to determine how well they "hang together" and whether they measure one concept or multiple concepts. But probably the most common reason for doing scaling is for scoring purposes. When a participant gives their responses to a set of items, we often would like to assign a single number that represents that person's overall attitude or belief. For the figure above, we would like to be able to give a single number that describes a person's attitudes towards immigration, for example." In Sec.A of [4] "Classification of Scales of Measurement" is written: "Paraphrasing N. R. Campbell (Final Report, p.340), we may say that measurement, in the broadest sense, is defined as the assignment of numerals to objects or events according to rules. The fact that numerals can be assigned under different rules leads to different kinds of scales and different kinds of measurement. The problem then becomes that of making explicit (a) the various rules for the assignment of numerals, (b) the mathematical properties (or group structure) of the resulting scales, and (c) the statistical operations applicable to measurements made with each type of scale."
Scales are possible in the first place only because there is a certain isomorphism between what we can do with the aspects of objects and the properties of the numeral series. In dealing with the aspects of objects we invoke empirical operations for determining equality (classifying), for rank-ordering, and for determining when differences and when ratios between the aspects of objects are equal. The conventional series of numerals yields to analogous operations: We can identify the members of a numeral series and classify them. We know their order as given by convention. We can determine equal differences, as $8-6 = 4-2$, and equal ratios, as $8/4 = 6/3$. The isomorphism between these properties of the numeral series and certain empirical operations which we perform with objects permits the use of the series as a model to represent aspects of the empirical world. The type of scale achieved depends upon the character of the basic empirical operations performed. These operations are limited ordinarily by the nature of the thing being scaled and by our choice of procedures, "...

The generalization of the scaling method, proposed in [1, 2], is to be done to satisfy the demands formulated above by authors of [3, 4], but the theory that will be constructed so is expected to be more flexible and general than the approaches to the scaling considered in works [3, 4]. In [1] the scaling is based on type of the information obtained from each scale, processed inside each scale, forwarded from each scale to the other ones and to the observer. The notion observer is necessary for the definition of the scaling in each concrete case. As it is considered in [1], the observer can be a human being or computer. The observer must be able to receive, process, and interpret the information. Such an interpretation includes the creation of laws (physical, chemical, biological, economical etc.) governing processes occurring within each scale. As it was considered in [1], the consideration of the information treatment within different scales allows one to
establish their hierarchy, and thereupon to order the set of all scales with respect to
increase or decrease of their hierarchies. Of course, the content, amount and structure of
the information depend on the nature of the considered system and the purpose of its
study.

In [1] one reveals that events occurring in scales of lower hierarchy may
influence the occurring in scales of higher hierarchy. If one considers the economy, such
phenomenon is important. This is not something new in the World economy, but the
mathematical scaling theory could help to study it. Probably, results of this study could be
used, in particular, for economical forecasts. Following [1], one must firstly introduce
concepts of measurements in the economy, the information provided by such experiments,
its processing, transfer from scale to scale, its interpretation, and establish hierarchy of
scales on the information treatment like it was done in [1]. Notice that scales should be
ordered and then defined so that they would be almost independent, in other words, that
connections (interactions) between neighbor scales could be such that would be
considered as relatively small perturbations. The author limits himself with these general
notes leaving their development and realization to economists.

In Sec. 2 the general mathematical formalism is represented.

In Sec. 3 one considers sets with different ways of ordering based on different
definitions of the hierarchy. The Eqn. (4) defines the hierarchy as a set (tuple) of numbers,
each of them is a partial hierarchy defined along a certain way. In the following sections
this definition will be generalized.

In Sec. 4 the proposed mathematical formalism is applied to the consideration of
molecular genetic information influence upon living organism, i. e., this is an attempt to
construct mathematical formalism allowing us to connect the properties and occurring on molecular level with the occurring on the level of the whole organism.

In Sec. 5 one considers the statistical distributions application to the genetic information step – by - step transfer from genes to the whole organism. The formalism developed in Secs. 4 & 5 is of the general character and can be applied not only to living system, but also to economical and other ones.

In Sec. 6 the concept of the level of organization (of a system) is considered in the framework of the set theory.

Sec. 7 is dedicated to the consideration of direct and indirect measurements in complex system. This concept firstly appeared in quantum mechanics, is important for the scaling as it was shown in [1].
2. GENERAL MATHEMATICAL FORMALISM

Try to formulate mathematically the problem of scaling, generally as possible.

Let there is the set \( \forall \alpha \{ x_{\alpha \beta} \} \) of variables \( x_{\alpha \beta} \) of arbitrary nature, where

\[
X_\alpha = \exists \alpha, \exists \beta \{ \alpha \in \mathbb{N}, \forall \beta \in \mathbb{N} \} \{ x_{\alpha \beta} \},
\]

(1)

and \( \mathbb{N} \) is the set of all natural numbers.

Let us consider the case when each \( X_\alpha \) defined by the expression (1) be a finite or infinite countable set of its subsets \( Y_{\alpha, \vartheta} \):

\[
X_\alpha = \exists Y_{\alpha, \vartheta} \subseteq X_\alpha \left( \forall \vartheta_{\alpha} \in \mathbb{N}, Y_{\alpha, \vartheta} \cap Y_{\alpha, \vartheta \neq \vartheta} = \emptyset \right) \{ Y_{\alpha, \vartheta} \},
\]

(2)

i. e., the variables \( x_{\alpha \beta} = Y_{\alpha, \vartheta} \), and, to emphasize it, \( \beta \) is replaced to \( \vartheta \).

Choose each system of subsets \( Y_{\alpha, \vartheta} \) so that the occurring within one of these subsets may influence the occurring within its neighbor subsets of Eqn. (2) only as small perturbation. Remind that the expression (2) plays in the present paper the same role that the well-ordered set \( S \) of well-ordered sets in Ref. [1]. However \( X_\alpha \) itself may not be a well-ordered set, in particular, it may be a poset.

Let us determine through what subsets \( Y_{\alpha, \vartheta} \) passes the information emitted from \( Y_{\alpha, \vartheta} \) up to \( Y_{\alpha, \vartheta} \), in other words, to find the way of the information transmission from a subset \( Y_{\alpha, \vartheta} \) to another subset \( Y_{\alpha, \vartheta} \). Thus, we accept that this way consists of subsets \( Y_{\alpha, \vartheta} \) such that among them are \( Y_{\alpha, \vartheta} \) and \( Y_{\alpha, \vartheta} \). Call them the first and the last subsets of the considered way \( G_{\alpha, \vartheta, \vartheta} \), denote \( \Omega_{\alpha, \vartheta, \vartheta} \) a set of subsets along the way \( G_{\alpha, \vartheta, \vartheta} \).
from $Y_{a,\alpha}$ up to $Y_{a,\alpha'}$. Following [1], order the set $\left\{ \exists Y_{a,\alpha,\alpha'}^{(\alpha,\alpha,\alpha')} \subset G_{a,\alpha,\alpha'} \right\}$ on the grounds of the hierarchy.

For this purpose let us establish the hierarchy among subsets $Y_{a,\alpha,\alpha'}^{(\alpha,\alpha,\alpha')}$ as it was done in [1], and thereupon order the set

$$G_{a,\alpha,\alpha'} = \left\{ Y_{a,\alpha,\alpha'}^{(\alpha,\alpha,\alpha')} \subset G_{a,\alpha,\alpha'} \right\} \left( \forall \partial'_{\alpha} \in \partial_{\alpha} \right) \left\{ Y_{a,\alpha,\alpha'}^{(\alpha,\alpha,\alpha')} \right\}$$

(3)

with respect to the hierarchies of the subsets $Y_{a,\alpha,\alpha'}^{(\alpha,\alpha,\alpha')} \subset G_{a,\alpha,\alpha'}$. A very important difference between this case and that considered in [1] is that in [1] were considered the cases (for example, for the elementary particle theory), when there is only one subset having the highest hierarchy, namely the set (containing the observer), as well as only one way, which leads from any subset to the observer. Only the observer is able to interpret the information, so just this set containing the observer has the highest hierarchy. In the general case several subsets $Y_{a,\alpha,\alpha'}$ may be able to interpret the information, so there is more than one way leading from a certain subset to those able to interpret the information (=observer). Observer certainly has the hierarchy higher than that of any other subset $Y_{a,\alpha,\alpha'}$, which is not observer, but it does not mean that all observers have the equal hierarchies.

This problem demands a special attention, as well as the problem of the establishing order (sometimes well-order) by several ways based on the hierarchy definition with respect to different subsets $Y_{a,\alpha,\alpha'}$ able to interpret the information, i. e., with respect to different observers.

**NOTATION.** Beginning from this place, for short, we shall call **OBSERVER** each subset able to interpret the information.
3. SETS WITH DIFFERENT WAYS OF ORDERING BASED ON DIFFERENT DEFINITIONS OF THE HIERARCHY

Thus, it is to establish the order (possibly, well-order) on the grounds of hierarchy for each way separately and thereupon to take into consideration crossings of different ways, which may demand to attribute more than one hierarchies to certain subsets. This means, each subset $Y_{\alpha,\vartheta}$ that is at the crossing of two or more ways should have a number $> 1$ of hierarchies each of them corresponds to a certain way. Denote $Y_{\alpha,\vartheta} (\kappa \in \mathbb{N})$ such a subset that is able to interpret the information (i.e., an observer).

There are various ways for a certain subset $Y_{\alpha,\vartheta}$ to exchange the information with the subset $Y_{\alpha,\vartheta} (\kappa \in \mathbb{N})$. Each of these ways consists of some subsets of the set $X_{\alpha}$ that are not of the type $Y_{\alpha,\vartheta} (\kappa \in \mathbb{N})$ (i.e., not observer). Following [1], one can define the hierarchy $h_{\vartheta',\vartheta} \left( Y_{\alpha,\vartheta} \right) = \left\{ g_{\alpha,\vartheta} \mid G_{\alpha,\vartheta} = \left( \forall \vartheta' \in \left[ \vartheta, \vartheta' \right] \right) \left\{ Y_{\alpha,\vartheta} \right\} \right\}$ of the subset $Y_{\alpha,\vartheta}$ with respect to $Y_{\alpha,\vartheta} (\kappa \in \mathbb{N})$ along the way $G_{\alpha,\vartheta} = \left\{ Y_{\alpha,\vartheta} \right\}$. Thus, to different $G_{\alpha,\vartheta} = \left\{ Y_{\alpha,\vartheta} \right\}$ will correspond different hierarchy.

**Important NOTE.** Taking into account the meaning, such as physical, economical etc. (but besides of the pure mathematical one) of a functioning way that connects the considered subset $Y_{\alpha,\vartheta}$ with the subset $Y_{\alpha,\vartheta} (\kappa \in \mathbb{N})$, it must strike off those of such ways that by any cause are not able to transmit the information relevant to the
meaning from $Y_{a,\alpha_0}$ to $Y_{a,\alpha_0,\alpha_\kappa}$ ($\kappa \in \mathbb{N}$) and vice versa. This is the difference of the real problems' consideration from the pure mathematical one.

It must find all possible (working!) ways connecting a subset with a certain subset able to interpret the information (observer), taking into account the "Important Note" written above. It is also to be taken into account that there are different kinds of the information and, correspondingly, different types of subsets able to interpret these types of the information. This fact influences directly the choice and number of ways connecting the considered subset to them.

Really, the situation is much more complicated. For example, a way connecting a subset $Y_{a,\alpha_0}$ with an observer $Y_{a,\alpha_0,\alpha_\kappa}$ ($\kappa \in \mathbb{N}$) may cross also another observer $Y_{a,\alpha_0,\alpha_{\kappa'}}$ ($\kappa' \in \mathbb{N}$) that treats and interprets another type of the information. How the information emitted from $Y_{a,\alpha_0}$ influences the observer $Y_{a,\alpha_0,\alpha_{\kappa'}}$ ($\kappa' \in \mathbb{N}$) and the treatment of the second type information by $Y_{a,\alpha_0,\alpha_{\kappa'}}$ ($\kappa' \in \mathbb{N}$) and what could do $Y_{a,\alpha_0}$ with the mixture of the information came from two observers $Y_{a,\alpha_0,\alpha_{\kappa}}$ ($\kappa \in \mathbb{N}$) and $Y_{a,\alpha_0,\alpha_{\kappa'}}$ ($\kappa' \in \mathbb{N}$) if the experiments be organized so that such a way is excluded (cf. "IMPORTANT NOTE" written above), this problem could be eliminated. For example, in economical researches a relevant choice from what locations one obtains the information on the economical state and processes and at what locations it is treated and interpreted could allow one to avoid the problem described above. This means, the mathematical study of an economical problem begins from the search of the relevant choice of the set (2) $X_\alpha = \left\{ \exists Y_{a,\alpha_0} \subseteq X_\alpha \mid \left( \forall \theta_{\alpha_0} \in \mathbb{N}, Y_{a,\alpha_0} \cap Y_{a,\alpha_0,\alpha_{\theta_{\alpha_0}}} = \emptyset \right) \{Y_{a,\alpha_0}\} \right\}$, including the positions of observers. This is not a mathematical operation, but the problem where economical
experimental data should be obtained, to what directions they should be sent, where they
must be treated and interpreted, which really means the solution of the problem how
instruments and personnel should be distributed and arranged throughout the considered
system. This first stage of the economical problem study that is the experiment planning is
extremely important for the continuation of the considered research determining the
statement of the problem.

Let us choose the set arrangement so that to each subset $Y_{\alpha,\alpha_k}$ the information
comes from only one observer $Y_{\alpha,\alpha_k,\alpha_k'} (\kappa' = \kappa_0 \in \mathbb{N})$ and can be sent to only one observer
that is the same $Y_{\alpha,\alpha_k,\alpha_k'} (\kappa' = \kappa_0 \in \mathbb{N})$, no matter how much different ways connect
$Y_{\alpha,\alpha_k}$ and $Y_{\alpha,\alpha_k,\alpha_k'} (\kappa' = \kappa_0 \in \mathbb{N})$. Now one can introduce the hierarchies of subsets
belonging to such a way $G_{\alpha,\alpha_k} = \left\{ Y_{\alpha,\alpha_k}^{(G_{\alpha,\alpha_k})} \right\}$ as it was done in [1], and order the set
$G_{\alpha,\alpha_k} = \left\{ Y_{\alpha,\alpha_k}^{(G_{\alpha,\alpha_k})} \right\}$ with respect to these hierarchies. It is important that in the considered
case there is a number $n$ of ways $G_{\alpha_k,\alpha_k} = \left\{ Y_{\alpha,\alpha_k}^{(G_{\alpha,\alpha_k})} \right\}$ instead only one way in [1]. Then the
question arises: what hierarchy should be attributed to a subset $Y_{\alpha,\alpha_k}^{(G_{\alpha,\alpha_k})}$ being on different
ways' crossings? We choose the following definition of the subset $Y_{\alpha,\alpha_k}^{(G_{\alpha,\alpha_k})}$ hierarchy: the
hierarchy $H_n$ of this subset is the set (one tuple) containing hierarchies defined for all
ways crossing at subset $Y_{\alpha,\alpha_k}^{(G_{\alpha,\alpha_k})}$, which can be written as
Thus, the hierarchy stopped to be a number, and becomes a set (tuple). The following problem is to find how one can compare hierarchies of two or more subsets $Y_{a,d}$. In real problems different ways in (4) may have different weights because of different subsets $Y_{a,d}$ meaning of the information transferred through them. It must find how to calculate the weight of the tuple (4) from the weights of $h_1, \ldots, h_n$. We denote this weight $W(H_n)$, and shall use it for different tuples to compare hierarchies of different subsets $Y_{a,d}$. In some cases, when there is a large number of such ways, the weighted statistical distribution of different ways may exist. Then the above consideration can be done by use of this probability distribution. If this distribution has narrow peaks, such an approximation could be used when only one way is chosen from each peak corresponding to the average or maximum value of the probability.

One of factors influencing the weight of a way could be the value of the information [8, 9] transmitted by this way. However, probably it is not the only factor. For example, the choice of the problem determines what types of the information are important, and, therefore, ways that transfer it have larger weights than the other ones. The said means that an observer must define what types of the information this observer is ready to accept and treat and what of them should come from each subset $Y_{a,d}$ connected with the considered observer by a number of ways. Remind that, as in [1], each subset
belonging to such a way is able to measure the occurring within itself and within its neighbor (next) subset belonging to the same way. Thus, we order separately each way connecting the considered subset $\mathbf{Y}_{\alpha,\vartheta}$ and observer, and only after all definitions and considerations concerning the information transferred and treated (cf. the written above) were done. On the grounds of the written above one can conclude that a universal ordering of the set (2) $X_\alpha = \exists \mathbf{Y}_{\alpha,\vartheta} \subseteq X_\alpha \left( \forall \vartheta_\alpha \in \mathbb{N}, Y_{\alpha,\vartheta_\alpha} \cap Y_{\alpha,\vartheta_\alpha} = \emptyset \right) \{ Y_{\alpha,\vartheta_\alpha} \}$ on the whole is impossible.

**4. MOLECULAR GENETIC INFORMATION INFLUENCE UPON LIVING ORGANISM**

A living organism is another object for the proposed theory application. Parts of this organism can be represented by subsets $\mathbf{Y}_{\alpha,\vartheta}$. Some of them could be able to treat and interpret the information, i.e., to be *observers*, but types of these observers and of the information [6] treated and interpreted by them could be different. This means, these observers themselves should be of different hierarchies. For example, even different parts of the human brain perform functions of different levels, so the information treated and interpreted by them is of different types [6]. Among other parts of the human organism there exist those able to interpret the information (observers), but on the level lower than the brain does. There are subsets possessing of the status "*observer*" also on the cellular and molecular level. The problem of living organism is much more complicated and difficult than the considered above (for example, for economical systems) because a living
organism is something given, created by the nature, so voluntary changes like described above for the case of economical systems are prohibited. In other words, it must consider the system (organism) as he is. Not each type of living organisms is able to be observer studying himself. A necessary condition of it is the quantum character of his thinking [5]. A human being certainly can be an observer and study himself [5]. However, it is not clear what kinds of other representatives of living organisms, for example, apes, monkeys, octopus etc., are also able to be observers, maybe yes, but maybe not. It is important to take into account also the existence and functioning of external observers, as, for example, human beings with experimental equipment, laboratories etc. who can study other human beings, animals etc. up to DNA molecules, genes, genome and so on. In the case of living beings able to be observers the comparison of results obtained by the self-study and those obtained by external observers is extremely important because in some cases they may be reciprocally complementary (cf. [5]).

A very important property of living organisms is the existence of different sets of scales from the molecular set of scales up to the macroscopic sets of scales, and of the information transfer between different scales. In particular, it includes the genetic information transfer from molecular to the macroscopic scales and the transfer of the information on changes created by learning within macroscopic scales (for example, in the brain) to the molecular scales. In this situation one can expect that observers that are within low level scales forward the information (as the output of their activity) to higher level observers that convert it to the one acceptable there.

Try to consider the process of the information transfer and conversion between different levels of the living organism organization. Note that in terms of the present work the set belonging to the next level of organization is obtained from the previous one by
means of a certain rearrangement of the corresponding set of subsets (taking into account that all living organisms are open systems!) and interactions between elements of sets, as well as between subsets of sets.

We shall begin from the molecular level, \textit{i.e.}, from DNA molecules, and shall continue this process step-by-step, level (of the organization) – by - level, corresponding scale – by - scale up to the whole macroscopic organism. Denote \( L \) organization level, and write the way between two levels as follows:

\[
G_{L', L}^{L', L} = \left\{ Y_{\alpha, d, L}^{(L')} \right\} \cup \left\{ Y_{\alpha, d, L}^{(L)} \right\} 
\]

(5)

As it is seen from Eqn. (4), notations and their set after the symbol \( \cup \) are changed, and differ from those before it. It is natural because the difference of the system at different levels of its organization that leads to a difference, for example, of the set of variables characterizing the system (if, of course, the system can be characterized by variables). Then after \( N \) steps one can write:

\[
G_{d, L}^{L_1, L_2, \ldots, L_N} = \left\{ Y_{\alpha, d, L}^{(L_1)} \right\} \cup \left\{ Y_{\alpha, d, L}^{(L_2)} \right\} \cup \ldots \\
\cup \left\{ Y_{\alpha, d, L}^{(L_N)} \right\}
\]

(6)

Let us consider the general problem of the scaling when there are scales of some different types. To make this problem clear we begin from the scaling with respect system organization levels and some kinds of scaling within each such scale, \textit{i.e.}, at each level of system organization. The written means that a subset \( Y_{\alpha, d} \) can be represented as a set

\[
Y_{\alpha, d}^{(L)} \overset{\text{def}}{=} \left\{ Z_{\alpha, d, \vartheta}^{(L)} \right\}
\]

(7)

Then the Eqn. (2) can be written as follows:
Now the Eqn. (6) should be written as follows:

\[
G_{d_a, d_b, n}^{L_a, L_b, \ldots, L_{n-1}} = \bigcup \left\{ \left\{ Z_{a, b, d_a, d_{b(i)}}^{(L_a, L_{b(i)})} \right\} \bigg| \left\{ Z_{a, b, d_a, d_{b(i)}}^{(L_a, L_{b(i)})} \right\} \right\} \cup \ldots
\]

\[
\ldots \bigcup \left\{ \left\{ Z_{a, b, d_a, d_{b(n)}}^{(L_a, L_{b(n)})} \right\} \bigg| \left\{ Z_{a, b, d_a, d_{b(n)}}^{(L_a, L_{b(n)})} \right\} \right\}
\]

The Eqn. (9) represents a certain way. Really, there are a number of ways including different choice of \( Y^{(L)}_{a, d_a} \) and different choice of \( Z^{(L)}_{a, d_a, \theta_{d_{b(i)}}} \) inside each \( Y^{(L)}_{a, d_a} \), which must be taken into account in Eqn. (4) for hierarchies.

At the consideration of the information transmission described above it is important to distinct between the active information that can initiate certain physical, chemical, psychological, physiological a. o. effects and potential information \([1, 5]\) that must be transformed into the active one to be able to produce any effect. This activation of the potential information can be performed by some natural processes, e. g., physical, chemical, psychological, physiological a. o. ones, but not in the pure mathematical framework. Possibly, not every subset \( Z^{(L)}_{a, d_a, \theta_{d_{b(i)}}} \) is able to perform such an operation, but only certain subsets denoted \( Z^{(L, A_{k})}_{a, d_a, \theta_{d_{b(i)}}} \) and called information activating systems (IAS).

Thus, there are different types of the information propagating through the considered set. Some of them are potential information, others are active information. Active information produced in IASs from corresponding potential information consists of different types of the information (notice that among "products" produced by active information are also
various types of the information). In other words, the existence of IASs leads to the multiplication of the information in numbers and in types. This process may be of the avalanche character. Generally speaking, the information multiplication is accompanied with the different physical, chemical, biological, psychological and so on multiplication processes initiated by the active information. The general study of all these processes is a very complicated, maybe impossible task, but systems with not so big number of processes could be considered.

It seems reasonable to consider separately the hierarchy of the potential information and the active one. Concerning the hierarchy of a subset receiving the potential information and converting it into the active one, its hierarchy must be determined on the grounds of this active information. Taking into account these kinds of the information, one can rewrite the Eqn. (4) as follows:

\[
H_n \left[ \forall n, m \in \mathbb{N} \right] \left( Y^{(G_{p_n}, \alpha_n)}_{a_n, \delta_n} \right)^{\text{def}} = \left\{ \begin{array}{c}
H_1^{(P)} \left( Y^{(G_{p_n}, \alpha_n)}_{a_n, \delta_n} \right) \\
H_1^{(A)} \left( Y^{(G_{p_n}, \alpha_n)}_{a_n, \delta_n} \right) \\
\vdots \\
H_1^{(P)} \left( Y^{(G_{p_n}, \alpha_n)}_{a_n, \delta_n} \right) \\
H_1^{(A)} \left( Y^{(G_{p_n}, \alpha_n)}_{a_n, \delta_n} \right)
\end{array} \right\}, \quad (10)
\]

where \( P \) and \( A \) denote potential and active information, correspondingly, and \( \otimes \) denotes the Cartesian product of sets. The number of tuples in Eqn. (10) can be more than two, if there are more than one types of the potential information or/and more than one types of the active information. It is not difficult to write Eqn. (10) in the general form, taking into account all types of the information \( P \) and \( A \). In the case of the active information obtained
by the potential information conversion within a IAS, the hierarchy of subsets is to be
determined along the way beginning at this IAS, to the direction from this IAS.

Try to write it in the general form:

\[
H_{n,m_0} \left[ \forall (e, \xi) \in \mathbb{N} ; \left\{ (e, \xi) \right\} \right] \left( Y_{a, d} \right)_{e.}^{(G_{n, r})_{f q}} = \\
\prod_{\forall \xi} \left( \left\langle Y_{a, d} \right\rangle_{e.}^{(G_{n, r})_{f q}} \right) \\
\prod_{\forall x} \left( \left\langle Y_{a, d} \right\rangle_{e.}^{(G_{n, r})_{f q}} \right) \\
\prod_{\forall \xi} \left( \left\langle Y_{a, d} \right\rangle_{e.}^{(G_{n, r})_{f q}} \right) \\
\prod_{\forall x} \left( \left\langle Y_{a, d} \right\rangle_{e.}^{(G_{n, r})_{f q}} \right),
\]

(11)

where symbols \( \otimes \) and \( \prod_{\forall \xi} \) mean Cartesian product of two sets and that of any number of
sets, correspondingly.

The meaning of Eqn. (11) is clear. The hierarchy defined by Eqns. (4), (10) and
(11) is a set, but not a number. If it is possible to put in correspondence to such a set a
certain number, e. g., the norm when the considered set is a normalized space, such
numbers could be used for comparison of hierarchies of different subsets with the purpose
to establish the scaling, but only under the condition that this number has the clear natural,
for example, physical, biological, economical etc. meaning.

How to use hierarchies for ordering the considered set? Hierarchies are
established along a certain way (see above). So each way should be ordered separately
following the order of subsets' hierarchies that were established namely along this way.

We propose to order the whole lattice using two numbers: number determining the
hierarchy of the considered way relatively other possible ways in this poset (see above),
and the subset hierarchy inside the chosen way (see above). It should implement the
condition that all subsets of a certain way possess the hierarchies higher than those in any way having the hierarchy lower than that of the considered way and vice versa. In other words, it is to order a lattice with respect to ways' hierarchies and thereupon with respect to hierarchies of subsets along each way.

Let us consider now the information transfer from DNA molecule to a living cell, from the living cell to organs of a living organism and so on. It is necessary to consider also the inverse process when the information obtained within a macroscopic scale is transferred to the molecular scale, for example, to forward brain learning results up to molecular genetic material, and by this to keep them as changes of the heredity (cf. Ch 7 of [5]).

Up to this place statistics was not used. However, when one begins the consideration of the information transfer between scales from DNA, it must remember that there are an enormous number of DNA molecules, and so the use of statistical methods is inevitable. There are two possibilities: A) to use DNA molecules statistical distribution and to consider the information transfer from the DNA molecules' collective distributed so, and B) the consideration of the information transfer from DNA molecules, and to apply statistics to the result at a certain scale (considered as the final result). The second way seems to be more complicated, but correctly than the first one.

The statistical distribution of DNA molecules includes their distribution with respect to rotational, vibration and electronic states. The role of these states in genetics was discussed in Ch.7 of [5]. Indeed, the behavior of a living system, its reactions to the influence of outside factors etc. occur as result of transitions between molecular quantum states. This means, these states must be taken into account at the consideration of the genetic information curried by DNA molecule and genome, as well. The approach
developing in the present work seems to be relevant to the direct consideration of the molecular quantum states’ influence of the macroscopic behavior and other macroscopic effects of living systems.

\section{5. Statistical Approach}

Try to rewrite the Eqn. (10) for the hierarchy taking into account the statistical distribution. We shall represent each subsystem $Y_{\alpha,\alpha'}$ as follows $Y_{\alpha,\alpha'} = (\forall \rho)\{Y_{\alpha,\alpha'}^{\rho}\}$, where $\rho \in \mathbb{N} \cup \mathbb{Z}$ is the index denoting states of this subsystem $Y_{\alpha,\alpha'}$. In other words, we represent this subsystem having a set of states as a set of subsystems. The next step is the introducing the distribution function $\Phi$ of this subsystem with respect to its states, or, which is the same thing, distribution in ensemble $(\forall \rho)\{Y_{\alpha,\alpha'}^{\rho}\}$. Now one can rewrite the Eqn. (11) as follows:

$$H_{n_{\alpha,\alpha'}} \left[ \forall n_{\alpha,\alpha'}, (\epsilon, \xi) \in \mathbb{N}; \left[ \forall (\epsilon, \xi) \right] \left[ \{\epsilon, \xi\} \right] \left[ (\forall \rho)\{Y_{\alpha,\alpha'}^{\rho}\} \right] \right] =$$

\begin{equation}
\prod_{\forall \epsilon} \left( \Phi \right) \left[ \left[ \forall \rho \right] \left[ Y_{\alpha,\alpha'}^{\rho} \right] \right] \left[ \left[ \forall \rho \right] \left[ Y_{\alpha,\alpha'}^{\rho} \right] \right] \left[ \left[ \forall \rho \right] \left[ Y_{\alpha,\alpha'}^{\rho} \right] \right] \left[ \left[ \forall \rho \right] \left[ Y_{\alpha,\alpha'}^{\rho} \right] \right] \left[ \left[ \forall \rho \right] \left[ Y_{\alpha,\alpha'}^{\rho} \right] \right]
\end{equation}

Note that the function $\Phi$ may be single-system $Y_{\alpha,\alpha'}^{\rho}$ distribution function, or a multi-system one, as it is well known from statistical theories, for example, statistical physics. The last version contains more information and, in particular, is fit for the correlations’
representation, but is very complicated and sometimes makes difficult to find problem solutions. However, it must be very careful when we choose to limit ourselves with only single-system \( Y_{\alpha,\varphi,\rho} \) distribution function or, in general, with the distribution function of a limited number of such systems instead of the maximum existing number of them.

Suppose that \( \Phi \) is the density matrix. Then, in distinct to the Eqn. (12) the hierarchy will be defined not in terms of sets \( h_n \) of sets of the type \( (\forall \rho) \{ Y_{\alpha,\varphi,\rho} \} \), but in terms of the density matrix \( \Phi \left( (\forall \rho) \{ Y_{\alpha,\varphi,\rho} \} \right) \) hierarchies at different scales. The density matrix should be defined differently for different scales. Let us introduce the index \( \Lambda \in \mathbb{N} \) as index of the scale. Then Eqn. (12) can be rewritten as follows:

\[
H_{(n_1),\Lambda} \bigg[ \left( \forall n_2, m_2, \xi, \Lambda \right) \in \mathbb{N}; \left( \forall (\epsilon, \eta) \right) \left( \{ \epsilon, \eta \} \right) \bigg] \left( (\forall \rho) \left( \{ Y_{\alpha,\varphi,\rho,\Lambda} \} \right) \right) =
\prod_{\forall \xi} \left( \bigotimes \right)
\begin{align*}
\begin{pmatrix}
\vdots \\
\prod_{\forall \xi} \left( \bigotimes \right)
\end{pmatrix}
\end{align*}
\]

Thus, Eqn. (13) takes into account those ways which can lead from a subset belonging to a scale \( \Lambda \in \mathbb{N} \) to another subset belonging to the same or another scale. Among possible such ways may be those beginning, for example, from molecular level (e. g., DNA molecule) and ending at macroscopic level (e. g., the brain). These ways, if they exist, are especially interesting because just they serve for the genetic information transfer from DNA directly to the living organism. However, it seems more realistic that the
information is transferred between close (maybe neighbor) scales with the close values of \( \Lambda \in \mathbb{N} \), and the genetic information reaches the macroscopic scales by step – by – step way, but not directly, not at once. They may serve also for the information transfer to the inverse direction, i.e., from the whole living organism to the molecular genetic material, fixing the learning results in genetics of the living organism. For example, the learning of the neural network model of the brain changes synapses (exactly, their representation in this model). It would be natural to suppose that in the real brain occurs something like that. Possibly, abovementioned structural changes on the macroscopic level may, at the end, influence the molecular genetic material.

Using the notations written above, one can introduce the density matrices \( \Phi_\Lambda \) for different levels (from molecular level up to the macroscopic one), i.e., different values of \( \Lambda \in \mathbb{N} \). These density matrices must satisfy a time-dependent equation, including transitions between density matrices with different values of \( \Lambda \in \mathbb{N} \). It would be interesting and practically useful to clarify whether something like the rate approximation could be valid under certain conditions. If the rate approximation is valid, one can represent the movement of the information, for example, from molecular to macroscopic level by equations like those of the chemical kinetics with transitions from a certain \( \Lambda \in \mathbb{N} \) value to the other one.

There is an important principal problem. This problem arises already on the lowest level of the system organization, i.e., at the lowest value of \( \Lambda \in \mathbb{N} \). Indeed, the genetic information is contained in each DNA molecule. But there are an enormous number of DNA molecules. How the genetic information influences this collective? How it is expressed by its statistical distribution \( \Phi_\Lambda \)? Moreover, how this information is
transferred to higher levels of the system organization and how it is expressed there, where
variables characterizing the system and determining its statistical distribution \( \Phi_{\Lambda\Lambda} \) are
quite different from those on the level \( \Lambda \) of the organization? Try to search for a relevant
mathematical formalism to express the conversion of the molecular genetic information to
that contained in the statistical system of such molecules and thereupon to its conversion
at transitions between different levels of the system organization. The success of this
try would mean that we shall find the explanation of the wonder: the genetic
information contained in each separated DNA molecule influences the whole macroscopic
living organism. This means, this information has a very big, maybe enormous value. How
it could be realized? One of possibilities is that there is a kind of "resonance" between all
DNA molecules containing the same information, in other words, their statistical
distribution with respect to the information carried by them should have a sharp peak in
the close environment of the considered information. But how it could be realized? How
and under what conditions the behavior of the DNA molecule collective differs so
profundly from the one of the gas of the same, but independent molecules? It is to clarify
also whether it is enough that this "resonance" would exist only for the active information,
or it is necessary that it would exist simultaneously for the corresponding potential
information. The connection between genes, genome, micro- and macroevolution are
discussed in detail in the article [7]. This consideration of the connection between micro-
and macroevolution means the consideration of small and large time-intervals (more
precisely, time-steps). The mathematical formalism proposed in the present paper can be
applied to the study how the occurring within each small time-interval creates the
occurring within a large time-interval formed of a number of such small time-steps.
However, the main question is whether the direct study a large time-interval (i. e., a step of
the macroevolution) leads to results identical to those obtained by is study as built of successive small time-intervals. The answer is not clear. This problem merits serious considerations because this is the key to the understanding the evolution. A very important element of such a consideration is the correct choice of measurements direct and indirect (see below Sec. 7) that could help to find the correct organization of paleontological works aiming to the evolution studies.

6. SET THEORETICAL APPROACH TO THE PROBLEM

Let us consider a well-ordered set

\[ S = \{ L \in \mathbb{N} \left| L \in [1, L_{\max} > \forall L] \right. \} \{ S^{(L)} \}, \tag{14} \]

where \( \forall S^{(L)} \) are well-ordered sets constructed as follows. Let \( S^{(0)} \) is the given well-ordered set under consideration and \( S^{(i)} \) is the set of all such subsets of \( S^{(0)} \) forming the set \( S^{(0)} \) at a certain(1st) level of organization. Here we introduced the notion organization of a set. This means, we suppose that only sets possessing a certain property are considered. We call it organization, but it may be called by another name. Continuing this process, we define the set \( S^{(2)} \) as the set of all such subsets of \( S^{(1)} \) forming the set \( S^{(0)} \) at the second level of organization etc…up to \( S^{(L_{\max} > \forall L)} \) that we shall call the highest level of the organization. Thus, the set \( S = \{ L \in \mathbb{N} \left| L \in [1, L_{\max} > \forall L] \right. \} \{ S^{(L)} \} \) represents all levels of the organization of the studied set \( S^{(0)} \). If possible to introduce a kind of the normalization, each set \( S^{(L)} \) must be normalized exactly as the set \( S^{(0)} \). Possibly the set measure can be used with this purpose. We shall return to this problem below.

What this means, the set organization? This is the system of connections between elements and subsets of the considered set creating the interdependence between them.
The choice of subsets forming a set $S^{(L)}$ is an important factor determining the level of the set $S^{(L)}$ organization. It is evident that there are different such choices, therefore one should consider ensemble of sets $S^{(L)}$. Members of this ensemble may have different levels of organization, as it was written above, and should be denoted as follows: $S^{(L)} (\forall s \in \mathbb{N}, s \in [1, s_{\text{max}}])$. Principally some of them may enter to the region of $L' < L$, i.e., to a previous level of the organization, and this possibility may have important consequences, for example, in biology, and should be considered seriously. If one considers the statistical distribution of such an ensemble, a certain level of its organization can be defined only when this distribution have a sharp peak at this level of organization, in other words, when the big majority of the ensemble's members have almost the same the level of organization. It could be realized by means of the abovementioned connections between subsets chosen.

As it is seen from the written above, probably, the concept level of organization can be defined only in each concrete case. It is not yet clear whether its general definition can be formulated.

It is important to remember that the concept of level of organization, generally speaking, does not coincide with the concept of scale, i.e., index $L$ has nothing to do with index $\Lambda$. Of course, in some particular cases they may accidentally coincide, but not in the general case.

Consider now the case of human beings. Each human being is able to be an observer for him. In this case his brain or some parts of the brain (relevant to the studied problems) must have the highest hierarchy. The hierarchy decreases along each way directed from the brain. For the sake of simplicity we accept that all laboratory tests, as for
example, blood tests, are performed by other observers, even in the case when really the considered person does them himself (e. g., blood glucose test, blood pressure measures etc.), but at the end the information, the whole or parts of it, is transferred to the main observer, i. e., the considered human being, influencing his decisions concerning his behavior. Remind that the very important problem of the mind self-measuring was considered in detail in [5].

Let us now begin from the highest level of the human being organization. This level is the one of the observer. Denote the corresponding scale $\Lambda = 0$. The following scales $\Lambda > 0$ and rise when self-measurements performing by the observer advance to lower-and-lower level of the organization $L$ (cf. [1]). Of course, $L$ decreases when $\Lambda$ increases. The Eqn. (13) determining the hierarchy corresponding to various ways remains valid for this case. In the case of a human being the information is obtained by the observer (brain or its certain parts) and sent by the observer to different parts of the body, for example, ordering them what is to be done. The most interesting is the consideration of the feedback, i. e., cases when the information sent by the observer (brain) to different parts of the body was created on the grounds of the information obtained by the observer from parts of the body with $\Lambda > 0$. By this way one can connect the human thinking and behavior, and find their interdependence. The second step should be the study of the information obtained from different scales of the body by means measuring instruments, and clarification how this information compatible/incompatible with that obtained by this human being mind without use measuring instruments (see above). The third step should be the consideration of the increase the organization level $L$ with the increase of the scale number $\Lambda$ beginning from DNA molecules. The consideration of all three steps together
is expected to bring an explanation of the molecular genetic influence thinking and behavior. However, it is a very complicated and difficult problem.

7. DIRECT AND INDIRECT MEASUREMENTS

In [1] the concepts of direct and indirect measurements in the quantum mechanics introduced and developed in [10, 11], was considered in the connection with the scaling in the HEP. Let us consider them in the connection with the complex system scaling. We shall focus our attention here on biological systems, but this consideration can be, at least in general outline, valid for other kinds of systems.

Let $Y_{\alpha, \vartheta, \kappa} \ (\kappa \in \mathbb{N})$ be an observer. Let a subset $Y_{\alpha, \vartheta}$ is on the way $G_{\vartheta, \kappa} = \{ Y_{\alpha, \vartheta}^{(G_{\vartheta, \kappa})} \}$, i.e., $Y_{\alpha, \vartheta}^{(G_{\vartheta, \kappa})} \in G_{\vartheta, \kappa} = \{ Y_{\alpha, \vartheta}^{(G_{\vartheta, \kappa})} \}$ within the scale $\Lambda$ relative to the observer $Y_{\alpha, \vartheta, \kappa} \ (\kappa \in \mathbb{N})$ defining along the way $G_{\vartheta, \kappa} = \{ Y_{\alpha, \vartheta}^{(G_{\vartheta, \kappa})} \}$. Measurements performed on the considered subset $Y_{\alpha, \vartheta}^{(G_{\vartheta, \kappa})} \in G_{\vartheta, \kappa} = \{ Y_{\alpha, \vartheta}^{(G_{\vartheta, \kappa})} \}$ can be made from outside of this subset $Y_{\alpha, \vartheta}^{(G_{\vartheta, \kappa})}$ by measuring instruments, but the obtained information will be transmitted to the observer along the way $G_{\vartheta, \kappa} = \{ Y_{\alpha, \vartheta}^{(G_{\vartheta, \kappa})} \}$. These are direct measurements. But there are measurements made within a certain scale $\Lambda$ with the purpose to obtain information on occurring within the scale $\Lambda + 1$ or following scales. These are indirect measurements (cf. [1]). Indirect measurements are necessary when direct measurements within the scale $\Lambda$ cannot be interpreted on the grounds of laws of the nature ruling within the scale $\Lambda$. Then it would be natural to suppose that the occurring within the scale $\Lambda + 1$ influences the occurring within the scale $\Lambda$. It could allow one to
find what occurs within the scale $\Lambda + 1$, which is the main purpose of this indirect measurement, as well as to find the interpretation of direct measurements within the scale $\Lambda$. The case when subset $Y_{\alpha, \theta}$ is on the crossing of two or more ways leading to the same or different observers demands the special attention on the grounds of the above made consideration, including Eqn.(13) for the subset hierarchy definition when there is a number $>1$ of ways.

8. CONCLUSIONS

In the present paper a method of scaling in complex systems is proposed and developed. This method is a generalization of the method of scaling proposed, developed and applied in previous works of the author [1, 2] for the scaling of the micro-World in the connection with HEP. The method proposed in the present paper is fit for applications, for example, to the mathematical study of economic systems, of living systems, of systems such as the human society (as the whole and the parts) demanding a kind of scaling, of the parliamentary and, in general, political activity etc..

The scaling is based on the definition of hierarchy of subsets of the considered set. In distinct from the consideration made in [1, 2] the hierarchy in complex systems, generally speaking, cannot be defined as a number, but as a set of numbers. It is a consequence of the existence of different ways for the information transmission from a certain subset to other ones and vice versa. The hierarchy definition of a certain subsystem is based on the information properties [1], i. e., on the type of the information obtained from each subsystem by a certain way, processed inside it, forwarded from the considered subsystem to the other ones, including the observer, by the ways connected them. Just because of it along each way one defines only a partial hierarchy of the considered
subsystem. The set of such partial hierarchies forms the hierarchy of the considered subset.

In the framework of the genetic information influence upon living organism the concept of organization level is introduced and the ways of the information transmission between different organization levels is considered and the relevant mathematical formalism is constructed. Really this consideration is of much more general character and can be applied also to economic a. o. complex systems. At this consideration and mathematical formalism two types of the information, potential and active ones are considered explicitly, and the definition of the subset hierarchy is extended to distinguish between its parts caused by the potential and active information.

The use of statistical approach and how the shape of statistical distribution could influence the genetic information transmission from genes to the macroscopic living organism is discussed.

The levels of the system organization are considered in terms of the set theory.

As it was done in [1] for elementary particles' problems, in the present paper direct and indirect measurements are considered for the general case of complex systems. Usually the concepts of direct and indirect measurements appear only in quantum mechanics, but we came to the conclusion that they are relevant and moreover necessary at the consideration of complex systems, as well.
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