Spectral structure of phase-induced intensity noise in recirculating delay lines

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Abstract

The dynamic range of fiber optic signal processors driven by relatively incoherent multimode semiconductor lasers is shown to be severely limited by laser phase-induced noise. It is experimentally demonstrated that while the noise power spectrum of differential length fiber filters is approximately flat, processors with recirculating loops exhibit noise with a periodically structured power spectrum with notches at zero frequency as well as at all other multiples of 1/(loop delay). The experimental results are augmented by a theoretical analysis.

1. Introduction

Recently, several fiber-optic signal processors have been produced which employ recirculating single-mode loops. These include the recirculating fiber-optic delay line and the fiber-optic systolic matrix-vector multiplier. The simplest recirculating delay line, Fig. 1, is made by closing a continuous single-mode fiber loop on itself using an adjustable directional coupler. A single optical pulse at the input of the device, gives rise to an infinite number of pulses at its output due to multiple recirculations of the injected radiation in the closed loop. This characteristic of the device makes it suitable to be used as a transient buffer memory. The recirculating delay line can be also used as a filter. In this application, an RF signal modulates the intensity of the light source. The injected wave is optically summed at the coupler with its various delayed versions to produce an output signal whose depth of modulation strongly depends on the modulation frequency. Since the laser driving current and the detector output current are, respectively, proportional to the injected and detected light intensities, overall linearity is ensured only if the above mentioned optical summation is linear in the intensities of the interfering waves (rather than in their wave amplitudes). This requirement is easily met by using low coherence sources with a coherence time much shorter than any relevant loop delay. When the coupling ratio of the directional coupler is set to approximately 38, the recirculating delay line exhibits uniform notch and bandpass characteristics: maxima occur at multiples of 1/τ (τ is the loop delay) where the injected modulated light intensity adds constructively with its delayed versions and very deep notches (in excess of 15dB) appear at 1/2τ, 3/2τ,... where destructive addition filters out most of the modulation and produces a constant unmodulated signal at the device output. A filter with notches at (0.5 + n)MHz (n is a non-negative integer) and uniform response through 1.3 GHz has been demonstrated with a 200m recirculating delay line (The upper limit was set by the measurement system). Due to the low coherence of the source, the device is practically insensitive to environmental effects (e.g. temperature changes, vibrations, etc.).

In another application, several recirculating loops are coupled to form a systolic matrix-vector multiplier (Fig. 2). Here, the device operates in the time domain, and bipolar inputs are generated by driving the laser source above and below a constant pedestal. Again, low coherence sources must be used to ensure that the coupling process in the directional couplers will be linear in the intensities of the input signals.
the guided waves.

It may appear, therefore, that the dynamic range of these wide-band devices will be restricted from above by nonlinearities in the fiber and from below by the laser intensity noise (assuming a relatively "quiet" detection system). It is the purpose of this paper to show that the interference process which takes place in the coupler between its two input waves, while being insignificant for the filtering operation, is responsible for the conversion of the laser phase noise into a strong intensity noise at the output of the detector. In fact, most frequently this phase-induced intensity noise will set the lower bound of the dynamic range characteristics of single-mode fiber-optic signal processors which employ evanescent field couplers and use low coherence semiconductor laser sources. We also demonstrate that the structure of the power spectrum of the phase-induced intensity noise at the output of the detection system is highly dependent on the device configuration. It is relatively flat for differential length, Mach-Zehnder type, configurations (see Fig. 8), in agreement with previous studies in nonrecirculating interferometric sensors as well as in homodyne and heterodyne communication systems. However, when recirculating loops are present the noise spectrum is characterized by minima at zero frequency as well as at other integer multiples of the inverse of the loop delay.

Experimental spectra are presented in Sec. 2 for both recirculating and non-recirculating single-mode fiber loops and Sec. 3 provides a theoretical explanation of the observed effects.

2. Experimental arrangements and results

2.1. A 27 cm recirculating loop:

Fig. 3 describes the experimental set-up. As shown, the mechanically polished evanescent field coupler closes the delay line with no splices. The loop length was 27 cm which corresponds to a delay of $r = 135$ nsec. The continuous-wave GaAlAs multi-mode laser (GENERAL OPTOELECTRONICS Model GO-ANA) operates with no external modulation. The PIN photodetector is followed by a 1-1000 MHz amplifier whose output is directly fed to the input of the spectrum analyzer. The bottom curve A in Fig. 4 describes the spectrum of the amplifier output when the laser is off. Curve B shows the output spectrum when the coupler is disassembled into its two halves, so that the laser light goes directly from the laser through the fiber to the detector. This spectrum is therefore characteristic of the intensity noise of the laser. The upper curve C was taken with an assembled coupler and a power coupling ratio of approximately 40%. Here we see a much stronger spectrum with two notches (within the 1-1000 MHz range): one at $r = 0$ and the other at a frequency of $1/r$ (≈ 740 MHz). Therefore, the dynamic range of the loop, when operating as a filter, is limited (from below) not by the laser intensity noise but by a 20 dB stronger noise which attains its maximum at the notch location $(1/2r)$. The observed noise spectrum was found to be highly dependent on the coupling ratio, see Curve D. In particular, when this ratio was either 0 or 100% the noise reduced to the level of curve B in Fig. 4, and a coupling ratio of ≈40% was found to give the highest noise.

It was experimentally determined that the above spectral structure was completely insensitive to loop length variations on the order of an optical wavelength, indicating that $r \gg r_s$ where $r_s$ is the coherence time of the laser (which is on the order of 200 nsec).

In order to investigate the hypothesis that the observed spectral structure arises as a result of optical reflections from the device back into the laser, a directional coupler was used to split the laser output into two parts. The output pigtailed were then butt-coupled to two recirculating delay lines with different characteristic delays (1.35 and 50 nsec). If optical feedback was important, one
should expect that changing the coupling ratio in one of the loops would affect the power spectrum at the output of the second loop. Since the loops appeared to behave completely independent of each other, the feedback hypothesis was rejected.

2.2. A 200m recirculating loop:

Fig. 5 shows the experimental set up for a 200m loop \( r = 1 \mu \text{sec} \). Again (Fig. 6), the spectrum is characterized by equally spaced notches located at all integer multiples of \( 1/r \). In this long loop it was possible to incorporate a manually adjustable polarization controller which can change the state of polarization of the propagating wave without altering its degree of polarization. As the state of polarization of the recirculating waves was varied, we observed a few dB vertical shifts of the noise power spectrum, see Fig. 6. Note that this spectral picture is complementary to the transfer function of the recirculating delay line when operating as a filter for RF modulation signals applied to the incident light.

Fig. 5: A recirculating delay line with a 200 ± 5m loop. We failed to close the loop on itself due to a break in the output pigtail of the upper half coupler which forms coupler 2. Instead we used an "extension" made of the above mentioned 27cm loop in its open configuration. The only disadvantage of this configuration is an additional loop loss. Coupler 2 was adjusted for maximum coupling.

2.3. A 200m non-recirculating differential delay line:

Fig. 7 shows the experimental set up for the 200m non-recirculating, Mach-Zehnder type, loop. Since the impulse response of this configuration is composed of two impulses separated by the loop delay \( r \), the transfer function of the device is proportional to \( 1 + \cos(2\pi f r) \), where \( f \) is the modulation frequency. Indeed, there is a setting of the polarization controller where the laser intensity noise, Curve 3 in Fig. 8, is filtered by the differential delay line according to \( 1 + \cos(2\pi f r) \), producing maxima at multiples of \( 1/r \) and minima in between, see Curve 4a in Fig. 8. There is, however, another setting of the polarization controller, Curve 4b, which is characterized by a much stronger yet spectrally flat power spectrum.

Fig. 7: The 200(± 5)m differential Mach Zehnder delay line.

Fig. 8: Experimental noise spectra for the set up of Fig. 7: (1) the spectrum analyzer noise, (2) the amplifier noise, (3) the laser intensity noise (the straight line) (4) phase-induced noise. The coupling ratios of #1 and #2 are adjusted to provide maximum noise and the setting of the polarization controller is varied [(4a) and (4b)].

2.4. A 70cm differential delay line with intensity addition:

In the set up of Fig. 7, coupler #1 added together the wave amplitudes from the two arms. Since the detector temporally averages the output light over periods much longer than the source coherence time, the output intensity is still the sum of the two input intensities. Alternatively, by pointing the outputs of the two arms directly onto the detector surface, Fig. 9, spatial averaging results in intensity summation even with a coherent source. While Fig. 10 clearly shows the filtering action of the device, we could not observe any enhanced noise, similar to that shown in Figs. 4, 6 and 8.

24
Fig. 9: A 70cm differential delay line with non-interferometric intensity addition.

![Diagram](image)

Fig. 10: Experimental noise spectra for the set up of Fig. 9. The loop delay is 3.5 nsec and the frequency spacing between successive notches is ≈ 300MHz. (1) the laser intensity noise (Coupling ratio=0), (2) the phase-induced noise (Coupler tuned to produce maximum noise).

3. Theoretical model

The experimental results of Sec. 2 show that whenever the input wave interacted with its delayed versions on an amplitude basis, the output intensity noise was much stronger than the laser intensity noise. This fact combined with the polarization sensitivity of the enhanced noise strongly suggest that the delay line, operating as a phase discriminator, interferometrically converts the laser phase noise into a strong intensity noise at the output of the detection system.

In order to simplify the analysis, we assume a single-mode laser emission of the form: $\exp[i(\omega t + \phi(t))]$, where $\omega$ is the center optical frequency of the laser, and $\phi(t)$ is the laser phase noise. Laser intensity noise is neglected (see Fig. 4), and we proceed now to analyze the differences between the recirculating and non-recirculating delay lines.

3.1. Laser phase noise in non-recirculating delay lines:

Here, see Fig. 7, the electric field vector at the detector input may be expressed as

$$ E(t) = V_i \exp(i[\omega t + \phi(t)]) + V_d \exp(i[\omega(t-r) + \phi(t-r)]) $$

where $V_i$ and $V_d$ depend on the coupling ratios of the two couplers as well as on the state of polarization of the outputs of the two arms. The detector output is proportional to

$$ I(t) = |E(t)|^2 = |V_i|^2 + |V_d|^2 + 2|V_i V_d| \cos(\omega r + \phi(t) - \phi(t-r)). $$

Obviously, if the polarizations of $V_i$ and $V_d$ are orthogonal to each other, the output intensity is the sum of the intensities in each arm, and the spectrum analyzer will display a filtered version of the weak laser intensity noise (which has not been taken into account in our model). Otherwise, the interference term in Eq. (2) converts the temporal fluctuations of $\phi(t)$ into fluctuations of $I(t)$ itself.

The output of the AC-coupled spectrum analyzer is related to its input, $w(t)$, by the Wiener-Kinchine theorem, namely: the autocovariance function of $w(t)$, $<w(t) w(t-t')>$, is equal to $<w(t) w(t-t')>$ where $< >$ denote ensemble average and the density, $S(f)$, of the displayed spectrum, are a Fourier transform pair. Since the spectrum analyzer input voltage is proportional to the output light intensity $I(t)$, $S(f)$ is related to the autocovariance function of $I(t)$ by

$$ \text{Cov}(t, t-t') = \frac{<I(t)I(t-t')>}{\sqrt{<I(t)> <I(t-t')>}} = \int_{-\infty}^{\infty} S(f) \exp[2\pi i(t-t')f] df. $$

(The spectrum analyzer display is actually given by $P(f)$ whereas the proportionality factor, $P$, depends on the detector responsivity and the amplifier gain, as well as on the resolution setting of the spectrum analyzer).

Expressions for Cov($t, t-t'$) were derived by several authors\(^{11,12}\). Assuming that $\phi(t)$ follows a Gaussian probability distribution and that the laser emission line has a Lorentzian shape, it may be shown\(^{11,12}\) that

$$ \text{Cov}(t, t-t') = \frac{4|V_i V_d|}{\sqrt{2\pi r}} \exp\left(-\frac{|t-t'|}{2r}\right) \left[\exp\left(-\frac{1}{2r}\right) - \frac{1}{2} \right]. $$

Eq. (4) correctly describes the phase-induced intensity noise in nonrecirculating interferometric sensors\(^3\) as well as in homodyne and heterodyne communication systems\(^1\). The characteristic delays in these devices are usually much smaller than $r$, and the covariance function of Eq. (4) is extremely sensitive to very small variations in $r$.

When $r >> r_s$, Eq. (4) reduces to

$$ \text{Cov}(t, t-t') \approx 2|V_i V_d| \exp(-|t-t'|/r) $$

which is no longer dependent on $\omega$ or $r$.

The power spectral density of $I(t)$, i.e., the Fourier transform of $\text{Cov}(t, t-t')$, Eq. (5), is given by

$$ S(f) = \frac{|V_i V_d|}{2\pi r_s} \frac{r_s}{1 + (2\pi f r_s)^2}. $$

It obtains its maximum at $f = 0$, and when $r_s$ is much smaller than the device delay $r$, the spectrum near $f = 0$ is relatively flat within several $1/r$ units, in agreement with Fig. 8. $S(f)$ is not sensitive to variations of $r$ which are on the order of the period of the optical carrier. However, it depends on the setting of the polarization controller through the scalar product of $V_i$ and $V_d$.

The laser phase noise is converted to intensity noise as
3.2. Laser phase noise in recirculating delay lines:

While the output of the differential delay line filter is the sum of two contributions, an infinite number of recirculations are added to the injected wave to form the output of the recirculating delay line. We now proceed to show that the experimentally observed periodic spectral structure can be actually predicted from a model which takes into consideration both the laser phase noise and the unitary nature of the directional coupler.

The input-output relationship of the directional coupler can be described by a 2x2 complex transfer matrix,

\[
\begin{bmatrix}
    E_i^1 \\
    E_i^2
\end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix}
    E_r^1 \\
    E_r^2
\end{bmatrix},
\]

where \(E_r^1, E_r^2\) and \(E_i^1, E_i^2\) are, respectively, the input and output complex amplitudes of the vector fields \(E_i^1, E_i^2, E_r^1,\) and \(E_r^2,\) which are assumed, for simplicity, to have the same state of polarization. This last assumption is experimentally verified using a polarization controller,\(^{25}\) which is set to compensate for the loop birefringence so that the polarizations entering the coupler from the laser \(E_i^1\) and the fiber loop \(E_r^2\) are the same. (The coupler itself exhibits very little dependence on the state of polarization of the input fields\(^ {23}\).) Since backward reflections are negligibly small, the coupler matrix can be approximated\(^{25}\) by the product of an over-all transmission factor, \((\gamma)^N\) (0.9 < \(\gamma < 1\)) and a unitary matrix. Therefore: \(|A|^2 + |B|^2 = |C|^2 + |D|^2 = 1\) and \(CA^* + DB^* = 0\) (The star denotes complex conjugation).

The field amplitude at the output of the recirculating delay line \(E_r\) is the sum of contributions from an infinite number of recirculations:

\[
E_r = (\gamma)^N \sum_{n=0}^{\infty} (i\omega t + \phi(t)) + \gamma DA \sum_{n=1}^{\infty} (i\omega t + \phi(n)) e^{-\alpha z_s} \exp[i(t-n\tau) + \phi(n-r)] ,
\]

where \(r\) and \(L\) are, respectively, the loop delay and loop length and \(\alpha_s\) is the fiber (amplitude) attenuation per unit length which was on the order of several dB/km.

The covariance function of the intensity fluctuations, Eq. (3), can be constructed from Eq. (8) and the ensemble average can be evaluated by repeated applications of Eq. (4). Assuming again that \(\tau \gg \tau_r\), one can show (to be published) that

\[
\text{Cov}_s(t_1, t_2) = \text{Cov}_s(t_2, t_1) \theta \exp[-(t_1 - t_2)/\tau_r] ,
\]

where \(\theta\) denotes a convolution operation and \(\text{Cov}_s(t_1, t_2)\) is given by

\[
\text{Cov}_s(t_1, t_2) = \sum_{M=-\infty}^{\infty} \text{M} \delta \{ (t_2 - t_1) - \text{M} \tau \} ,
\]

\(k(t)\) is the Dirac delta function and

\[
G_M = \begin{cases} 
2 |A| \gamma^M |D| \exp[-2\alpha_s] \exp[-2|B|^2] & \frac{1}{1 - \gamma \exp(-2\alpha_s) |B|^2} \\
2 \gamma^{M+1} |A| |C| |B| \gamma M \exp[-2|B|^2] & \frac{1}{1 - \gamma \exp(-2\alpha_s) |B|^2} \\
2 \gamma^{M+1} |A| |C| |B| \gamma M \exp[-2|B|^2] & \frac{1}{1 - \gamma \exp(-2\alpha_s) |B|^2} \\
\end{cases} 
\]

Since \(A, B, C\) and \(D\) are the elements of a unitary matrix, it follows that all the \(G_M\) with \(M \neq 0\) are negative. Therefore, \(\text{Cov}_s(t_1, t_2)\) is a symmetrical function of \(t_1 - t_2\). It is the sum of a positive impulse at \(t_1 = 0\) and infinite number of equally spaced negative impulses at \(t = M \tau\) (\(M \neq 0\)).

The power spectral density is the product of the two Fourier transforms of the components of Eq. (9). Hence,

\[
S(f) = S_i(f) \frac{2\tau_r}{1 + (2\pi f \tau_r)^2} .
\]

\(S_i(f)\) which is the Fourier transform of \(\text{Cov}_s(t_1, t_2)\), is shown in Fig. 11 for various values of \(B\) and \(\alpha_s\). When the loop is lossless, infinitely deep notches appear, Fig. 11a. But their depths decrease as the loop loss increases. It is interesting to compare the loop behavior with the low coherence source to its performance as a resonator with a highly coherent HeNe laser\(^ {2}\). In the resonator case, a lossless loop is characterized by an infinite finesse. However, the loop transmission with the HeNe laser is extremely sensitive to micron-size variations in the loop length, while with our relatively incoherent source the characteristic spectrum is environmentally insensitive. Another interesting feature in Fig. 11 is the dependence of the form of the spectrum on the coupling ratio: the higher the coupling ratio, the flatter is the power spectral density within any given period. This dependence can be correlated with the fact that the number of recirculations increases with the coupling ratio.

The above analysis was limited to guided waves with a single polarization and such cannot predict the polarization dependence of the spectrum in Fig. 6. An intuitive explanation parallels the one given previously in Sec. 3.1 for the differential length filter with an important distinction. While it is possible to adjust the polarization controller such that the polarization of the first recirculated wave is made orthogonal to the polarization of the input wave, the polarization of the second recirculation will be again parallel to that of the input wave. Thus, it is impossible to completely eliminate the phase induced intensity noise, see Curve 4a in Fig. 6.

4. Conclusion

In this paper we discussed the phase-induced intensity noise at the output of single-mode fiber-optic signal proces-
sors with particular emphasis on the spectral structure of this noise for processors with recirculating loops. Experimental results were presented for loops with different lengths along with a theoretical treatment which confirmed most of the experimental data. Current investigations deal with (a) the dependence of the power spectral density on the setting of the polarization controller in processors with recirculating loops; (b) the extension of the theoretical analysis to multi-mode lasers and (c) the effects of the (so far neglected) source intensity noise on the total noise at the output of single-mode fiber-optic signal processors.


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References
