Signal-to-Noise Ratio in Raman Active Fiber Systems: Application to Recirculating Delay Lines

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Abstract—An analysis of photon count statistics of noise in Raman fiber amplifiers and passive optical components is presented. The results are used for evaluating the output signal-to-noise ratio (SNR) of Raman active recirculating delay lines. It is found that the quadratic dependence of the noise on the number of recirculations causes the SNR to decay with increasing optical delays. Ultimate system performance capabilities, as determined by the SNR quantum limit, are discussed.

I. INTRODUCTION

OVER the past few years, increasing interest has been given to Stimulated Raman Scattering (SRS) in optical fibers. Phase-insensitive optical amplification by SRS has a great potential for application in ultra-long distance, high-speed optical transmission systems [1], as well as in the fields of fiber sensing and signal processing [2]. In active recirculating delay lines (ARDL), loss compensation by Raman gain has been shown to enhance system performance [2]. However, many noise factors degrade the output signal-to-noise ratio (SNR) of such Raman active systems, namely: the amplified spontaneous scattering (Stokes noise), the photon number fluctuations inherent to the statistics of the amplification and loss processes [3], and the noise introduced by the system components, from the light source to the receiver. Output SNR of linear optical amplifier chains (LOAC) using semiconductor amplifiers have been extensively studied for optical communications [4]-[6]; Raman ARDL systems in which a recirculating signal undergoes successive amplification and attenuation stages, are expected to exhibit SNR properties similar to LOAC systems. However, in-line amplification by SRS results in a distribution of the gain along the fiber (caused by pump propagation loss), which is expected to affect the photon count statistics. In addition, it has been shown that in the case of a distributed gain the Stokes noise is dependent on the relative propagation directions of the pump and the signal [2]. It is the purpose of this paper to study the output SNR of Raman ARDL systems for both forward and backward pumping configurations, when a direct detection scheme is utilized, and to evaluate their ultimate performance capabilities in SNR and optical delay.

In the first section, the expectation value and variance of the output photon count of Raman fiber amplifiers is analyzed in the linear amplification regime for the two pumping schemes, using a photon rate equation model [3] and some results from a former work [2]. The effects of passive optical components on output intensity noise are considered in the second section in terms of a novel transfer matrix formulation. In the third section, the previous results are used for deriving the expectation value and the variance of the ARDL output photon count. The fourth section expresses in a canonical form the video output SNR of the ARDL, as it can be obtained with an avalanche photodetector, which yields a quantum limit for the system performance. It is shown that the quadratic dependence of the noise on the number of recirculations causes the SNR to decay with increasing optical delays.

II. OUTPUT INTENSITY NOISE IN RAMAN FIBER AMPLIFIERS

Stimulated Raman scattering in single-mode fibers can be studied through a unidimensional quantum model of a two-energy level system [7]. In this model, the evolution of the pump and Stokes fields with the fiber longitudinal coordinate is described through a photon-rate equation formalism. In the case of a weak interaction (small-signal amplification regime), the effect of pump depletion by SRS can be neglected, and the spatial evolution of the pump photon number is only determined by propagation loss. As a result, a linear equation is obtained for the Stokes photon population, which can be solved exactly in the forward or the backward pumping configurations [2]. However, these solutions represent mean photon numbers or expectation values, and therefore do not contain sufficient information for characterizing the output photon count statistics. For this purpose, one needs the corresponding solutions for the higher photon number moments. In the fundamental paper by Shimoda, Takahashi, and Townes [3], a photon statistics theory for linear optical amplifiers is used to characterize such probability distribution moments. As established in this reference, the
The photon-rate equation for the mean-square photon number is solved in this section for the two pumping schemes, and the results are used for evaluating the output photon number variances of forward and backward fiber Raman amplifiers.

In the steady-state regime, the expectation value of the Stokes photon number \( \langle n_{\pm} \rangle \), corresponding to the forward (+) or to the backward (−) interactions, follows the linear rate equation

\[
\frac{d\langle n_{\pm}\rangle}{dz} = a^{\pm} (\langle n_{\pm}\rangle + 1) - b \langle n_{\pm}\rangle
\]

where \( z \) is the longitudinal coordinate, \( a^{\pm} = \gamma_0 \langle n_p^{\pm}(z) \rangle \) is a Raman gain factor proportional to the mean pump photon number \( \langle n_p^{\pm}(z) \rangle \), \( \gamma_0 \) is a gain coefficient (assumed to be independent of the pumping scheme), and \( b \) is the fiber attenuation coefficient. Due to the fiber attenuation, the number of pump photons varies with the fiber length as \( \langle n_p^{\pm}(z) \rangle = \langle n_p^{\pm}(0) \rangle \exp(-bz) \) for forward SRS and \( \langle n_p^{\pm}(z) \rangle = \langle n_p^{\pm}(0) \rangle \exp(-b(L-z)) \) for backward SRS. \( \langle n_p^{0}(0) \rangle \) is the mean number of pump photons coupled into the fiber at \( z = 0 \) or at \( z = L \), \( L \) the fiber length, and the attenuation coefficient \( b \) is taken to be identical for both the pump and the Stokes wavelengths. The solutions of (1) for the two interaction cases may be written

\[
\langle n_{\pm}(z) \rangle = K^{\pm}(z) \langle n_0 \rangle + N^{\pm}(z)
\]

where the gain \( K^{\pm}(z) \) is defined as

\[
K^{\pm}(z) = \int_0^z [a^{\pm}(z') - b] \, dz'.
\]

The substitution of \( a^{\pm}(z) \) in (3) yields

\[
K^{+}(z) = \exp \left\{ a_0 L_{+}(z) - bz \right\}
\]

\[
K^{-}(z) = \exp \left\{ a_0 L_{-}(z) - b(z - L) - bz \right\}
\]

with \( a_0 = \gamma_0 \langle n_p^{0}(0) \rangle \). \( L_{\pm}(x) = [1 - \exp(-bx)]/b \) is an effective interaction length accounting for the pump attenuation. In (2), \( \langle n_0 \rangle \) is the mean number of photons coupled into the fiber at \( z = 0 \), or input signal.

\( N^{\pm}(z) \) is the contribution to the output of the amplified spontaneous scattering (ASS) or Stokes noise, as given by

\[
N^{\pm}(z) = K^{\pm}(z) \int_0^z \frac{a^{\pm}(z')}{K^{\pm}(z')} \, dz'.
\]

In the two interaction cases, the ASS can be expressed from (6) as [2]

\[
N^{+}(z) = K^{+}(z) \frac{a_0}{b} e^{-(a_0 b z)} \left\{ E_i \left( \frac{a_0}{b} \right) - E_i \left( \frac{a_0}{b} e^{-b z} \right) \right\}
\]

\[
N^{-}(z) = K^{-}(z) - 1 + \frac{b e^{b L}}{a_0} (K^{-}(z) - e^{-b z})
\]

where \( E_i(x) \) is the exponential-integral function.

Equations (4) and (5) show that the forward and the backward gains, though equal at \( z = 0 \) and \( z = L \), vary differently along the fiber, or \( K^{+}(z) \gtrless K^{-}(z) \) due to a lower effective length in the backward case. Integration in (6) then yields \( N^{-}(L) > N^{+}(L) \), i.e., the ASS photon number is greater in the backward case than in the forward case. On the other hand, since \( K^{+}(L) = K^{-}(L) = K \), the amplified signal \( K \langle n_0 \rangle \) is independent of the pumping scheme.

The next step is to obtain corresponding solutions for the expectation value of the square photon number, i.e., \( \langle n_{\pm}^2 \rangle \). The rate equation for \( \langle n_{\pm}^2 \rangle \), as derived from the photon statistics master equation [3], is

\[
\frac{d\langle n_{\pm}^2 \rangle}{dz} = 2(a^{\pm} - b) \langle n_{\pm}^2 \rangle + (3a^{\pm} + b) \langle n_{\pm} \rangle + a^{\pm}
\]

(9)

Substituting the expression for \( \langle n_{\pm}(z) \rangle \) from (2) and integrating (9) yields at \( z = L \) the following solution:

\[
\langle n_{\pm}^2(L) \rangle = K^{2}(\langle n_0 \rangle) + K(4N^{\pm} - K + 1) \langle n_0 \rangle + N^{\pm}(2N^{\pm} + 1)
\]

with, by definition, \( K = K^{+}(L) = K^{-}(L) \) and \( N^{\pm} = N^{\pm}(L) \) for the rest of the paper. The variance of the ASS output, i.e., \( \sigma_2^{\pm} = \sigma_2^{\pm}(L) = \langle n_{\pm}^2(L) \rangle - \langle n_{\pm}(L) \rangle^2 \), can be expressed then with (2) and (10) as:

\[
\sigma_2^{\pm} = K^{2}(\langle n_0 \rangle - \langle n_0 \rangle) + (K \langle n_0 \rangle + N^{\pm})
\]

\[
+ 2KN^{\pm}(\langle n_0 \rangle + \langle n_0 \rangle^2)
\]

\[
= \sigma^{\text{excess}}_2 + \sigma^{\text{shot}}_2 + \sigma^{\text{beat}}_2
\]

(11)

with \( \sigma_0^2 = \langle n_0^2 \rangle - \langle n_0 \rangle^2 \) being the input signal variance, and \( \sigma^{\text{excess}}_2, \sigma^{\text{shot}}_2, \sigma^{\text{beat}}_2 \) being the signal excess noise, the shot noise, and the beat noise, respectively. The expression of \( \sigma^{\pm}_2 \) in (11) is similar to the one obtained in [3] in which a constant gain factor \( a \) is assumed (see also [4]).

In the present case however, the effect of pump power attenuation is taken into account, which yields modified expressions for the gain \( K \) and the amplified spontaneous scattering \( N^{\pm} \) (as given by (4), (7), and (8) with \( z = L \), respectively). In particular, (11) shows that the variance of the output photon count is quadratic in the ASS photon number \( N^{\pm} \), and since \( N^{-} > N^{+} \), the variance corresponding to the backward case is greater than in the forward case, i.e., \( \sigma^{\pm}_2 > \sigma^{\pm}_2 \).

III. Effect of Passive Optical Components on Output Intensity Noise

In this section we propose to consider the effect on the photon statistics of the passive optical components which can be part of an overall fiber system, such as for instance: attenuators, beam splitters, coupling lenses or fiber couplers. The intensity noise introduced by these elements is due to the statistical character of the process in which a certain number of photons pass "successfully" through them. Such a selection process is characterized by the Bernoulli’s distribution [3], [8]. If \( \eta \) is the component transmission (or the probability for one input photon to pass successfully through the component in ques-
tion), then it is found with the distribution that the mean and variance of the output photon count \( \langle n \rangle, \sigma^2 \) are related to the initial conditions \( \langle n_0 \rangle, \sigma_0^2 \) as follows:

\[
\langle n \rangle = \eta \langle n_0 \rangle
\]

\[
\sigma^2 = \eta^2 \sigma_0^2 + \eta(1 - \eta) \langle n_0 \rangle.
\]

Actually, the same result is obtained in (2) and (11) by setting \( \sigma_0 = 0 \), which is equivalent to considering the fiber as being a passive component with transmission \( \eta = K = e^{-bl} \) [3].

It is convenient to summarize the results of (2), (11), (12), and (13) by using a matrix form. We define \( P_0 \) as a vector with components \( \langle n_0 \rangle, \sigma_0^2 \), which characterizes the input statistical parameters, and the corresponding output vector \( P^\pm \) having components \( \langle n_\pm \rangle, \sigma_\pm^2 \). Then (2), (11), and (12), (13) become respectively,

\[
P^\pm = T^\pm \eta \eta P_0 + N^\pm
\]

with the following definitions for the transfer matrices

\[
T^\pm = K \begin{pmatrix} 1 & 0 \\ 1 - K + 2N^\pm & K \end{pmatrix}
\]

\[
N^\pm = N^\pm \begin{pmatrix} N^\pm + 1 \\ 1 - N^\pm \end{pmatrix}
\]

\[
T_\eta(\eta) = \begin{pmatrix} 1 & 0 \\ 1 - \eta & \eta \end{pmatrix}
\]

The set of equations (14)–(18) characterize the statistical transformations of the photon number mean and variance that are caused by passing through active or passive optical elements. As shown in the following section, this matrix form is found to be practical for the calculation of the overall transfer matrix of fiber systems including combinations or iterations of such elements.

IV. APPLICATION TO ACTIVE RECIRCULATING DELAY LINES

Fig. 1 shows a recirculating delay line system constituted of: 1) a length \( L \) of single-mode fiber that is pumped to form a Raman fiber amplifier characterized by a net gain \( K \), 2) a fiber coupler of coupling ratio \( \eta \) at the Stokes wavelength and insertion loss \( 1 - \gamma \), which closes the fiber upon itself to form a reentrant loop, and 3) a direct detection setup including coupling lenses (of overall coupling efficiency \( \eta_\gamma \)), a polarizer, an optical filter of bandwidth \( B_0 \), an avalanche photodiode (APD), and an electronic amplifier. The overall system is assumed to be polarization-preserving and the signal to propagate in one polarization.

The output statistical parameters can be characterized by the transfer matrix of the whole system. One has first to consider the transfer matrix of the reentrant loop, then to iterate the result for the case of multiple signal recirculations. The transfer matrices of the fiber coupler are yet to be defined. Depending upon which optical path is considered, as pictured in Fig. 2, the transfer equations for the fiber coupler are either \( P = C_p P_0 \) or \( P' = C_D P_0 \). The matrices \( C_p \) and \( C_D \) are defined by using (19)

\[
C_p = T_p(\eta) T_\eta(\eta) T_\eta(1 - \eta) = A_\eta \gamma \begin{pmatrix} 1 & 0 \\ 1 - A_\eta \gamma & A_\eta \gamma \end{pmatrix}
\]

where \( A = (1 - \eta)/\eta \) is the coupler branching ratio.

Forward or backward internal Raman gain \( K \) in the fiber is provided by coupling at either loop input end, a pump signal of frequency upshifted by 440 cm\(^{-1}\) with respect to the Stokes frequency [2]. For simplicity, we assume that the multiplexing effect of the fiber coupler [2] yields a null coupling ratio at the pump wavelength, so that there are no resonant effects nor pump recirculations in the loop. The input pump power is stabilized and adjusted to a value for which the internal Raman gain compensates for the loop losses, so as to provide a unity net gain for the signal with negligible fluctuations. We assume the signal spectrum to be peaked at the Stokes wavelength and to have a linewidth \( B \). In the time domain, it is also assumed that the input signal consists of a train of pulses of width \( \tau \) which have a low repetition rate \( 1/T \), such that \( \tau < T_{\text{loop}} \) being the loop transit time, as pictured in Fig. 3(a). In addition, we assume that the input pump signal consists in a pulse train of equal repetition rate, but with a pulselength \( T_{\text{pump}} \) slightly lower than \( T \) (Fig. 3(b)). With such a configuration, there is only one signal pulse propagating into the loop at any time, which can recirculate with constant intensity due to a unity net gain in
Equations (22)–(24) can be expressed in terms of mean and variance \( \langle n_{\pm}^{(n)} \rangle, \sigma_{\pm}^{2(n)} \) of the photon numbers of the output signal pulses

\[
\langle n_{\pm}^{(n)} \rangle = \frac{A^2}{K} \langle n_0 \rangle + nN_{\pm}^{\mp} \frac{A}{K} \quad (25)
\]

\[
\sigma_{\pm}^{2(n)} = \frac{A^4}{K^2} \left( \sigma_0^2 - \langle n_0 \rangle \right) + \left\{ \frac{A^2}{K} \langle n_0 \rangle + nN_{\pm}^{\mp} \frac{A}{K} \right\}^2
\]

\[
+ \left\{ 2 \frac{A^2}{K} \langle n_0 \rangle nN_{\pm}^{\mp} \frac{A}{K} + n^2 \left( N_{\pm}^{\mp} \frac{A^2}{K} \right) \right\}. \quad (26)
\]

The three terms on the right-hand side of (26) correspond, respectively, to the excess noise, shot noise, and beat noise. As seen in the equation the output variance \( \sigma_{\pm}^{2(n)} \) is a quadratic function of the number of recirculations \( n \). An identical result is found for an optical transmission line using \( n \) optical repeaters at regular spacings [5], [6], which is actually a system quite similar to the active recirculating delay line.

As compared with (2)–(11), the results of (25)–(26) show that, when the loop net gain is unity, the active reentrant fiber loop is equivalent to an optical amplifier having a signal gain \( A/K \), and an amplified spontaneous noise output \( nN_{\pm}^{\mp} A/K \), which is proportional to the number \( n \) of recirculations. The number of recirculations \( n_c \) for which the ASS intensity is about equal to the signal intensity is, from (25), the integer number nearest to \( A(n_0)/N_{\pm}^{\mp} \). The optical delay capability of the recirculating delay line can be arbitrarily defined as \( T_{\max} = n_c T_{\text{loop}} \).

The optical delay capability of the system, as determined by the parameter \( n_c \), has been analysed in detail in [2].

\[
\mathcal{N}_{\pm}^{(n)} = nN_{\pm}^{\mp} \frac{A}{K} \left( \frac{1}{1 + nN_{\pm}^{\mp} \frac{A}{K}} \right). \quad (24)
\]

V. SIGNAL-TO-NOISE RATIO OF THE ACTIVE RECURRENT DELAY LINE

An optical signal-to-noise ratio (SNR) for the recirculating delay line has been defined in [2] as the power ratio of the output signal to the ASS. In this paper however, we consider a system which includes a photodetector. In this case, the quantity of practical interest is the video SNR, as measured by the electric power ratio of the signal to the noise outputs which are generated by the photodetector through a load resistance.

The input signal is assumed to be completely coherent with Poisson statistics, for which \( \sigma_{\text{excess}}^2 \) in (26) vanishes. The input signal linewidth \( B_0 \) and the detection bandwidth \( B_0 \) are assumed to be narrower than the Raman line width, so that the frequency-dependent Raman gain factor \( \gamma_0 \) can be considered to be constant over the frequency interval \( B_0 \). The photodetector is assumed to be an avalanche photodiode (APD) of quantum efficiency \( \eta_d \) and multiplication factor \( \langle g \rangle \).
The electrical baseband power SNR can be defined as [9]

$$\text{SNR} = \frac{\langle n_{S+N} - n_N \rangle^2}{\sigma_{S+N}^2 + \sigma_N^2}$$

where $\langle n_{S+N} \rangle$ and $\langle n_N \rangle$ are the number of photoelectron counts during a sampling interval $T_s$ obtained when an output signal pulse has been detected or has not been detected, respectively, with $\sigma_{S+N}^2$ and $\sigma_N^2$ being the corresponding variances. Using (25), the quantity $\langle n_{S+N} - n_N \rangle$ has for expression

$$\langle n_{S+N} - n_N \rangle = \eta_e \eta_d \frac{\langle g \rangle}{K} \langle g \rangle_0.$$  

On the other hand, the variance $\sigma_{S+N}^2$ corresponding to $\langle n_{S+N} \rangle$ is written [5], [9]

$$\sigma_{S+N}^2 = \eta_e \eta_d \langle g^2 \rangle \sigma^2_{\text{shot}} + \eta_e^2 \eta_d^2 \langle g^2 \rangle^2 \sigma^2_{\text{beat}} + \sigma^2_{\text{DC}} + \sigma^2_{\text{TH}}$$

with $\sigma^2_{\text{shot}}$ and $\sigma^2_{\text{beat}}$ being defined in (26). $\sigma^2_{\text{DC}} = \langle g^2 \rangle N_{\text{DC}}$ is the variance of the APD dark electron count $N_{\text{DC}}$ [9], and $\sigma^2_{\text{TH}}$ is the variance of the thermal noise electron count introduced by the electronic amplifier. The factors $\eta_e \eta_d$ and $\eta_e^2 \eta_d^2$ for the shot noise and the beat noise variances come from the convolution of the signal photon number distribution with the Bernoulli distribution, accounting for the optical coupling of the signal with the APD, and the statistics of photoelectron emission [6]. As seen in Section II, such a convolution is done by multiplying the vector $P$ in (14) by the transfer matrix $T_{\text{P}}(Q_d) \times T_{\text{P}}(Q_d)$, which can be obtained with (18). The variance $\sigma_N^2$ is given by (29), with $\langle n_0 \rangle = 0$. The factor $\langle g^2 \rangle$ can be approximated by $g^2$, where $g$ is the APD excess noise exponent [9]. One obtains then for the SNR corresponding to the two pumping configurations

$$\text{SNR}_{\text{p}} = \frac{K \langle g \rangle (\langle n_0 \rangle + 2n \langle N \rangle B_0 / A + 2 \langle N \rangle N_{\text{DC}} / A^2 \eta_d \eta_d) / \sigma^2_{\text{TH}}}{A^2 \eta_d \eta_d}.$$  

In the present study, we shall consider only the case of a perfect detector, i.e., $\eta_d = g = 1$ and $N_{\text{DC}} = 0$. In addition, we shall assume a 100-percent coupling efficiency between the fiber loop end and the detector ($\eta_e = 1$) and that the thermal noise is negligible as compared to other noise contributions. Introducing the ASS noise equivalent input photon number $\mu_0^2$, as defined by $N^2 = K \mu_0^2$ [2], and considering the optimal case $B_0/B = 1$, the SNR becomes

$$\text{SNR}_{\text{p}} = \frac{K}{A^2} \left( \langle n_0 \rangle + 2n \frac{K}{A} \mu_0^2 \right) + 2n \frac{K}{A} \mu_0^2 \langle n_0 \rangle + 2n^2 \left( \frac{K}{A} \mu_0^2 \right)^2.$$  

It can be verified with (31) that the SNR decays as a function of the loop coupling ratio. This is explained by the following considerations. First, the amount of signal entering the loop decays as $1 - \eta$; thus, a high coupling ratio ($\eta \approx 1$) is a penalty on the input SNR. In addition, the detected (electrical) signal varies as $A^2/K^2$ or $(1 - \eta)^2/\eta^2$, as shown in (27), (28), which decays rapidly with $\eta$. On the other hand, increasing coupling ratios result in reduced loop loss, lower internal gain, and therefore lower ASS noise; however, the noise variance having a dependence on $A/K$, $A^2/K$, $A^4/K^2$ and $A^4/K^4$ (cf., (26) for coherent input signals), has thus a slower decay with $\eta$ than the detected signal. As a result, the ratio of the two quantities, or output SNR, is a monotonical decaying function of the loop coupling ratio.

For very high gains, or very low loop coupling ratios (since $K = 1/\eta \gamma$), using $A = (1 - \eta)/\eta$, $\mu_0^2 \approx 1$ [2] and the approximation $\gamma \approx 1$, the SNR is maximized and becomes

$$\text{SNR}_{\text{p}} \approx \frac{1}{1 + \frac{\langle n_0 \rangle}{2n}}.$$  

Equation (32) shows that for high input signals such that $\langle n_0 \rangle \gg n$, the maximum achievable SNR, or quantum limit is $\langle n_0 \rangle / 2n$. Thus, the best active recirculating delay line degrades the SNR by the reciprocal of the number of recirculations, and by at least 3 dB ($n = 1$), which is the quantum limit of the ideal optical amplifier [9]. Such a limit is due to the signal-spontaneous beat noise term in (31).

On the other hand, for large numbers of recirculations or small input signals (i.e., $n \gg \langle n_0 \rangle$), the spontaneous-beat noise term dominates and the SNR in (32) decreases as the square of the number of recirculations. The signal-spontaneous and spontaneous-spontaneous beat noise levels are thus fixing a limit in the SNR performance of the ARDL, as is likewise found in the analysis of linear repeater systems [5].

Fig. 4 shows the SNR, as defined in (31), plotted as a function of the number of recirculations (bottom X-axis), or the signal optical delay (top X-axis). In this numerical example, the parameters have been chosen to represent an optimal case. The input signal power is assumed to be 10 mW [2]. The signal wavelength is assumed to be 1.55 $\mu$m, for which the fiber loss can be as low as 0.2 dB/km.

$$\text{SNR}_{\text{p}} = \frac{K}{A^2} \left( \langle n_0 \rangle + 2n \frac{K}{A} \mu_0^2 \right) + 2n \frac{K}{A} \mu_0^2 \langle n_0 \rangle + 2n^2 \left( \frac{K}{A} \mu_0^2 \right)^2.$$  

The gain required for loop loss compensation is thus minimized, which results is lower ASS noise and lower pump power thresholds. The coupler insertion loss is assumed...
to be 5 percent (or $\gamma = 0.95$), and the coupler coupling ratio to be $\eta = 0.9$. The choice of such a high coupling ratio, though not optimal for the SNR, is meant to lower the pump power threshold [2]; with a loop length of $L = 1$ km, the above parameters and typical values for the effective fiber core area and the silica Raman gain coefficient [2], this critical pump power can be reduced to less than 100 mW.

With these parameters, one finds with (7) and (8) a negligible difference between the forward and the backward ASS noise levels, namely $(N^- - N^+/N^- \approx 0.3$ percent, and thus the SNR's corresponding to both pumping schemes are very nearly equal, with $\text{SNR}^+$ being slightly greater than $\text{SNR}^-$. The last parameter of interest is the optical filter bandwidth $B_0$, which for SNR optimization has to match the input signal bandwidth $B$. Since the Raman gain is frequency-dependent, $B$ has to be much lower than the gain linewidth $\Delta\nu_R$, for instance $B \leq 5.10^{-2} \Delta\nu_R \approx 300$ GHz, so as to avoid signal decay with recirculations [2]. On the other hand, the lower bound for $B$ is determined by: 1) the frequency stability of the signal source, and 2) the stimulated Brillouin Scattering (SBS) linewidth. First, the temperature dependence of longitudinal-mode wavelength for 1.55-µm laser diodes being 10 GHz/°C [10], the choice of $B \geq 3$ GHz imposes moderate temperature stabilization constraints. In addition, this limit of 3 GHz is about two orders of magnitude above the SBS linewidth (16 MHz at $\lambda = 1.55$ µm [11]), which guarantees that no sizeable signal depletion by SBS can occur. Therefore, the signal and optical filter bandwidths have to be chosen with the 3-300 GHz range.

In Fig. 4, the SNR is plotted for $B = 3$, 30, and 300 GHz (solid lines); the quantum-limited SNR corresponding to each case, as defined by (32), is plotted in dashed lines. It can be seen from the figure that the SNR, though not optimized by the choice of a high loop coupling ratio, is fairly close to the quantum limit (about 3-dB difference), which is expected since the detector has been assumed to be ideal. However, this shows that the operation of the ARDL with lower loop coupling ratios would not result in a significant SNR improvement.

Three regions can be distinguished in the figure: 1) within the first ten recirculations, the SNR undergoes moderate decay due to a low signal-spontaneous beat noise; 2) then, within 10 to 100 recirculations, the SNR decays as $1/n$, due to the buildup of the ASS shot noise and the signal-spontaneous beat noise; 3) with a large number of recirculations ($n > 10^5 - 10^7$), the SNR decay, proportional to $1/n^2$, is due to the buildup of the spontaneous-spontaneous beat noise.

The figure shows that a 0-dB quantum limit for the SNR is attained for $B = 3$ GHz at $n = 10^5$, which in the example sets the ultimate performance capability of the active recirculating delay line. The number of recirculations $n = 10^6$ for which the SNR is 10 dB, corresponds to an optical delay of about 5 s or an effective optical path length of $10^9$ m. These values can be viewed as defining the practical performance limits of the ARDL.

VI. CONCLUSION

An analysis of intensity noise in Raman fiber amplifiers has been made for both the forward and the backward pumping configurations. Analytical expressions for the output signal and Stokes photon number variances have been obtained. The contribution to the intensity noise of passive optical elements has also been considered.

With these results, the intensity noise in an active recirculating delay line was characterized, and the system output signal-to-noise ratio was analyzed. It was found that the output SNR decays as a quadratic function of the number of signal recirculations. In the quantum limit, the buildup of beat noise between the signal and the amplified spontaneous scattering components was shown to set an upper bound to the system SNR performance. An optimized system operating at $\lambda = 1.55$ µm was taken as an example, in which optical delays as high as 5 s (corresponding to $10^6$ signal recirculations) are theoretically achievable with a 10-dB output SNR.

Such high potential performance capabilities of Raman active recirculating delay lines makes them suitable candidates for a broad range of applications in the domains of fiber-optic sensing, fiber-optic memories, and tapped delay line signal processing.

APPENDIX

CALCULATION OF $P^{(n)}$

The following matrices in (21) have to be calculated

$$T^{(n)} = C_\odot C_\odot^{-1}(C_\odot T_\odot^{+})^n C_\odot$$ \hspace{1cm} (A1)

$$N^{(n)} = C_\odot C_\odot^{-1} \sum_{k=0}^{n-1} (C_\odot T_\odot^{+})^k C_\odot N_a.$$ \hspace{1cm} (A2)

For simplicity, the subscripts and superscripts $\pm$ will be omitted. Let $q = K\eta$, and $\mu_0 = N/K$. Then, using (16) and (19), the $n$th power of $C_\odot T_\odot^{+}$ is written

$$(C_\odot T_\odot^{+})^n = q^n \left( \frac{1}{(2\mu_0 + 1 - q) Q_n} \right)$$ \hspace{1cm} (A3)
with $Q_n = (1 - q^n)/(1 - q)$. Using (A1), (A3), the inverse matrix of (19), and (20), it is found

$$T^{(n)}_{\pm} = \frac{A^2}{K} q^{n+1} \left( \begin{array}{cc} 1 & 0 \\ 2A\mu_0 Q_n + 1 - \frac{A^2}{K} q^{n+1} & \frac{A^2}{K} q^{n+1} \end{array} \right)$$

(A4)

The sum in (A2) can be expressed with (A3) as

$$\sum_{k=0}^{n-1} (C_k T_{\pm}^{(n)})^k = \frac{1}{1 - q} \left( \frac{2q\mu_0 + 1 - q}{1 - q} \right)^n \left( \frac{q}{1 + q} \right)^{Q_n-1}$$

(A5)

Using (A2), (A5), (18), the inverse matrix of (19), and (20), one finds

$$N^{(n)}_{\pm} = A\mu_0 qQ_n \left( \frac{1}{1 + A\mu_0 + (2q\mu_0 + 1 - q) A\mu_0 Q_{n-1}} \right)$$

(A6)

REFERENCES


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