Measurement and analysis of light transmission through a modified cladding optical fiber with applications to sensors

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The effect of a modified cladding on the transmission of light through a step-index optical fiber is investigated using 3-D geometrical optics. Measurements of the light transmission of the optical fiber as a function of the modified cladding refractive index and length are presented for the case of focused illumination and compared with 3-D ray theory. The effect of defocus on the transmission of the modified fiber is also studied. Applications to intensity sensors are discussed.

I. Introduction

In recent years, several fiber sensors have been suggested, where a section of the fiber cladding is replaced by an environmentally sensitive material\(^1\)\(^2\) (see Fig. 1). For example, by using a material with temperature sensitive index of refraction, a fairly sensitive set temperature sensor is constructed\(^3\) whose transition point can be controlled by varying the wavelength of the light source.\(^4\)

Analysis of the effect of the modified cladding was carried out in Refs. 3 and 4 using 2-D slab geometry and assuming that the optical power is equally distributed among the propagating modes. However, close agreement between this theory and the experimental results could be obtained only after the Fresnel reflection formulation was empirically augmented by an excess loss factor. In this work, we overcome this problem by using a model based on a 3-D initial power distribution formed by a focused beam. This distribution, together with Fresnel's reflection formula and proper choice of the effective number of the ray core-cladding encounters, was then used to calculate the effect of the modified cladding's refractive index and length on the transmitted intensity. These theoretical predictions were found to be in close agreement with our experimental results without further adjustment.

The theoretical analysis of the dependence of the fiber transmission on the parameters of the modified cladding is given in Sec. II. Experiments in which part of the fiber's original cladding is replaced by an index matching oil acting as a modified cladding are described in Sec. III followed by a summary and suggestions for future work (Sec. IV).

II. Transmission Variations due to the Modified Cladding

A. Mathematical Formulation

Consider Fig. 1 again: part of the cladding of a large-core (several hundreds microns) step-index fiber is replaced by another material—the modified cladding. Since the size of the core is large compared with the optical wavelength, this problem can be accurately handled using geometrical optics. The total guided power, carried by the bound rays in the fiber, \(P_{b}(0)\) is

\[
P_{b}(0) = \int_{0}^{\pi} d\phi \int_{0}^{\pi} r dr \int_{0}^{2\pi} d\theta_{c} \int_{0}^{\theta_{c}} I(r, \theta, \phi, \theta_{c}) \sin \theta_{c} d\theta_{c}.
\]  \(1\)

\(\theta_{\text{max}}\) is the largest angle in air between a bound ray and the fiber's axis, as determined either by the fiber's numerical aperture or by the input distribution of rays. \(\phi, r, \theta, \theta_{c}\) are shown in Fig. 2.\(^5\) \(I(r, \theta, \phi, \theta_{c})\) is the intensity distribution in air.

When a ray enters the modified cladding section, either \(\theta_{c} \leq \theta_{c}\), and the ray remains guided, or \(\theta_{c} > \theta_{c}\), and the ray is only partially reflected from the core-cladding boundary. \(\theta_{c}\) is the angle between the ray and fiber axis inside the fiber, and \(\theta_{c}\) is the critical angle for which a ray is still bound.

\[
\theta_{c} = \sin^{-1}(n_{\text{core}}/n_{\text{cl}}).
\]  \(2\)
As long as the illuminating cone of light is cylindrically symmetrical around the core axis, all propagating rays are meridional, no tunneling rays are excited, and the reflection coefficient at the core–cladding boundary is given by

$$R = \left( \frac{n_{\text{core}} \cos \alpha - n_{\text{clad}} \cos \alpha}{n_{\text{core}} \cos \alpha + n_{\text{clad}} \cos \alpha} \right)^2,$$

(3)

$$\theta_i > \theta_i \text{ (refracting ray, Fresnel formula)},$$

where $\alpha$ is the angle between the incident ray (in the fiber) and the normal to the core–cladding interface. $\alpha_i$ is the angle between the transmitted ray and the normal to the core–cladding interface.

Each ray has a characteristic length it passes between two successive reflections, denoted by $z_p$. In a step-index fiber, $z_p$ is given by

$$z_p = \frac{2 \rho \delta \theta_i}{n_{\text{core}} - n_{\text{clad}}},$$

(4)

where $\beta$ and $l$ are the ray invariants:

$$\beta = n_{\text{core}} \cos \theta_i,$$

(5a)

$$l = \frac{r}{\rho} n_{\text{core}} \sin \theta_i \cos \phi_i,$$

(5b)

$$g(\rho) = n_{\text{core}}^2 - \beta^2 - l^2,$$

(5c)

B. Effective Number of Core–Cladding Encounters

The number of reflections (i.e., core–cladding encounters) in a modified cladding region of length $L_0$ is determined by the relation $N = L_0/z_p$, and by the initial conditions in which a ray enters the modified cladding section. While the initial conditions are not known a priori, the number of reflections must be an integer. We have chosen to treat the fractional part of $N$ in statistical terms: if $N = 3.2$ we shall say that the ray has an 80% probability of undergoing three reflections and a 20% probability of undergoing four reflections.

More generally, if a ray suffers $N$ reflections, an effective reflection coefficient can be defined in terms of $N$ and $R$:

$$R = [(N - [N])R^N + [1 - (N - [N])R^N].$$

(6)

$[N]$ denotes the integer part of $N$.

These considerations are important since the ray undergoes very few reflections in the modified cladding region. For a typical meridional ray with $\theta_i = 0.1$ rad, entering a 1-cm modified cladding section with a core diameter of 300 $\mu$m, the number of reflections is three or four. Our particular choice of the number of reflections is experimentally justified in Sec. III.

If only meridional rays are considered, Eqs. (3) and (4) can be considerably simplified to obtain

$$z_p = \frac{2 \rho \delta \theta_i}{n_{\text{core}}},$$

(7)

and the power remaining in the fiber after the modified cladding section of length $L_0$ is

$$P_{\text{in}}(L_0) = \int_0^{\pi} \phi \int_0^{\beta_x} r \, dr \, d\theta_i \int_0^{\theta_{\text{max}}} I(r, \theta_i, \phi, \theta_i)(r sin \theta_i) d\theta_i,$$

(9)

While $\langle R \rangle$ can be calculated from Eqs. (6)–(8), $I(r, \theta_i, \phi, \theta_i)$ is determined by the input illumination conditions.

C. Intensity Distribution of the Incident Beam

One of the most common ways to couple light into the fiber is by focusing a collimated laser beam onto the fiber axis. Only meridional rays are excited. Let us suppose that the collimated beam carries uniform power $P_i$ per unit cross-sectional area. The intensity distribution that is excited is given by

$$I = \frac{2 \pi \rho \delta \theta_i}{2 \pi \rho \delta \theta_i} \delta(r/r_0)$$

(10)

for $0 \leq \theta_i \leq \theta_{\text{max}}$

$$I = 0 \text{ for } \theta_{\text{max}} \leq \theta_i \leq \frac{\pi}{2},$$

where $f$ is the lens focal length, and $\delta(r)$ is the Dirac delta function. From Eqs. (1) and (9) the ratio of the output power in the fiber at $z = L_0$ to the power at $z = 0$ (which equals the input launched power) is

$$\frac{P_{\text{out}}(L_0)}{P_{\text{out}}(0)} = \frac{2 \pi \rho \delta \theta_i}{\tan^4(\theta_{\text{max}})} \langle R \rangle,$$

(11)

and $N$, Eq. (6), is given by

$$N = \frac{L_0}{2 \rho \cot(\sin^{-1}(\sin(\theta_i)/n_{\text{core}}))}.$$

We have also examined the effect of defocus, i.e., the situation in which the optical axes of the lens and fiber coincide, but the fiber endface is not in the focal plane.
of the (same) lens. As long as the defocusing distance \( a \) [see Fig. 4(b)] is smaller than \( \rho - \cot(sin^{-1}(N.A.)) \) (N.A. is the lens numerical aperture), the light transmission through the fiber, Eq. (11), remains unchanged, since this slight defocusing is equivalent to a small axial translation of the modified cladding section, and Eq. (11) is independent of the absolute axial location of the modified section.

Considerable defocusing [see inset in Fig. 4(b)] means that not all the light is coupled into the core. However, this situation is equivalent to a slightly defocused illumination from a lens with a smaller numerical aperture [see inset in Fig. 4(b)]. This effective numerical aperture is

\[
N.A_{\text{eff}} = \sin\arctan(\rho/a),
\]

and the relative change in power is

\[
P_{\text{rel}}(L_0) = \frac{2}{P_{\text{rel}}(0)} \int_{\theta_0}^{\theta_{\text{max}}} |R| \sin \theta_0 (\cos^2 \theta_0) d\theta_0
\]

\[= \tan^2(\theta_{\text{max}})
\]

### III. Experimental Results

We used the setup shown in Fig. 3 with a \( \approx 2\)-m long, \( \phi 300-\mu m \) plastic cladding silica step-index fiber having a numerical aperture of 0.4. The spatially filtered He-Ne expanded beam was coupled into the fiber using a \( \times 10 \) lens with N.A. of 0.25. Being smaller than the fiber's N.A., it is the lens which determines \( \theta_{\text{max}} \) of Eq. (1). The beam splitter and TV camera were used to form an image of the fiber's entrance face thereby enabling proper focusing and centering of the input beam. An ideally centered illuminating spot will excite only meridional rays.

The light travels in the fiber for a length of \( \approx 1 \) m before it reaches the modified cladding. The fiber is held straight so that there would be no perturbations that might change the initial intensity distribution.

A short section of the original cladding was removed. The material we used as a modified cladding was an index matching liquid.\(^7\) This liquid has \( n = 1.468 \) at \( 25^\circ C, 0.6328 \mu m, \) and a temperature coefficient \( \frac{dn}{dT} = -0.00037^C^{-1}. \) The modified cladding was held on a metallic mount with a diameter of 5 mm. The mount diameter determines the length of the modified cladding region. Experiments were also made with other mount diameters. The mounts were heated with a blower that changed the oil's temperature and, subsequently, the modified cladding's refractive index according to

\[
n_{\text{mod}}(T) = n_{\text{mod}}(25^\circ C) + \frac{dn_{\text{mod}}}{dT} [T - 25^\circ C]
\]

\[= 1.468 - 0.00037^C^{-1} [T - 25^\circ C],
\]

where \( T \) is the temperature in degrees celsius. The oil was initially heated to a point where the oil refractive index is lower than \( n_{\text{core}}, \) and all measurements were taken on the free slow-cooling cycle of the mount. The temperature of the mount, and hence of the oil, was measured by a digital thermometer placed on the mount. Simultaneously with the temperature measurement, the light coming out of the fiber end was also monitored by a power meter. Both meters were controlled by a GPIB computer/controller.

The dependence of the fiber transmission on the refractive index of the modified cladding is shown in Fig. 4(a) and compared with Eq. (11) of the previous section [solid line in Fig. 4(a)]. The index of the core is taken as the index that gives minimum transmission; \( n_{\text{core}} = 1.46 \pm 0.001. \) Both the experimental and theoretical curves are referenced to their values at the lowest measured refractive index.

There is good agreement between the theoretical and experimental results except near \( n = n_{\text{core}}. \) We believe that this discrepancy is due to small inhomogeneities in the oil's temperature that tend to average the true steeper behavior of the measured curve.

The broken curves in Fig. 4(a) show two alternate ways to treat the fractional part of the number of reflections \( L_0/\xi, \) [see Eq. (6)]. In curve 1, Eq. (11) was evaluated based on Kopera's expression for \( (R), \)

\[
(R) = R^{1/2},
\]

while in curve 2, \( (R) \) follows Eq. (7-3) in Snyder and Love:\(^8\)

\[
(R) = \exp(-(1-R)L_0/\xi).
\]

As can be seen, our formulation, Eq. (6), gives a much better fit.

Transmission of a defocused incident beam was measured by moving the fiber in an axial direction, thereby increasing \( a, \) see inset in Fig. 4(b), from 0 to 1.5 mm. The effective numerical aperture was experimentally determined from the far-field pattern at the fiber output end.\(^5\) Two sets of measurement have been made, results of which are given in Fig. 4(b) for \( N.A._{\text{eff}} = 0.15 \) and \( N.A._{\text{eff}} = 0.085. \) These results were compared with Eq. (14) with the appropriate \( \theta_{\text{max}} \) as determined by \( N.A._{\text{eff}}. \)

Depending on the amount of defocusing, Fig. 4(b) shows that the transmitted power becomes saturated below a certain value of \( n_{\text{mod}}, \) for which the fiber N.A. is equal to the input N.A. Evidently, smaller values of
Fig. 4. Fiber transmission vs oil index of refraction and temperature: (a) L0 = 5 mm. Dotted curve—experimental result; solid curve—theoretical results based on Eq. (6); broken curves 1 and 2—theoretical calculations based on Eqs. (16a) and (16b), respectively. (b) Defocusing effects for N.A. eff = 0.15 and N.A. eff = 0.085.

n_{mod.cl} will increase the fiber N.A., and all the rays become completely guided with no attenuation. This saturation region does not appear in Fig. 4(a) because larger input angles were involved, and the presented range of n_{mod.cl} does not extend far enough to the left to show this region.

Experimental and theoretical results for the dependence of the fiber transmission on the length of the modified cladding region at a constant oil temperature (25°C) is shown in Fig. 5(a). The length of the oil was varied by replacing the mount on which the oil is placed. (The L0 = 0 point is the transmission without oil.) When n_{mod.cl} < n_{core}, curve 1 in Fig. 5(a), the introduction of the oil just reduces the fiber’s N.A., and after a short transition period the transmission saturates at a level determined by the new N.A. and the input illumination. The power remaining in the core is actually the power carried by the bound rays alone. On the other hand, if n_{mod.cl} > n_{core}, curve 2 in Fig. 5(a), light is no longer guided, and all of it can be extracted out of the fiber provided the modified cladding region is long enough. Figure 5(b) depicts more theoretical results for various values of n_{mod.cl} and n_{core}.

IV. Conclusions

We have analyzed and measured the effect of a modified cladding region on the optical fiber transmission. While previous work did require an additional core-cladding attenuation factor to fit the theory with the experiment, it was shown here that a 3-D ray theory, which takes into account the input intensity distribution, as well as a proper choice of the effective number of core-cladding encounters [Eq. (6)], is indeed sufficient for a fairly accurate prediction of the experimental results.

Now that transmission of the modified cladding fiber is understood with respect to its dependence on the refractive index and length of the modified cladding and also with respect to errors in the launch conditions (e.g., defocus); better designs can be achieved for fiber sensors incorporating modified cladding segments. The results of this study can also be applied to the design of linear mode strippers.8
4,558,925  17 Dec. 1985 (Cl. 350-358)
Multi-functional acousto-optic signal processor.
M. W. CASSIDAY, N. J. BERG, and A. N. PILIPOV. Assigned to
U.S.A. as represented by Secretary of the Army. Filed 2 Aug. 1984.
This processor combines two well-known acoustooptic architectures, the coherent
time-integrating correlator and the rf spectrum analyzer in a manner
retaining all the disadvantages of a discrete two-armed interferometer with
respect to operation in a harsh environment.

I.J.A.

4,558,925

4,560,243  24 Dec. 1985 (Cl. 350-469)
Projection lens.
H. TERASAWA. Assigned to Nippon Kogaku K.K. Filed 26 June
Four fully symmetrical copying lenses of the Plasmat type are described.
The aperture is //1, the semidield is 23°, and the aberration corrections are
elegant. The design is controlled by seven conditions.

R.K.

4,560,253  24 Dec. 1985 (Cl. 350-428)
Zoom lens system.
S. OGINO. Assigned to Minolta Camera K.K. Filed 4 Oct. 1983 (in
Japan 5 Oct. 1982).
A remarkably simple two-component zoom lens is described covering a
range of focal lengths from 36 to 68 mm at //4. The front negative component
contains two or three elements and a single aspheric surface. The rear
positive component has four elements (+-+). Two embodiments are
given controlled by two conditions. The published aberration graphs indicate
that, as would be expected, the corrections are not as good as in some other
more complex two-component zooms.

R.K.

4,560,254  24 Dec. 1985 (Cl. 350-427)
Zoom lens system.
Y. DOI and K. SADO. Assigned to Fuji Photo Optical Co., Ltd.
A complex twenty-element zoom lens is described having a 12:1 zoom range
at an aperture of f//1.59. The semidield ranges from 2.7 to 29.5° during a zoom.
The front and rear components are fixed, and four internal airspaces are
varied during a zoom. The fixed front component has four elements; the
negative variator has four elements; the positive compensator is a cemented
doublet; and a second positive compensator has three elements. Following
the stop is a fixed rear component with seven elements. A single example is
given.

R.K.

4,561,739  31 Dec. 1985 (Cl. 350-432)
Synthetic resin lens system for imaging apparatus.
J. A. LAWSON, M. R. KUEHNLE, and J. D. KNOX. Assigned to
A four-element molded plastic f//6.6 copying lens covering a 64° field of
view is described. It is a compact symmetric arrangement of aspheric
meniscus lenses about a central stop. There are twenty specific design examples
given some embodying PMMA and polystyrene elements and others
embodying PMMA and polycarbonate elements. There are twenty-one claims.

D.C.G.

4,561,736  31 Dec. 1985 (Cl. 351-169)
Eyeglass lenses for persons suffering from severe ametropia.
G. FURTER and H. LAHRES. Assigned to Carl-Zeiss-Stiftung.
An eyeglass lens for persons requiring a very strong correction is described.
The shape claimed is said to give good vision, reduced lens weight, and better
cosmetic appearance. The aspheric curves required may make the lens quite
costly.

J.J.J.S.

4,563,060  7 Jan. 1986 (Cl. 350-414)
Microscope objective.
M. YAMAGISHI. Assigned to Olympus Optical Co., Ltd. Filed 2
Three embodiments are given of a ten-element flat-field microscope objective
having a magnification of 20X and a numerical aperture of 0.4. The
working distance is long, being ~1.4 times the focal length. The front element
is a positive meniscus concave to the front; this is followed by another positive
element and a series of cemented doublet, two of which are negative.
The design is controlled by seven conditions.

R.K.

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