Phase-induced intensity noise in an incoherent Fabry–Perot interferometer and other recirculating devices

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Laser phase-induced intensity noise limits the dynamic range of all kinds of recirculating structures under short coherence illumination. The shape of the power spectrum of this noise, while it is always periodic, depends on the device type. Recirculating delay lines and the reflecting Fabry–Perot interferometer (F–P) exhibit minimum noise at integer multiples of 1/(round-trip time), whereas the transmitting F–P shows maximum noise at exactly these frequencies. This phenomenon is theoretically analyzed, and expressions are given for the standard deviation, the autocovariance, and the power spectrum of that noise for a general recirculating structure. The important problem of laser phase-induced intensity noise for the F–P structure is studied in detail, theoretically and experimentally, in both reflecting and transmitting modes.

1. INTRODUCTION

Recirculating structures are devices in which the optical radiation traverses the same optical path over and over again. At each round trip, some of the power carried by the optical radiation escapes the cavity. These sampled fields add up to form the optical output(s) of the structures. When the coherence time of the optical radiation is much longer than the round-trip time, the sum of the infinite series of sampled complex field amplitudes exhibits resonance behavior. Here the output has peaks (or notches) whenever the round-trip length is an integral multiple of the optical wavelength. Thus the output intensity is critically dependent on the cavity length, and when the round-trip loss is very small, the peaks (or notches) are very sharp, making the device an excellent spectral filter. On the other hand, when the source coherence time is much shorter than the round-trip time (incoherent recirculating structure), the average output intensity is the sum of the individual average intensities of the recirculating fields, and its value is independent of the cavity length (provided that the cavity loss remains the same).

Until recently, only coherent resonators were investigated and used for a variety of purposes. Lately, however, incoherent recirculating structures have found some interesting applications, the main one being rf notch filtering. Other uses include transient buffer memory and data rate transformers, building blocks for single-mode-fiber lattice filters, and amplitude to rf phase conversion in fiber optic sensors. While the work described in Refs. 2–5 made use of single-mode-fiber recirculating delay line, an incoherent Fabry–Perot interferometer (F–P) served as the optical cavity in Ref. 6.

It has been also found that the fiber-optic recirculating delay line, when driven by a low-coherence semiconductor laser, exhibits excess noise at the output, which significantly limits the dynamic range of the device. This excess noise results from the conversion of the laser phase noise to intensity noise at the output of the loop, which combines time-delayed versions of the laser output on an amplitude basis. The resulting power spectrum is periodic and is characterized by deep notches at zero frequency as well as at other multiples of 1/(loop-delay).

Since this phase-induced intensity noise is due to nonzero correlations among various intensity terms of the recirculating fields, it is anticipated that this kind of noise will also show up in other types of incoherent recirculating structures. Indeed, Ref. 11 theoretically discusses phase-induced intensity noise in a transmitting F–P, but it does not analyze the structure of the power spectrum of that noise.

The aim of this paper is to treat theoretically this phase-induced intensity noise in a general incoherent recirculating structure, with an emphasis on the detailed shape of its power spectrum, as well as to present experimental results that support the theoretical study. Section 2 provides the mathematical formulation, and Section 3 describes our experiments with both reflecting and transmitting Fabry–Perot cavities. We conclude in Section 4 with a discussion of the results.

2. MATHEMATICAL FORMULATION

A. General

Let the input, unit amplitude laser field be given by

$$E(t) = \exp[i(\omega_0 t + \phi(t))]$$

where $\omega_0$ is the center optical frequency and $\phi(t)$ represents the source phase noise. The field at the output of the recirculating device is

$$E_{\text{out}}(t) = \sum_{n=0}^{\infty} F_n E(t - nr)$$

$$= \sum_{n=0}^{\infty} F_n \exp[i(\omega_0(t - nr) + \phi(t - nr))]$$

where $F_n$ are the complex amplitudes of the sampled fields and $r$ is the round-trip time of the structure. For all recirculating structures, the ratio between two consecutive fields is independent of $n$, except, maybe, for the first two terms in...
We get

\begin{align}
F_0 &= \delta_{0c} \delta_{1a} \delta_{1b} t_{1a} t_{1b}, \\
F_1 &= \delta_{0c} \delta_{1a} \delta_{2a} t_{1a} t_{2b} t_{2a}, \\
F_2 &= \delta_{0c} \delta_{1a} \delta_{2b} t_{1a} t_{2b} t_{2b},
\end{align}

where \( \delta_{0c} \) is the single-path cavity loss. Notice that in this case \( F_n = TR^n \) for all \( n \). This is no longer true for the reflection mode, where the first \( F_n \)'s are

\begin{align}
F_0 &= \delta_{0c} \delta_{1a} \delta_{1b} t_{1a} t_{1b}, \\
F_1 &= \delta_{0c} \delta_{1a} \delta_{2a} t_{1a} t_{2b} t_{2a}, \\
F_2 &= \delta_{0c} \delta_{1a} \delta_{2b} t_{1a} t_{2b} t_{2b},
\end{align}

and thus

\[ F_n = \delta_{0c} t_{1a} t_{1b} \]

Finally, let us consider the recirculating fiber-optic delay line [Fig. 1(b)]. For this device we have (see Ref. 8)

\begin{align}
F_0 &= \delta_{0c} C, \\
T &= \delta_{0c} B D^{-1} A, \\
R &= \delta_{0c} B \exp[-\alpha_0 L],
\end{align}

where \( \delta \) is the coupler loss, \( L \) is the loop length, and \( \alpha_0 \) is the fiber loss per unit length. \( A, B, C, \) and \( D \) are the entries of the unitary matrix,

\[ \begin{bmatrix} A & B \\ C & D \end{bmatrix} \]

The similarity between the reflecting F-P and the fiber-optic recirculating delay line has already been recognized, the second mirror playing the role of the feedback fiber that couples field \( E_3 \) to field \( E_2 \) of the directional coupler.

C. The Theory

We follow the development of Ref. 8, thus assuming a stationary normal process with zero mean and flat spectrum for the source frequency noise and neglecting amplitude noise as well as chromatic dispersion. Ignoring polarization, we find that, for a round-trip time \( r \) much longer than the source coherence time \( \tau_c \), the autocovariance function of the output intensity, which is defined by

\[ C(t_1, t_2) = \langle (I(t_1) - \langle I \rangle)(I(t_2) - \langle I \rangle) \rangle, \]

can be expressed as a convolution [see Eqs. (33)-(35) of Ref. 8]:

\[ C(t_1, t_2) = C_0(t_1, t_2) \otimes \exp[-|t_1 - t_2|/\tau_c]. \]

Here

\[ C_0(t_1, t_2) = \sum_{M=\pm} A_M \delta[(t_1 - t_2) - M\tau_c], \]

where \( \delta[(t_1 - t_2) - M\tau_c] \) is the Dirac \( \delta \) function, centered around \( M\tau_c \), and \( A_M \) is given by
\[ A_M = \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} F_m F_m^* F_{m-M} F_{m-M}^* = 0, \]  
(12)\

where \( \langle \cdot \rangle \) denotes an ensemble average.

Substituting Eq. (3) into expression (12) yields two nesting infinite geometrical series that sum up to give

\[ A_0 = 2|F_o|^2 T^2 - \frac{U}{1-U} + 2|T|^4 \frac{U^5}{(1-U)(1-U^2)}, \]  
(13)\n
\[ A_M = kU^{M}, \quad M \neq 0, \]  
(14)\n
\[ \kappa = 2 \text{Re}[F_0^* T]|T|^2 - \frac{U}{1-U} + 2|T|^4 \frac{U^5}{(1-U)(1-U^2)}, \]  
(15)\n
where \( U = |R|^2. \)

From Eqs. (9)-(11) we find

\[ \langle [I(t) - \langle I \rangle]^2 \rangle = C(t_1 = t_2) = A_0. \]  
(16)\

Thus \( A_0 \) is the variance of the output phase-induced intensity noise. The average output light intensity is given by Ref. 8:

\[ \langle I \rangle = \sum_{n=0}^{\infty} |F_n|^2 = |F_0|^2 + |T|^2 - \frac{U}{1-U}. \]  
(17)\

The normalized standard deviation of the output can now be explicitly written:

\[ \rho = \sqrt{\frac{\langle [I(t) - \langle I \rangle]^2 \rangle}{\langle I \rangle^2}} = \sqrt{A_0}. \]  
(18)\

\( A_0 \) and \( \langle I \rangle \) are given in Eqs. (13) and (17). For the case of the transmitting F-P, Eq. (18) reduces to the result of Ref. 11. It should be emphasized that this normalized standard deviation, which is independent of \( \tau \), measures the noise all over the spectrum, which extends beyond \( 1/\tau \). Of course in actual systems the detection bandwidth is usually much smaller (see below); thus it will be advantageous to investigate the spectral distribution of this noise. The power spectrum of the output intensity is the Fourier transform of the autocovariance function [Eq. (10)]. Thus we obtain for the power spectrum

\[ S(f) = S_0(f) \frac{2\tau}{1 + (2\pi\tau f)^2}. \]  
(19)\n
where \( S_0(f) \) is the Fourier transform of \( C(t_1, t_2) \):

\[ S_0(f) = \sum_{n=0}^{\infty} A_M \exp(-2\pi i M \tau f). \]  
(20)\n
The full spectrum \( S(f) \) in Eq. (19) can be obtained by multiplying \( S_0(f) \) by a decreasing envelope (with a characteristic scale of \( 1/\tau \approx 10 \text{GHz} \)), which is determined by the actual line shape of the laser mode. Substituting \( A_M \) from Eqs. (13) and (14) yields our final result,

\[ S(f) = A_0 + 2kU - \frac{\cos(2\pi f \tau) - U}{1 + U^2 - 2U \cos(2\pi f \tau)}. \]  
(21)\n
Hence the power spectrum of the phase-induced intensity noise is periodic, with a period of \( 1/\tau \), regardless of the specific configuration of the recirculating structure. Two parameters dictate the shape of the power spectrum: \( U \) and \( \kappa \). \( U \) is the round-trip intensity loss of the cavity, while \( \kappa \) bears all the information on the specific configuration of the device. For two different configurations with the same round-trip loss (U), the shapes of the power spectra may differ in the modulation depth and/or in the location of minima and maxima, depending on the magnitude and sign of \( \kappa \).

We consider now a few examples:

(a) The Reflecting Fabry–Perot Interferometer. After substituting Eqs. (7) into Eq. (15) we get

\[ \kappa = 2\delta_{01}^4 \text{Re} \left[ \frac{r_{1b}}{r_{1a}} \frac{t_{1a}^{*} t_{1b}}{r_{1b}^{*}} \right] \frac{U^3}{(1-U)(1-U^2)}, \]  
(22)\n
where

\[ U = |R|^2 = \delta_0^{*} \delta_{01}^{*} \delta_{02}^{*} \delta_{04}^{*} r_{3b} r_{1b}^{*}. \]  
(23)\n
Using the unitarity property, Eqs. (4a) and (4b), it is easily shown that \( \kappa < 0 \), resulting in minima at all integer multiples of \( 1/\tau \) [see curve (a) of Fig. 2].

(b) The Recirculating Delay Line. Here,

\[ \kappa = 2\delta_{01}^4 \text{Re} \left[ \frac{DA^*}{B} \right] \frac{DA^2}{B} \frac{U}{1-U} \]  
(24)\n
where

\[ U = |R|^2 = \delta_0^{*} |D|^2 \exp(-2\alpha L). \]  
(25)\n
Equation (24), after some manipulations, yields the same expression for \( S(f) \) as in Eq. (47) of Ref. 8. Again, when the
coupling matrix is unitary, $k < 0$, curve (a) of Fig. 2 applies, and its predictions are supported well by the experimental evidence.8

(c) The Transmitting Fabry-Perot Interferometer. Making use of Eqs. (6), we find that $[U$ is given by Eq. (23)]

$$k = \frac{2\theta_0 e^{i\phi_0} e^{i\phi_2}}{1 - (1 - U)(1 - U^2)}.$$  (26)

Here, for the first time, we get a positive $k$, so that all $A_M$'s are positive [Eq. (14)], and the shape of the power spectrum has the appearance of curve (b) of Fig. 2, with minima at $(N + 1/2)/T$.

3. EXPERIMENTAL RESULTS

In order to check the validity of the general theory, a bulk-optics Fabry-Perot structure was constructed (see Fig. 3). The device was driven with a low-coherence 0.82-μm multimode semiconductor laser, whose coherence length was previously found to be approximately 30 mm. After spatial filtering and collimation, a 10% transmittance beam splitter was placed in order to minimize backreflections into the laser. This beam splitter served also to direct most of the radiation, reflected from the device, into a detector. Since this beam splitter deflected most of the input laser power, it also permitted direct monitoring of the laser output in order to exclude the possibility that the measured excess noise is due to backreflections into the laser. The dielectric mirrors, which formed the cavity, had intensity reflectances and transmittances of 0.38 and 0.58, respectively (the same values were assumed for the calculated solid curves of Fig. 2).

The detection system consisted of a fast detector, a 1–500 MHz, 45-dB preamplifier, and a spectrum analyzer. The 3-dB point of the detection system was at 110 MHz. The detector was alternately placed at one of the three different locations, designated I, II, and III (see Fig. 3), in order to analyze the reflected, transmitted, and direct laser output beam, respectively.

Figures 4–6 show the spectrum of the detected radiation, at locations I and II, for different system parameters. Figure 4 shows the noise power spectrum of the beam reflected from the cavity (a) together with the reflection when the cavity was internally blocked (b) and the inherent noise of the detection system (laser turned off) (c). The length of the cavity was $2L = 375$ cm, corresponding to $\tau = 12.5$ nsec or to a spectral period of 80 MHz. The detector dc voltages, measured with a 100-Ω load resistor, were 0.84, 0.74, and 0.010 V for curves (a), (b), and (c), respectively. These dc measurements indicate that the increase in measured noise when the cavity is properly tuned cannot be attributed to an additional source intensity noise because of an increase in the average intensity that falls upon the detector. Rather, the excess intensity noise is generated by the interferometric conversion of the source phase noise to an output intensity.
noise. Curve (a) of Fig. 5 shows the noise of the transmitted beam at exactly the same system situation as in curve (a) of Fig. 4. Curves (b) and (c) of Fig. 5 were taken with untuned cavity and no input illumination, respectively. As was expected, the spectra are periodic with period $1/\tau$. In the reflected beam, the first minimum is located at zero frequency, whereas the spectrum of the transmitted beam has its first minimum at $1/2\tau$. In Fig. 6, the system has been modified: the rear partial mirror was replaced by a silvered mirror with higher reflectivity ($\approx 0.9$), and the cavity was shortened to $2L = 250$ cm. Comparison of Figs. 4 and 6 shows that the depth of the minima depends on the reflectance of the mirrors, in agreement with the theory, as indicated by the dashed curves of Fig. 2. Again, the spacing ($\Delta f$) between the minima follows the expected relation, $\Delta f/\tau = 1$. In all the above cases, the noise level of the input laser beam, as monitored at location III, was stable and independent of the cavity settings, showing no measurable feedback.

4. DISCUSSION AND CONCLUSIONS

This paper has considered phase-induced intensity noise in a general incoherent recirculating structure. This noise is important because it limits the dynamic range of these devices. The power spectrum of the noise was calculated, as were its covariance function and the normalized standard deviation. Experimental results were given for some specific devices. These results, which agree well with the theory, show a phase-induced intensity noise of as much as $15$ dB above the source intensity noise—a fact that approximately justifies the neglect of the laser amplitude noise in the theoretical analysis.

As expected, the power spectrum shows a periodic structure with period $1/(\text{round-trip time})$, but the exact shape of a single period, e.g., the location of the minimum, was found to depend on the specific structure of the device.

One possible limitation of the analysis is that a single-mode laser was assumed [Eq. (1)], whereas the experimental system used a multimode laser. This subject has been discussed in Ref. 8.

In conclusion, whenever a recirculating optical structure is considered with a low-coherence laser as its source, one has to take into account the excess noise, resulting from the laser's phase noise, that will be present at the output. The noise power that will enter the detection system can be minimized by choosing the system's operating frequency response to overlap one of the minima in the power spectrum. Another possibility for reducing the phase-induced intensity noise can be drawn from Eq. (19), from which it can be seen that a lower source coherence time ($\tau_r$) will tend to spread the noise power to higher rf's, thus decreasing the noise power entering the finite bandwidth of the detection system. As a consequence, lasers with very broadband emissions should be used in these devices.

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REFERENCES AND NOTES

13. Superluminescent diodes and other quasi-monochromatic sources have extremely short coherence times. However, being thermal sources, their intensity noise is significantly higher than that of lasers.