Phase-Induced Intensity Noise in Optical Interferometers Excited by Semiconductor Lasers with Non-Lorentzian Lineshapes

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Abstract—Previously obtained results for the covariance, power spectrum, and variance of the phase-induced intensity noise (PIIN) at the output of optical systems with multiple paths are modified to include the effect of non-Lorentzian source lineshapes. In the coherent limit (delay $\ll$ source coherence time), the source relaxation oscillations significantly change the spectral shape of the PIIN at the output of such multiple-path architectures.

Optical systems with multiple paths in which a semiconductor-injection laser source is being split, and after experiencing different time delays recombined on an amplitude basis, are widely used in fiber-optic interferometric sensors [1] and fiber-optic signal processing systems [2], as well as in systems that characterize lasers for coherent communication applications [3]. One of the major noise sources in these systems is an excessive amount of output intensity noise, which is the result of the interferometric conversion of the laser-phase noise. Being directly proportional to the input power, this phase-induced intensity noise (PIIN) may become much stronger than the source-intensity noise and tends to severely limit the dynamic range of vulnerable fiber-optic sensors and signal processors [4], [5].

Laser-phase noise in optical interferometers was studied by many authors, e.g., [3]–[9]. Common to all their treatments is the assumption that the deviation of the laser instantaneous optical frequency $\omega(t)$ from its mean $\bar{\omega}$ can be modeled by a stationary normal (Gaussian) random process with a flat spectrum [7]. Thus, if one represents the output of an intensity-noiseless, single-mode laser, emitting at a center frequency $\omega_0$ by

$$E(t) = \sqrt{I_0} \exp \left[ i(\omega_0 t + \phi(t)) \right]$$  \hspace{1cm} (1)

where $I_0$ is the laser output power, the laser phase noise $\phi(t)$ is a Wiener-Levy random process whose structure function $D_\phi(\tau)$ is proportional to $|\tau|$:}

$$D_\phi(\tau) = \left\langle \left[ \phi(t + \tau) - \phi(t) \right]^2 \right\rangle \propto |\tau|. \hspace{1cm} (2)$$

The angle brackets denote ensemble average. Since the laser lineshape is the Fourier transform of the laser field autocorrelation function, $\langle E(t) E^*(t + \tau) \rangle = I_0 \exp \left( -i\omega_0 \tau \right) \exp \left[ -0.5 D_\phi(\tau) \right]$, the linear dependence of $D(\tau)$ on $\tau$ in (2) corresponds to a Lorentzian lineshape [10].

However, careful measurements of the lineshape of single-mode semiconductor lasers made by Vahala et al. [11] and by Daino et al. [12] showed that the lineshape is non-Lorentzian, due to side peaks at the relaxation-oscillation frequency. As a result, the structure function $D_\phi(\tau)$ can no longer be assumed to be proportional to $|\tau|$, and it is the purpose of this paper to study the implications of the appropriately modified structure function, with its associated non-Lorentzian lineshape, on the power spectral density of the PIIN at the output of relevant optical systems.

The main intrinsic source of noise in lasers is the random spontaneous emission. Every such spontaneous event disturbs the steady-state populations of the laser energy levels, but the system quickly recovers in a short series of relaxation oscillations of both the output intensity and the carrier concentration. However, since the refractive index of the semiconductor lasing medium is a relatively strong function of the carrier concentration [13], the spontaneous emission event not only generates intensity fluctuations but also affects the laser phase noise both through the noncoherent spontaneous photon and through the induced-carrier concentration fluctuations [10]. Thus, (2), which only represents the random walk of the phases of the randomly emitted spontaneous photons, has to be modified to properly describe those phase fluctuations related to the relaxation oscillations.

Although several alternative formulations are available, we follow the formulation of C. H. Henry [10]
\[ D_\alpha(\tau) = \left( \frac{R}{2l} \right) \left[ (1 + \alpha^2)|\tau| + \frac{\alpha^2 \cos(3\delta) - \exp(-\Gamma|\tau|) \cos(2\pi f_{n0}|\tau| - 3\delta)}{2\Gamma \cos(\delta)} \right] \]  

(3)

where \( R \) is the spontaneous emission rate into the lasing mode, \( l \) is the number of photons per mode, and \( \alpha \) is the linewidth enhancement factor, which relates the phase fluctuations with those of the carrier concentration: \( \alpha = (\partial \text{Re}(n)/\partial G)/|\partial \text{Im}(n)/\partial G| \) (\( n \) is the complex refractive index of the lasing medium and \( G \), the gain, is proportional to the carrier concentration). Although in most other lasers \( \alpha = 0 \), semiconductor lasers are characterized by \( 4 < \alpha < 7 \) [10]. Carrier injection in semiconductor diodes gives rise to band filling, which in turn shifts the optical absorption edge towards lower wavelengths and consequently also modifies the imaginary part of the material refractive index. According to the Kramers–Kronig relations, the imaginary part of the refractive index is coupled to its real part, and since the laser frequency depends on the optical length of the cavity, fluctuations in the carrier concentration result in excessive phase noise [13]. The relaxation-oscillation frequency, \( f_{n0} \), \( \Gamma \) is the damping rate of relaxation oscillation, and \( \delta \) is defined by \( \cos(\delta) = 2\pi f_{n0}/(2\pi f_{n0})^2 + \Gamma^2 \). Equation 3 ignores the relatively small effects of spectral hole burning and the shot noise related to the generation and recombination of minority carriers [10]. The structure function for the non-Lorentzian lineshape (3) is shown in Fig. 1. Typical values for the parameters in (3) were taken from [10, table I]: \( R = 1.5 \cdot 10^{12} \text{ s}^{-1}, I = 3.1 \cdot 10^9 \text{ photons per mode}, \alpha = 5.3, f_{n0} = 1.52 \text{ GHz}, \Gamma = 0.4 \cdot 10^9 - 1.8 \cdot 10^9 \text{ s}^{-1}, \) and \( \delta = 2.4^9 - 10.8^8 \). For comparison, we show the limiting case where \( \Gamma \) reaches infinity: the relaxation oscillations are completely quenched, and \( D_\alpha(t) \) is again proportional to \( \tau \), as in the Lorentzian case.

The coherence time is defined using the normalized field coherence function as [14]:

\[ \tau_c = \int_{-\infty}^{\infty} \frac{\langle E(t)E*(t - \tau) \rangle^2}{\langle |E(t)|^2 \rangle} \, d\tau. \]  

(4)

Since \( \phi(t) \) in (1) is a Gaussian random process, the integral in (4) becomes

\[ \tau_c = \int_{-\infty}^{\infty} \exp\left[ -D_\alpha(\tau) \right] \, d\tau. \]  

(5)

In the limit where the damping rate \( \Gamma \) reaches infinity, the lineshape is again proportional to \( \tau \), as in the Lorentzian case, with a coherence time

\[ \tau_c = 4\Gamma/(R(1 + \alpha^2)). \]  

(6)

However, when we use a finite value of the damping rate, the coherence time may deviate from this value. One can see from Fig. 1 that for low damping rates, the coherence time becomes meaningless due to the oscillations in the structure function [15]. Nonetheless, the coherence times for the three structure functions of Fig. 1 were calculated using (5), and they are indicated on the graph.

When the laser field of (1) is injected into an optical system with interferometrically interacting multiple paths, the output field can be generally expressed by [16]:

\[ \tilde{E}_{out}(t) = E_a(t) \tilde{P}_a + E_b(t) \tilde{P}_b = \sum_{n=1}^{N_a} E_{a,n} \exp\left[ i(\omega_0(t - T_{a,n}) + \phi(t - T_{a,n})) \right] \tilde{P}_a + \sum_{n=1}^{N_b} E_{b,n} \exp\left[ i(\omega_0(t - T_{b,n}) + \phi(t - T_{b,n})) \right] \tilde{P}_b \]  

(7)

where \( \tilde{P}_a, \tilde{P}_b \) are the unit-intensity polarization vectors of the two orthogonal polarization modes at the system output, \( E_a(t) \) and \( E_b(t) \) are their respective amplitudes, \( N \) is the number of different path delays, and \( E_{a,n}(E_{b,n}) \) is the complex amplitude of the \( P_a^i(P_b^i) \) polarized field that has experienced a delay of \( T_{a,n}(T_{b,n}) \). The intensity covariance function is

\[ \text{cov}(\Delta T) = \langle [I(t + \Delta T) - \langle I \rangle][I(t) - \langle I \rangle] \rangle. \]  

(8)

and since \( I(t) = I_a(t) + I_b(t), \text{cov}(\Delta T) = \text{sum of } C_{aa}(\Delta T), C_{ab}(\Delta T), C_{ba}(\Delta T), \) and \( C_{bb}(\Delta T) \), where \( C_{aa} \) and \( C_{bb} \) are, respectively, the autocovariances of \( I_a \) and \( I_b \), whereas \( C_{ab} \) and \( C_{ba} \) are the crosscovariances. Using the definition \( C_{aa}(t_1, t_2) = \langle I_a(t_1)I_a^*(t_2) \rangle - \langle I_a \rangle \langle I_a^* \rangle \), it was shown in [16] that

\[ C_{aa}(\Delta T) = \sum_{n=1}^{N_a} \sum_{m=1}^{N_b} \sum_{n=1}^{N_a} \sum_{m=1}^{N_b} E_{a,n} E_{a,m}^* E_{b,m} E_{b,n}^* \]  

\[ \cdot \exp\left[ i(\omega_0(T_{a,n} - T_{a,m} + T_{b,m} - T_{b,n})) \right] \]  

\[ - 0.5[D(\Delta T + T_{b,m} - T_{a,n}) - D(\Delta T + T_{a,m} - T_{b,n})] \]  

\[ - D(\Delta T + T_{b,m} - T_{a,m}) \]  

\[ + D(\Delta T + T_{b,m} - T_{a,n}) \]  

\[ - D(T_{a,n} - T_{a,m}) - D(T_{b,m} - T_{b,n}) \]
- \exp \left[ -0.5 \left[ D(T_{\phi,m} - T_{\phi,n}) + D(T_{\beta,m} - T_{\beta,n}) \right] \right].

(9)

For simplicity, we shall study the case of a two-beam interferometer (N = 2) with a delay of \( \tau \), in which the light remains fully polarized in the \( P_0 \) polarization, and the field output coefficients of the undelayed and delayed output fields are \( K_1 \sqrt{I_0} \) and \( K_2 \sqrt{I_0} \), respectively. In this case, the covariance \( \text{cov}(\Delta T) = C_{\phi \phi}(\Delta T) \) can be simplified to give

\[
\text{cov}(\Delta T) = 2 \left[ K_1 K_2^* \right]^2 \sqrt{I_0} \left[ \exp \left[ 0.5 \left( -2D(\Delta T) - 2D(\tau) \right) \right] - \exp \left( -D(\tau) \right) \right] \left( K^*_1 K_2 - 2i\omega_0 \tau \right) + \left( K^*_1 K_2 \right)^2 \left[ \exp \left[ 0.5 \left( 2D(\Delta T) - 2D(\tau) \right) \right] - D(\Delta T + \tau) - D(\Delta T - \tau) \right] - \exp \left( -D(\tau) \right) \].
\]

(10)

According to the Wiener-Khinchine theorem [17], the power spectrum of the PIN, \( S(f) \), is the Fourier transform of the autocovariance \( \text{cov}(\Delta T) \). Fig. 2 shows the power spectrum of the PIN, in the case of in-quadrature \( (\omega_0 \tau = \pi / 2 + n\pi) \) and maximum output \( (\omega_0 \tau = 2n\pi) \), for both Lorentzian (Fig. 2(a)) and non-Lorentzian (Fig. 2(b) and (c)) lineshapes. Both arms are identically excited \( (K_1 = K_2 = 1/2) \). The coherence time for the Lorentzian case was chosen to be \( \tau_L \) (6), which corresponds to a damping rate \( \Gamma = \infty \) in the non-Lorentzian lineshape. For the non-Lorentzian lineshapes, two values of the damping rate \( \Gamma \) were chosen: \( \Gamma = 1.8 \cdot 10^9 \text{ s}^{-1} \), (Fig. 2(b)), and \( \Gamma = 0.4 \cdot 10^9 \text{ s}^{-1} \), (Fig. 2(c)), which correspond, respectively, to relaxation-oscillation-related satellite side peaks in the lineshape of approximately 1 and 10 percent (with respect to the peak intensity at the central frequency \( \omega_0 \)). The results obtained from (10) using an FFT algorithm are shown for three different values of \( \tau/\tau_L \), covering the coherent and incoherent limits \( \tau/\tau_L = 0.1 \) and \( \tau/\tau_L = 10 \), respectively, as well as an intermediate case \( \tau/\tau_L = 1 \). Fig. 3 is the same as Fig. 2 except that instead of the real frequency \( f \), a scaled frequency \( f_\tau \) is used to annotate the frequency axes. These two different representations help to identify the important characteristic scales in \( S(f) \). Note that for low values of \( \tau/\tau_L \), Fig. 3 encompasses a wider frequency span, whereas the converse is true for high values of \( \tau/\tau_L \).

In the coherent regime \( (\tau/\tau_L = 0.1) \), the noise power spectrum of a non-Lorentzian source at the output of the interferometer (Fig. 2(b) and (c)) and Fig. 3(b) and (c) considerably deviates from the spectrum obtained under the assumption of a Lorentzian lineshape (Figs. 2(a) and 3(a)). At relatively low frequencies, the characteristic frequency in the non-Lorentzian case is the relaxation-oscillation frequency (see Fig. 2(b) and (c)), whereas for
higher frequencies, it becomes $1/\tau$, as in the case of the Lorentzian lineshape, (Fig. 3(b) and (c)). In the in-quadrature case, we get a peak in the PIN spectrum at the relaxation-oscillation frequency $f_{ro}$, as observed by Daino et al. [12]. However, in case of maximum output, we get a peak at twice the relaxation-oscillation frequency. The existence of this peak is clearly supported by Fig. 4, which shows the autocovariance $\text{cov} (\Delta T)$ for the in-quadrature and maximum-output cases. To the best of our knowledge, this peak has not been observed as yet. If confirmed, it may serve as an indication for the state of the interferometer, being in-quadrature, out of quadrature, or somewhere in between.

It is also interesting to note that the power spectrum of the non-Lorentzian source in the high-frequency range gets its minima at frequencies that are multiples of $1/\tau$ in the case of in-quadrature, whereas in the case of maximum output, maxima are obtained at the same frequencies (Fig. 3(b) and (c)). The $1/\tau$ periodicity, which is also evident in the coherent Lorentzian case, originates from discontinuities in the derivative $d \text{cov} (\Delta T)/d \Delta T$ at $\Delta T = \pm \tau$ (see the insets in Fig. 4) $(d \text{cov} (\Delta T)/d \Delta T$ is antisymmetric in $\Delta T$). The second derivative $d^2 \text{cov}$

Fig. 3. Normalized phase-induced intensity noise power spectrum $S(f)$ as a function of the scaled frequency $f/\tau$. All other parameters are the same as in Fig. 2.

Fig. 4. Intensity autocovariance function (10) at the output of a symmetrically excited MZDI. We used $R = 1.5 \cdot 10^{12}$ s$^{-1}$, $I = 3.1 \cdot 10^9$, $\alpha = 5.3$, $f_{ro} = 1.52 \cdot 10^9$ Hz, $\tau_{cl} = 4/(R(1 + \alpha^2)) = 2.84$ ns, $\Gamma = 1.8 \cdot 10^6$ s$^{-1}$, and $\tau/\tau_{cl} = 0.1$. The insets show the discontinuity of the derivative of the autocovariance at $\Delta T = \tau$, which accounts for the $1/\tau$ period at the higher frequencies of the noise spectrum (see Figs. 3(b) and (c)). (a) In-quadrature; (b) maximum output.

$(\Delta T)/d^2 \Delta T$ will exhibit, therefore, Delta function discontinuities at $\Delta T = \pm \tau$. $+\delta (\Delta T - \tau)$ and $-\delta (\Delta T + \tau)$ for the in-quadrature case but $-\delta (\Delta T - \tau)$ and $+\delta (\Delta T + \tau)$ at maximum output. These sign differences are responsible for the different locations of minima and maxima for these two cases.

As we get into the incoherent regime, the above-mentioned peaks at multiples of the relaxation-oscillation frequencies tend to disappear, and the noise spectrum becomes similar to that of the Lorentzian source. This result is consistent with the fact that the structure functions for the Lorentzian and non-Lorentzian lineshapes are different only in the coherent regime and become similar when $|\tau|$ goes to infinity (see Fig. 1).

Note that even relatively small relaxation-oscillation-related satellite side peaks (1 percent of the peak intensity at $f_{ro} = 1.8 \cdot 10^9$ s$^{-1}$) have a very sizable effect on the spectrum of the PIN noise (Figs. 2(b) and 3(b)). The difference between the Lorentzian and non-Lorentzian
spectrum becomes smaller when the damping rate of the relaxation oscillations $\Gamma$ becomes higher. A high damping rate may explain the results of [3], where the PIN of a DCPBH InGaAsP DFB laser at the output of a Mach-Zehnder interferometer could be fully accounted for by a Lorentzian model.

The variance of the output intensity $\sigma^2 I_{\text{out}}(\tau)$, which represents the total amount of the PIN noise, is obtained by evaluating the covariance function at its origin, $\text{cov}(\Delta \tau = 0)$ (see Fig. 5). Again, the difference between the variance functions in the Lorentzian and non-Lorentzian cases is very significant, mainly in the coherent regime.

In summary, the phase-induced intensity noise at the output of optical interferometers driven by semiconductor lasers whose lineshape exhibit relaxation-oscillations peaks cannot be accurately predicted using a Lorentzian model. For the non-Lorentzian structure function of (3), the deviations from the Lorentzian-based results are mainly in the coherent regime, where the PIIN spectrum at the output of a two-beam interferometer is characterized by a peak at the relaxation-oscillation frequency for the in-quadrature case and by a peak at twice this frequency at maximum output setting. In the incoherent limit, the Lorentzian model can be used as a fairly good approximation. More general multiple-path architectures, including polarization effects, can be treated by inserting the structure function (3) into the covariance equation (8). To obtain a more accurate description of the source-induced noise at the output of multiple-path optical systems, the source-intensity noise and its correlation with the source-phase noise [11] should be also taken into account. These subjects are now under current investigation.

REFERENCES

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