POLARISATION-INDUCED VISIBILITY LIMITS IN INTERFEROMETRIC FIBRE-OPTIC SENSOR ARRAYS

Indexing terms: Fibre-optics, Polarisation, Optical fibres

Polarisation-induced fading is analysed in Mach-Zehnder type interferometric fibre-optic sensor arrays. Given a sensor array configuration and the polarisation properties of individual elements, a procedure is presented to find an optimal input state of polarisation which maximises overall sensor visibility. It is shown that for both serial and parallel arrays with N sensor elements, the visibility of the worst-case sensor configuration can be maximised to a value of \( n/(2N) \).

Introduction: Polarisation-induced-fading (PIF) is a major problem in interferometric fibre-optic sensors. Since the sensor scale factor and the output S/N ratio critically depend on the polarisation states of the interfering beams, environmentally related random polarisation drifts in the fibre arms may severely degrade the sensor performance. Polarisation-maintaining fibres and couplers could, in principle, eliminate the PIF problem. However, their high prices and insufficient polarisation-holding characteristics, especially when splices are involved, prevent them from replacing the commonly used non-polarisation-maintaining single-mode fibres.

A few approaches have been proposed to overcome and control the PIF phenomenon in single sensors of the Mach-Zehnder type (or its Michelson equivalent) made of regular single-mode fibres. These include: (i) adjustment of the sensor arms to produce parallel output polarisations (a solution which obviously requires access to the sensor itself); (ii) polarisation scrambling at the input which solves the stability issue at the expense of less than optimal visibility; (iii) the simultaneous use of several polarisation analysers at the sensor output, thereby complicating the detection system; and (iv) proper choice of the state of polarisation (SOP) of the input light which can completely overcome the PIF penalties, although careful monitoring of the drifting polarisation state of the sensor itself is an obvious prerequisite.

Recently, multi-element fibre-optic sensor arrays have drawn considerable attention due to their advantages in many present and future applications, e.g. smart skis. Many multiplexing techniques, both serial and parallel, have been proposed and demonstrated but the PIF problem has been only partially addressed.

For an N-element sensor array and a given input SOP (C_{inp}, N visibility, \( \{V_{s_{i}}\}, i=1, \ldots, N \)), will be observed at the output of the N individual sensors. Array performance will be determined by the smallest visibility, \( V_{array} = \min_{\{X_{i}\}} \{V_{s_{i}}\}, i=1, \ldots, N \). For a particular polarisation state of the sensor array, as defined by the environmentally-dependent polarisation transformation properties of its individual elements, C_{inp} can be adjusted to maximise the value of \( V_{s_{i}} \). However, this maximum, \( V_{array} \), will vary as the polarisation states of the individual sensor elements drifts. In this letter we determine the worst-case optimal visibility

\[
V_{array}(N) = \min_{\{X_{i}\}} \{V_{s_{i}}\}
\]

as a function of the number of elements N, where the polarisation states of the individual array sensors, \( \{D_{i}\} = \{X_{i}\} \), are allowed to assume all their physically possible realisations.

Representation of parallel and serial array of configurations: Both the reference and signal arms of a Mach-Zehnder fibre-optic interferometric sensor will modify an input SOP. C_{inp} into two new SOPs given by C_{r} = R \ast C_{inp} and C_{s} = S \ast C_{inp}, where R and S represent the polarisation transformations induced by the reference and signal arms, respectively. For a lossless interferometer with 1:1 polarisation-insensitive couplers, and using the Poincaré sphere representation for all involved SOPs, the fringe visibility is given by the absolute value of the cosine of half the angle subtended by the great arc C_{r} - C_{s}. But a great arc of the same magnitude is also subtended between C_{inp} and [RS^{-1}]. Thus, for the purpose of visibility estimation, the two-arm sensor is equivalent to a single unitary transformation, RS^{-1}. On the Poincaré sphere, the same sensor is represented by a diameter, D, whose tips are the two orthogonal eigen-polarisations of RS^{-1}. The output SOP, [RS^{-1}], C_{out}, is obtained by rotating C_{inp} around D through an angle \( \Omega = \phi_{1} - \phi_{2} \), where \( \phi_{1} \) and \( \phi_{2} \) are the phases of the two eigenvalues, exp[i\phi_{1}] and exp[i\phi_{2}]. Environmentally induced drifts will change both the direction of D and the value of \( \Omega \). Given C_{inp} and D, the worst obtainable visibility is realised when \( \Omega = \pi \) and its value is

\[
\cos[\pi(D, C_{inp})] = \cos[\pi(D, C_{out})],
\]

where \( \pi(D, C_{inp}) \) is the angle between D and C_{inp}.

Fig. 1 shows typical parallel and serial configurations. On its way to the ith sensor the common input SOP, C_{inp}, is transformed by the down-lead fibres to become C_{inp} = P_{i} C_{inp}. For a system of N sensors the visibility at the output of the ith sensor is determined by C_{inp} and the individual \( \{S_{i}\} \), i = 1,..., N. Alternatively, on the Poincaré sphere, this visibility is related to the great arc between C_{inp} and \( \{R S_{i}^{-1}\} \), C_{out}. But the same great arc is subtended by C_{inp} and \( \{P_{i} R S_{i}^{-1} P_{i}\} \). C_{inp} = \{R S_{i}^{-1}\}, C_{out}. Thus, the sensor array is represented on the sphere by a set of N diameters, \{D_{i}\}, with respective rotation angles \( \Omega_{i} \), in correspondence with the N unitary transformations \( \{R S_{i}^{-1}\} \). The worst visibility for the ith sensor is obtained, as before, when \( \Omega = \pi \), and can, therefore, be geometrically expressed as

\[
\cos[\pi(D_{i}, C_{inp})] = \min_{i=1,...,N} \{V_{s_{i}}\} = \min_{i=1,...,N} \{V_{array}(N)\}
\]

One should remember, though, that in arrays, where the individual outputs are collected and guided in a common output fibre, a coding technique must be implemented in order to identify the contributions of the individual sensors.

Determination of V_{array}(N): Given \( \{D_{i}\}, i = 1,..., N \), we wish first to find \( \max_{i=1,...,N} \{V_{s_{i}}\} \). Let us pick an arbitrary point on the Poincaré sphere. This point defines a possible input SOP, to be denoted by C_{inp}, together with a set of N visibilities \( \{V(C_{inp})\} = \cos[\pi(D_{i}, C_{inp})] \), i = 1,..., N. There will always be at least one particular sensor, say D_{j}, for which \( V(C_{inp}) = \max_{i=1,...,N} \{V_{s_{i}}\} \), i = 1,..., N throughout a finite area around C_{inp}, and possibly throughout a few more non-contiguous areas (A_{k}, k = 1,..., K) on the sphere. Denoting the union of these areas by A_{j} we find

\[
A_{j} = \bigcup_{k=1}^{K} A_{k}
\]

\[
= \{C_{inp}, V(C_{inp}) \leq V(C_{inp}), i = 1,..., N, i \neq j\}
\]
The set \( \{ A_j \} \), \( j = 1, \ldots, N \), spans the entire sphere. For each \( A_k \), an input SOP \( C_{k} \) can be found which maximises \( V \) in \( A_k \):

\[
C_k \in A_k \quad V(C_k) \geq V(C_{\text{cap}}) \quad \text{for all} \quad C_{\text{cap}} \in A_k
\]  

(4)

In this Letter we assume that for a given set of \( \{ D_i \} \), \( i = 1, \ldots, N \), we can always choose the best possible SOP for which min, \( V(D_i) \) (i.e., \( V \) maximised). Following the definition of eqn. 4, this best possible input SOP will coincide with a particular \( C_{D_i} \) say \( C_{D_i} \). Since \( V(C_{D_i}) = \text{cos} [\varphi(D_i, C_{\text{cap}})] \) is a monotonically decreasing function of the angle \( \varphi(D_i, C_{\text{cap}}) \) it follows that \( C_{D_i} \) must lie on the border line between \( A_j \) and \( A_{j'} \) with \( j \neq j' \), which is defined by \( \varphi(D_i, C_{\text{cap}}) = \varphi(D_{j'}, C_{\text{cap}}) \). Therefore, we conclude that this optimal input SOP, \( C_{\text{cap}} \), for which \( \text{max}_{C_{\text{cap}}} \text{min}, \{ V(D_i) \} \) is realised, resides on one of the two bisector planes of the diameters \( D_i \) and \( D_{j'} \).

We still have to find that particular configuration, \( \{ D_i \} \), \( i = 1, \ldots, N \), which will minimise \( \text{max}_{C_{\text{cap}}} \text{min}, \{ V(D_i) \} \). Geometrical considerations, backed by extensive numerical simulations, indicate that the worst-case configuration is the one in which all \( \{ D_i \} \), \( i = 1, \ldots, N \), lie in a plane, dividing it symmetrically into equal sectors. The optimal SOP also lies in the same plane and either coincides with one of the \( D_i \) (odd \( N \)) or with a bisector between two neighbouring \( D_i \) (even \( N \)). Accordingly, the worst-case optimal visibility of eqn. 1 is given by

\[
V_{\text{cap}}(N) = \text{sin} \left( \pi/2N \right)
\]  

(5)

Discussion: Since the output \( S/N \) ratio is a linear function of the visibility, it is very important to ensure that the lowest visibility in a multi-element sensor array is not too low. While the input SOP can be conditioned to maximise this lowest visibility, it is extremely difficult to remotely control the polarisation states of the individual elements. Allowing random drifts in these polarisation states, eqn. 5 indicates that the worst-case optimal visibility quickly decreases with \( N \) and attains fairly unacceptable values for \( N > 5 \). As long as the sensor array is made of only a few elements, techniques should be developed for the remote real-time determination of the optimal input SOP. Once this optimal SOP is found, our analysis shows that small deviations of the input SOP actually used from its optimal value will not affect the output visibility by much. For a larger number of sensors, individual adjustments of each sensor must be made to assure adequate \( S/N \) or other, less optimal stabilisation algorithms must be utilised.

Y. S. BOGER
M. TUR
Faculty of Engineering
Tel-Aviv University
Ramat Aviv, Tel-Aviv, Israel 69978

References
1 Okoshi, T.: 'Polarization-state control schemes for heterodyne or homodyne optical fiber communications', J. Lightwave Technol., 1985, LT-3, p. 1232
5 Cusimano, G., and D'akki, J. (Eds.): 'Optical fiber sensors: principles and applications' (Artech House, Norwood, MA, 1988)

COHERENT DETECTION OF MULTIAMPLITUDE MSK UNDER IMPERFECT PHASE SYNCHRONISATION

Indexing terms: Digital communication systems, Detectors and detector circuits

Simple formulas for the BER performance achieved through coherent detection of MAMSK signals, under a carrier phase error, are presented. It is shown that the MAMSK schemes are much more sensitive to imperfect carrier recovery than the corresponding QAM schemes, this advantage being especially clear when serial detection is employed. Possible applications to future TDMA satellite systems are emphasised.

Introduction: In modern digital radio, extensive use is made of MSK (minimum shift keying) and other offset binary QAM modulations. In Reference 1, a receiver for coherent detection of MSK signals was proposed, which is based on a two-branch (I–Q) orthogonal demodulator, and gives optimal performance if perfect synchronisation and an AWGN channel are assumed. It was shown in Reference 2 that MSK is reducible to a form of biphasic keying: in fact binary data can be processed serially by a biphasic modulator, since the MSK signal can be described as an asymmetrically filtered BPSK signal; binary decisions can also be made serially through a single-branch, coherent biphasic demodulator.

In this Letter, we deal with offset M-ary QAM (OQAM) modulation schemes, i.e. those leading to signals that can be written as follows:

\[
s(t) = \text{Re} \left[ \sum_{k \in \mathbb{Z}} \left. a_k x(t - 2kT) \right) \right] + (Q(t)) \text{exp} \{i \pi s(t) \}
\]  

(1)

where

\[
I_\ell(t) = \sum_{k} a_k x(t - 2kT)
\]  

(2a)

\[
Q_\ell(t) = \sum_{k} b_k x(t - \tau - 2kT)
\]  

(2b)

with \( a_k \), \( b_k \) \( = \pm(2l - 1), \) \( i = 1, 2, \ldots, M/2 \), and an offset \( \tau = T \) \( (\tau = 0 \) for an M-ary QAM scheme). More specifically, we consider OQAM signals in which \( x(t) \) has duration \( 2T \). A useful choice is the following: \( \varphi(t) = \cos (\pi t/2T) \) for \( |t| \leq T \) and zero otherwise. This leads to a class of modulation schemes known as multi-amplitude MSK (MAMSK), the case where \( M = 2 \) corresponding to MSK (see Reference 3) for \( M = 4 \). In the following, we study the effect of imperfect phase synchronisation on the coherent detection of MAMSK signals, for several values of \( M = 2^n \), each M-ary symbol representing \( m \) bits according to some encoding rule. It is shown that 'serial detection', first described in Reference 2 and extended here to M-ary schemes, provides, for any \( M \), a clear performance advantage over the classical 'parallel detection' whenever a phase error occurs. This is mainly due to the possibility of compensating for the phase error by shifting the sampling instants.

Parallel detection of MAMSK signals: This is the classical OQAM-type detection, which can be achieved through the receiver in Fig. 1a. M-ary decisions are made, alternately, in both receiver branches, and the frequency of the local carrier is \( f_0 \) for a received signal consisting of \( s(t) \) defined in eqns. 1 and 2, plus white Gaussian noise.

Let us suppose that there is a phase error \( \Delta \theta = \theta - \theta \) (see Fig. 1a) and that the threshold levels are at the centre of the eye openings, for the sampling instants (see Fig. 2a). Then, for high SNR, it can be shown that the bit error probability is closely approximated by

\[
P_b = k_1 \frac{1}{M} \left[ \sqrt{\left( \frac{k_2}{N_0} \right) \cos (\Delta \theta) + \sum_{i=1}^{M-1} \left[ \left( \sqrt{\left( \frac{k_2}{N_0} \right) \cos (\Delta \theta) - 2ik_2 \sin (\Delta \theta)} \right) \right]} \right] + \sum_{i=1}^{M-1} \left[ \left( \sqrt{\left( \frac{k_2}{N_0} \right) \cos (\Delta \theta) + 2ik_2 \sin (\Delta \theta)} \right) \right]
\]  

(3)