A MICRO-MACRO MODEL FOR THE EFFECTS OF THERMO-MECHANICAL FIELDS ON OPTICAL FIBRES EMBEDDED IN A LAMINATED COMPOSITE PLATE WITH APPLICATIONS TO SENSING

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SUMMARY

A coupled micro–macromechanical approach is employed to model an optical fibre in a reinforced composite. As a result, the opto-thermo-mechanical behaviour of the optical fibre is determined in such a way that the phase–strain relationship is based on the micromechanical behaviour of the host media. The microstructures of the carbon fibre reinforced composite and the optical fibre inclusion are analysed, and their mutual influence on the performance of the entire composite structure is estimated. Different cases of laminated composite plates subjected to several cases of external loadings and temperature fields are studied. In-plane strains at the optical fibre, as well as in the composite are presented. The results are also applied to the study of embedded Fabry–Perot and radio-frequency fibre-optic strain and temperature sensors.

INTRODUCTION

Smart materials and structures have recently become the focus of fairly intensive research world wide. Mainly divided into three categories: sensing, actuation and control, this interdisciplinary subject combines the fields of mechanics (the structures), electrics (piezoelectric sensors and actuators), optics (sensors) and computers (control), and each field may play its role in more than one category. In this paper we concentrate on fibre-optic sensing and study the interactions between the structure and the optical fibre using a novel micro–macro model. Fibre-optic, sensors, attached to or embedded within the structure may provide real-time information, at the fabrication process or later in service, about temperature, strain, vibration, pressure, etc. Besides being compatible with the composite structure, optical fibres offer many other advantages: (i) silica-based optical fibres are mechanically compatible with composite materials both in dimensions and strength; (ii) the fibres are electrically non-conductive and, therefore, insensitive to electromagnetic interferences; (iii) they are easily shaped into almost any geometrical layout of interest; (iv) the same technology supports the intrinsic measurement of many physical phenomena; (v) many sensors can be conveniently multiplexed along the same fibre, and finally (vi) fibre-optic sensing and data flow between them and their control centre could and should benefit from the widespread success of its twin technology of
fibre-optic communication. While embedded (or surface-attached) optical fibres can sense the local thermo-mechanical field through a variety of optical processes, we chose first to apply the derived micro-macro model to those sensors based on induced changes in the optical length of the fibre. The treated family of sensors include the Fabry–Perot sensor, as well as its less sensitive and somewhat simpler radio-frequency interferometer. Both sensors are interferometric and their description and predicted performance is deferred to a later discussion.

Butter and Hocker were the first to relate the optical length of a fibre to its strain in an interferometric set-up. They derived the phase-strain ($\Phi - \epsilon$) relationship for a single-mode fibre and showed that $\Delta \Phi / \Delta \epsilon$ is mainly due to the actual length change, as well as to the strain-induced refractive index change while its dependence on the effect of strain on the guiding properties of the fibre can be neglected. Later, Sirkis and Haslach extended this theory such that an arbitrary strain field was considered, together with an arbitrary fibre configuration. Next, Sirkis and Haslach came up with the analysis of an embedded optical fibre in which an integral equation modelled the strain induced optical phase shift such that the entire strain along the fibre was related to the entire phase shift. Kim, Koller and Springer have modelled the problem of embedded intrinsic and extrinsic Fabry–Perot optical fibre sensors in an anisotropic medium bridging over most of the assumptions and approximations offered by previous investigators. Yet, the utilization of Lekhnitskii’s theory for inclusions embedded in infinite anisotropic media by the latter, lacked the ability to pin-point local geometrical, thermal and mechanical effects. These are needed when microstructures involving the interactions of the composite fibres and matrix and the optical fibre sensor are examined and their mutual influences are essential in establishing the solution. It is important to mention at this stage that there is no direct translation of the measured optical data into the strain field. The application of such sensors is based on the ability to mathematically relate the mechanical phenomena to the measured optical data from the sensor through the establishment of the phase-strain relationship.

The choice of a micromechanical approach enables one to examine the mechanical behaviour of the optical fibre and its geometrical changes when subjected to external loading and/or thermal field. In the latter case of temperature effects, the three-dimensional temperature distribution is computed, thereby allowing one to take into account very local influences of different temperature values at the three major constituents: graphite fibres, epoxy matrix and the optical fibres. When micromechanically analysed, the mutual effects of the host media and the hosted inclusion can be precisely investigated. As a result, local geometrical changes in the optical fibre such as changes in its cross-section which may introduce optical anisotropy can be established. Furthermore, phase changes which are the essence of this method of sensing can be accurately determined at any point. The importance of such surveillance lies in the fact that these changes, minor as they seem to be, may affect the optical behaviour of the sensor. This analysis is formulated within the framework of the generalized plane strain assumptions. This approach enables one to examine problems of any type of mechanical and/or thermal fields at any geometrical conditions.

The accuracy of the employed micro to macro model was verified by several authors. Pinder and Dunn conducted extensive comparisons with the predictions of the finite element method. It was found that the current model is an accurate and efficient analytical tool. For example, only four subcells per generic unit cells, used by the current model, achieved the same accuracy as a very fine mesh used by the finite element approach. Goldberg and Hopkins used the boundary element method for their verification of the accuracy of the model.
The governing equations

The illustration in Figure 1 shows the representation of the analysed medium constructed by a set of three-dimensional cells. Each of the cells may be used for the representation of the constituents combining the entire medium. The development of the equations is done on the representative element which is the generic unit cell that consists of eight subcells as shown in Figure 2. These representative basic units combine an entire material configuration according to the dictation of each cell's mechanical and geometrical properties. Thus, a row of cells in a certain direction, may represent a graphite fibre or an optical fibre surrounded by a group of cells representing the matrix. The subcells are indexed by the three Greek symbols $\alpha$, $\beta$, $\gamma$ having values of 1 to 2 (repeating $\alpha$, $\beta$, $\gamma$ does not obey the tensorial summation rule). The generic unit cell in the $x_1$ direction is indexed by $p$ which ranges from 1 (top surface) to $M$ number of layers (bottom surface). The development takes into account the entire thermo-mechanical behaviour in terms of the behaviour of the optical fibre as an inclusion with mechanical and thermal properties which are different from those of the surrounding material, as well as the formulation of the changes of phase due to applied mechanical and/or thermal field.

Figure 1. The micromechanical model of the analysed medium containing an optical fibre
Figure 2. The repeating generic unit cell consists of eight subcells

First, the thermal behaviour is formulated to obtain the temperature distribution in a subcell to be included in the overall mechanical formulation. Following Reference 8 we start with the heat conduction equation,

$$ q^{(\alpha\beta\gamma)}_{i,j} = 0 $$

in which $q^{(\alpha\beta\gamma)}_{i,j}$ is the heat flux vector in the subcell indexed $\alpha\beta\gamma$, related to the temperature field as follows,

$$ q^{(\alpha\beta\gamma)}_{i,j} = -k_{i,j}^{(\alpha\beta\gamma)}T^{(\alpha\beta\gamma)} $$

in which $T^{(\alpha\beta\gamma)}$ is the potential representing the temperature field at subcell $\alpha\beta\gamma$. $k_{i,j}^{(\alpha\beta\gamma)}$ is the material $i$th coefficient of heat conductivity at subcell $\alpha\beta\gamma$ and the brackets around subscript $i$ mean no summation over this index.

Following Aboudi, Pindera and Arnold$^8$ the relevant heat flux continuity conditions, thermal continuity conditions and thermal boundary conditions are considered to form the entire system required for the solution of thermal distribution at a subcell $\alpha\beta\gamma$. The temperature field at a subcell within a generic unit of cells $p$, is developed into a second order polynomial,

$$ T^{(\alpha\beta\gamma)} = T_0^{(\alpha\beta\gamma)} + \bar{x}_1^{(\alpha)}T_1^{(\alpha\beta\gamma)} + \frac{1}{2}\left(3\bar{x}_1^{(\alpha)}\right)^2 - \frac{d_{\alpha}^{(p)}}{4}T_2^{(\alpha\beta\gamma)} $$

$$ + \frac{1}{2}\left(3\bar{x}_2^{(\beta)}\right)^2 - \frac{k_{\beta}}{4}T_3^{(\alpha\beta\gamma)} + \frac{1}{2}\left(3\bar{x}_3^{(\gamma)}\right)^2 - \frac{l_{\gamma}}{4}T_4^{(\alpha\beta\gamma)} $$

in which $T_i^{(\alpha\beta\gamma)}$, $i = 0, \ldots, 4$, are unknown coefficients where the case $i = 0$ represents the temperature at the centre of the subcell $\alpha\beta\gamma$. 
The system of algebraic equations in terms of the unknown coefficients \( T_i^{(\alpha \beta \gamma)} \), consists of the steady-state heat equation and all the continuity and boundary conditions are satisfied in a volumetric sense in which the heat flux is presented volumetrically as,

\[
Q_i^{(\alpha \beta \gamma)} = \frac{1}{\delta_i(\alpha \beta \gamma)} \int_{-l_i/2}^{l_i/2} \int_{-a_i/2}^{a_i/2} \int_{-h_i/2}^{h_i/2} (\bar{x}_1^{(\alpha)})^m (\bar{x}_2^{(\beta)})^n (\bar{x}_3^{(\gamma)})^q q_i^{(\alpha \beta \gamma)} d\bar{x}_1^{(\alpha)} d\bar{x}_2^{(\beta)} d\bar{x}_3^{(\gamma)}
\]

in which \( l_i, a_i, h_i \) are 0, 1 or 2 and their sum must not exceed 2. \( \delta_i^{(\alpha \beta \gamma)} \) is the volume of the subcell \( \alpha \beta \gamma \) in the \( p \)th generic unit cell computed as \( d_i^{(\alpha \beta \gamma)} = h_i l_i l_i \).

A lengthy algebraic manipulation leads the entire system of equation (1) and the continuity and boundary conditions in terms of the temperature field via equation (2) turned into the coefficients as shown in equation (3) altogether presented in the volumetric sense via equation (4), to its final form of a set of algebraic equations.

The steady state-equation heat is,

\[
\left[ \frac{Q_i^{(\alpha \beta \gamma)}}{d_i^{(\alpha \beta \gamma)}} + \frac{Q_i^{(\alpha \beta \gamma)}}{h_i^{(\alpha \beta \gamma)}} + \frac{Q_i^{(\alpha \beta \gamma)}}{l_i^{(\alpha \beta \gamma)}} \right]^{(p)} = 0
\]

The continuity of heat fluxes at all subcells’ surfaces, within a generic cell unit and between them, through the thickness (the \( x_1 \) direction) are,

\[
\left[ \frac{12Q_i^{(12 \gamma)}}{h_i^{(12 \gamma)}} + \frac{12Q_i^{(13 \gamma)}}{h_i^{(13 \gamma)}} \right]^{(p)} + 6 \frac{d_i^{(1 \gamma)}}{d_i^{(p)}} \left[ \frac{Q_i^{(2 \gamma)}}{h_i^{(2 \gamma)}} + \frac{Q_i^{(3 \gamma)}}{h_i^{(3 \gamma)}} \right]^{(p-1)} + \frac{1}{d_i^{(p)}} \left[ Q_i^{(1 \gamma)} - Q_i^{(2 \gamma)} - Q_i^{(3 \gamma)} \right]^{(p-1)} = 0
\]

and

\[
Q_i^{(1 \gamma)} = \frac{1}{2} Q_i^{(2 \gamma)} + \frac{1}{2} Q_i^{(3 \gamma)} + \frac{1}{2} Q_i^{(4 \gamma)}
\]

Same types of conditions in the in-plane directions, \( x_2 \) and \( x_3 \) are,

\[
\left[ \frac{Q_i^{(2 \gamma)}}{h_i^{(2 \gamma)}} + \frac{Q_i^{(3 \gamma)}}{h_i^{(3 \gamma)}} \right]^{(p)} = 0
\]

and

\[
\left[ \frac{Q_i^{(3 \gamma)}}{l_i^{(3 \gamma)}} + \frac{Q_i^{(4 \gamma)}}{l_i^{(4 \gamma)}} \right]^{(p)} = 0
\]

Thermal continuity conditions in the same manner as done with heat conductivity leads to,

\[
\left[ T_i^{(1 \gamma)} + \frac{1}{4} d_i^{(p)} T_i^{(1 \gamma)} + \frac{1}{4} d_i^{(p)} T_i^{(2 \gamma)} \right]^{(p)} = \left[ T_i^{(2 \gamma)} + \frac{1}{4} h_i^{(2 \gamma)} T_i^{(1 \gamma)} + \frac{1}{4} d_i^{(p)} T_i^{(2 \gamma)} \right]^{(p)}
\]

\[
\left[ T_i^{(2 \gamma)} + \frac{1}{4} h_i^{(2 \gamma)} T_i^{(1 \gamma)} \right]^{(p)} = \left[ T_i^{(3 \gamma)} + \frac{1}{4} h_i^{(3 \gamma)} T_i^{(1 \gamma)} \right]^{(p)}
\]

\[
\left[ T_i^{(3 \gamma)} + \frac{1}{4} l_i^{(3 \gamma)} T_i^{(1 \gamma)} \right]^{(p)} = \left[ T_i^{(4 \gamma)} + \frac{1}{4} l_i^{(4 \gamma)} T_i^{(1 \gamma)} \right]^{(p)}
\]
While temperature continuity between neighbour generic unit cells is,

\[
[T_0^{(1\beta \gamma)} - \frac{1}{2} d_1^{(p+1)} T_1^{(1\beta \gamma)} + \frac{1}{2} d_2^{(p+1)2} T_2^{(1\beta \gamma)}]^{(p+1)} = [T_0^{(2\beta \gamma)} + \frac{1}{2} d_1^{(p)} T_1^{(2\beta \gamma)} + \frac{1}{2} d_2^{(p)2} T_2^{(2\beta \gamma)}]^{(p)}
\]  \(11\)

Equations (5)–(11) provide a system of 40 linear algebraic equations in 40 unknowns per one generic \(p\)th unit cell consists of eight subcells. The temperature distribution is obtained once this system is solved, and may be used in the formulation of the mechanical behaviour of the material.

The material within each cell has to fulfill the equilibrium equation and the constitutive relations as follows.

The equations of equilibrium in the absence of body forces,

\[
\sigma_{i,j} = 0, \quad i, j = 1, 2, 3
\]  \(12\)

in which \(\sigma_{ij}\) are the components of the stress tensor.

The generalized Hooke's law for linearly elastic materials,

\[
\sigma_{ij} = C_{ijkl} \epsilon_{kl} - \Gamma_{ij}^{(a\beta \gamma)} T^{(a\beta \gamma)}, \quad i, j, k = 1, 2, 3
\]  \(13\)

in which \(C_{ijkl}\) is the fourth order stiffness tensor of the elastic coefficients, \(\epsilon_{ij}\) are the components of the strain tensor, \(\Gamma_{ij}^{(a\beta \gamma)}\) are the products of the stiffness tensor and the thermal coefficients and \(T^{(a\beta \gamma)}\) is the temperature distribution computed via the system of equations (5)–(11).

The linear strain–displacement relations are,

\[
\epsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i})
\]  \(14\)

in which \(u_{i}\) are the components of the mechanical displacement vector.

The displacement field in the subcell \((a\beta \gamma)\) of the \(p\)th cell, developed into a second order expansion, taking into account symmetry and periodicity in the \(x_2\) and \(x_3\) directions, can be written in the form,

\[
\begin{align*}
U^{(a\beta \gamma)}_1 &= U_0^{(a\beta \gamma)} + \bar{x}_1^{(a\beta \gamma)} U_1^{(a\beta \gamma)} + \frac{1}{2} (3 \bar{x}_1^{(a\beta \gamma)} - \frac{1}{2} d_1^{(p)2}) U_2^{(a\beta \gamma)} \\
&+ \frac{1}{2} (3 \bar{x}_2^{(a\beta \gamma)} - \frac{1}{2} h_2^{(a\beta \gamma)}) U_3^{(a\beta \gamma)} + \frac{1}{2} (3 \bar{x}_3^{(a\beta \gamma)} - \frac{1}{2} l_2^{(a\beta \gamma)}) U_4^{(a\beta \gamma)} \\
U^{(a\beta \gamma)}_2 &= \bar{x}_2^{(a\beta \gamma)} V_1^{(a\beta \gamma)} \\
U^{(a\beta \gamma)}_3 &= \bar{x}_3^{(a\beta \gamma)} W_1^{(a\beta \gamma)}
\end{align*}
\]  \(15\)

here, \(U_0^{(a\beta \gamma)}\), the displacement in the centre of a subcell, along with \(U_1^{(a\beta \gamma)}\), \(U_2^{(a\beta \gamma)}\), \(U_3^{(a\beta \gamma)}\), \(U_4^{(a\beta \gamma)}\), \(V_1^{(a\beta \gamma)}\), \(W_1^{(a\beta \gamma)}\) are the unknown coefficients to be determined.

The governing equations are satisfied in a volumetric sense. To this end, let the average value of the stressess in a subcell \((a\beta \gamma)\) be given as,

\[
\bar{\sigma}_{ij}^{(a\beta \gamma)} = \frac{1}{\nu^{(p)}_i} \int_{-l/2}^{l/2} \int_{-h/2}^{h/2} \int_{-d_3/2}^{d_3/2} (\bar{x}_i^{(a\beta \gamma)})^j (\bar{x}_j^{(a\beta \gamma)})^m (\bar{x}_l^{(a\beta \gamma)})^n \sigma_{ij}^{(a\beta \gamma)} \, d\bar{x}_i^{(a\beta \gamma)} \, d\bar{x}_j^{(a\beta \gamma)} \, d\bar{x}_l^{(a\beta \gamma)},
\]  \(16\)

in which \(\nu^{(p)}_i = d_1^{(p)} h_{\beta} l_{\gamma}\), is the volume of a subcell as indexed, and \(l + m + n \leq 2\).

The non-vanishing zero and first order stress components in terms of the unknown
mechanical displacements are,

\[
\bar{\tau}_{120(0,0,0)}^{(a\beta\gamma)} = C_{11}^{(a\beta\gamma)} U_1^{(a\beta\gamma)} + C_{12}^{(a\beta\gamma)} V_1^{(a\beta\gamma)} + C_{13}^{(a\beta\gamma)} W_1^{(a\beta\gamma)} - \bar{T}_1^{(a\beta\gamma)}  \\
\bar{\tau}_{230(0,0,0)}^{(a\beta\gamma)} = C_{22}^{(a\beta\gamma)} U_1^{(a\beta\gamma)} + C_{23}^{(a\beta\gamma)} V_1^{(a\beta\gamma)} + C_{23}^{(a\beta\gamma)} W_1^{(a\beta\gamma)} - \bar{T}_2^{(a\beta\gamma)}  \\
\bar{\tau}_{310(0,0,0)}^{(a\beta\gamma)} = C_{33}^{(a\beta\gamma)} U_1^{(a\beta\gamma)} + C_{33}^{(a\beta\gamma)} V_1^{(a\beta\gamma)} + C_{33}^{(a\beta\gamma)} W_1^{(a\beta\gamma)} - \bar{T}_3^{(a\beta\gamma)}  \\
\bar{\tau}_{111(1,0,0)}^{(a\beta\gamma)} = \frac{1}{2} d_{1}^{(p)} C_{11}^{(a\beta\gamma)} U_1^{(a\beta\gamma)} - \frac{1}{2} d_{1}^{(p)} C_{12}^{(a\beta\gamma)} V_1^{(a\beta\gamma)} - \frac{1}{2} d_{1}^{(p)} C_{13}^{(a\beta\gamma)} W_1^{(a\beta\gamma)} - \bar{T}_1^{(a\beta\gamma)}  \\
\bar{\tau}_{220(1,0,0)}^{(a\beta\gamma)} = \frac{1}{2} h_{2}^{(p)} C_{11}^{(a\beta\gamma)} U_1^{(a\beta\gamma)}  \\
\bar{\tau}_{330(0,1,0)}^{(a\beta\gamma)} = \frac{1}{2} l_{3}^{(p)} C_{11}^{(a\beta\gamma)} U_1^{(a\beta\gamma)}  \\
\bar{\tau}_{440(0,0,1)}^{(a\beta\gamma)} = \frac{1}{2} l_{4}^{(p)} C_{11}^{(a\beta\gamma)} U_1^{(a\beta\gamma)}
\]

The equations of equilibrium, equation (12), in terms of the volumetric stress averages, equations (17a–f), result in the following eight relations,

\[
\left[ \frac{\bar{\tau}_{111(1,0,0)}^{(a\beta\gamma)}}{d_{1}^{(p)}} + \frac{\bar{\tau}_{220(1,0,0)}^{(a\beta\gamma)}}{h_{2}^{(p)}} + \frac{\bar{\tau}_{330(0,1,0)}^{(a\beta\gamma)}}{l_{3}^{(p)}} \right]^{(p)} = 0
\]

Continuity conditions

There are seven unknowns in the expressions shown in equation (16) which is substituted into equation (18), for each subcell within the generic unit cell. Altogether, a generic unit cell contributes 56 unknowns. The total number of unknowns is, then, \(56\) for \(M\) layers (number of generic unit cells through the thickness). Equation (18) provides eight equations for the \(p\)th generic unit cell. The rest of the conditions necessary to complete the system is achieved through continuity conditions and boundary conditions as follows.

Traction continuity within the first generic unit cell \((p = 1)\),

\[
C_{11}^{(1\beta\gamma)} (U_1^{(2\beta\gamma)}) + \frac{1}{2} d_{1}^{(1)} U_1^{(1\beta\gamma)} + C_{12}^{(1\beta\gamma)} V_1^{(1\beta\gamma)} + C_{13}^{(1\beta\gamma)} W_1^{(1\beta\gamma)} - \bar{T}_1^{(1\beta\gamma)}  \\
\left( T_1^{(2\beta\gamma)} + \frac{d_{1}^{(p)}}{2} T_1^{(1\beta\gamma)} + \frac{d_{1}^{(p)}}{4} T_1^{(1\beta\gamma)} \right) + C_{12}^{(2\beta\gamma)} V_1^{(2\beta\gamma)} - \frac{3}{2} d_{1}^{(1)} U_1^{(2\beta\gamma)}
\]

Traction continuity within a generic unit \(p\)th cell as well as between neighbouring cells \((p > 1)\) associated with \(x_i\) direction,

\[
\left[ \frac{12 \bar{\tau}_{111(1,0,0)}^{(a\beta\gamma)}}{h_{2}^{(p)}} + \frac{12 \bar{\tau}_{220(1,0,0)}^{(a\beta\gamma)}}{l_{3}^{(p)}} \right]^{(p)} + 6 \frac{d_{1}^{(p)}}{d_{1}^{(p-1)}} \left[ \frac{\bar{\tau}_{111(1,0,0)}^{(a\beta\gamma)}}{h_{2}^{(p)}} + \frac{\bar{\tau}_{220(1,0,0)}^{(a\beta\gamma)}}{l_{3}^{(p)}} \right]^{(p-1)} + \frac{1}{d_{1}^{(p)}} \left[ \bar{\tau}_{111(1,0,0)}^{(a\beta\gamma)} \right]^{(p)} = 0
\]
and
\[
\bar{\sigma}_{11(0,0,0)}^{(2\beta\gamma)}|^\mathcal{P}\ = \frac{1}{2} \bar{\sigma}_{11(0,0,0)}^{(2\beta\gamma)}|^\mathcal{P} + \frac{1}{2} \bar{\sigma}_{11(0,0,0)}^{(2\beta\gamma)}|^\mathcal{P-1}
\]
\[+ 3d_2^{-2} \left( \frac{\bar{\sigma}_{2\beta\gamma}(2\beta\gamma)}{h^2} + \frac{\bar{\sigma}_{2\beta\gamma}(2\beta\gamma)}{l^2} \right) - 3d_2^{-2} \left( \frac{\bar{\sigma}_{2\beta\gamma}(2\beta\gamma)}{h^2} + \frac{\bar{\sigma}_{2\beta\gamma}(2\beta\gamma)}{l^2} \right) \]
\[
(21)
\]

Traction continuity within a generic unit \( p \)th cell associated with the \( x_2 \) direction,
\[
\left[ \frac{\bar{\sigma}_{1(2\beta\gamma)}(0,0,1)}{h_1} + \frac{\bar{\sigma}_{1(2\beta\gamma)}(0,0,1)}{h_2} \right]^\mathcal{P} = 0
\]
(22a)
\[
\bar{\sigma}_{2(2\beta\gamma)}(0,0,0)^\mathcal{P} = \bar{\sigma}_{2(2\beta\gamma)}(0,0,0)^\mathcal{P}
\]
(22b)

Traction continuity within a generic unit \( p \)th cell associated with \( x_3 \) direction,
\[
\left[ \frac{\bar{\sigma}_{1(2\beta\gamma)}(0,0,1)}{l_1} + \frac{\bar{\sigma}_{1(2\beta\gamma)}(0,0,1)}{l_2} \right]^\mathcal{P} = 0
\]
(23a)
\[
\bar{\sigma}_{2(2\beta\gamma)}(0,0,0)^\mathcal{P} = \bar{\sigma}_{2(2\beta\gamma)}(0,0,0)^\mathcal{P}
\]
(23b)

Displacement continuity conditions within a generic unit \( p \)th cell associated with each direction,
\[
\left[ U_0^{(2\beta\gamma)} + \frac{1}{2} \bar{d}_1^{(2\beta\gamma)} U_1^{(2\beta\gamma)} + \frac{1}{2} \bar{d}_1^{(2\beta\gamma)} U_2^{(2\beta\gamma)} \right]^\mathcal{P} = \left[ U_0^{(2\beta\gamma)} + \frac{1}{2} \bar{d}_1^{(2\beta\gamma)} U_1^{(2\beta\gamma)} + \frac{1}{2} \bar{d}_1^{(2\beta\gamma)} U_2^{(2\beta\gamma)} \right]^\mathcal{P}
\]
(24a)
\[
\left[ U_0^{(2\beta\gamma)} + \frac{1}{2} \bar{d}_1^{(2\beta\gamma)} U_1^{(2\beta\gamma)} + \frac{1}{2} \bar{d}_1^{(2\beta\gamma)} U_2^{(2\beta\gamma)} \right]^\mathcal{P} = \left[ U_0^{(2\beta\gamma)} + \frac{1}{2} \bar{d}_1^{(2\beta\gamma)} U_1^{(2\beta\gamma)} + \frac{1}{2} \bar{d}_1^{(2\beta\gamma)} U_2^{(2\beta\gamma)} \right]^\mathcal{P}
\]
(24b)
\[
h_1 V_1^{(2\beta\gamma)}|^\mathcal{P} + h_2 V_2^{(2\beta\gamma)}|^\mathcal{P} - (h_1 + h_2) \bar{\epsilon}_{22} = 0
\]
(24c)
\[
\left[ U_0^{(2\beta\gamma)} + \frac{1}{2} \bar{d}_1^{(2\beta\gamma)} U_1^{(2\beta\gamma)} + \frac{1}{2} \bar{d}_1^{(2\beta\gamma)} U_2^{(2\beta\gamma)} \right]^\mathcal{P} = \left[ U_0^{(2\beta\gamma)} + \frac{1}{2} \bar{d}_1^{(2\beta\gamma)} U_1^{(2\beta\gamma)} + \frac{1}{2} \bar{d}_1^{(2\beta\gamma)} U_2^{(2\beta\gamma)} \right]^\mathcal{P}
\]
(24d)
\[
l_1 W_1^{(2\beta\gamma)}|^\mathcal{P} + l_2 W_2^{(2\beta\gamma)}|^\mathcal{P} - (l_1 + l_2) \bar{\epsilon}_{33} = 0
\]
(24e)

where \( \bar{\epsilon}_{22} \) and \( \bar{\epsilon}_{33} \) are two new unknowns resulting from the concept of generalized plane strains in two directions \( (x_2 \text{ and } x_3) \) as presented by Aboudi, Pinder and Arnold\(p\) in which instead of assuming these strains to vanish they are considered constant over the thickness in the relevant directions as indexed. The additional two unknowns require two more equations in addition to the 56\(M\) discussed before. The two additional equations are in terms of external in-plane loads in conjunction with the \( x_2 \) and \( x_3 \) directions being discussed under the topic of boundary conditions.

Displacement continuity between neighbouring generic unit \( p \)th cells in the \( x_1 \) direction excluding the interface between the \( (M - 1) \) and the \( M \)th generic unit cells \( i.e., \ p \neq M \),
\[
\left[ U_0^{(2\beta\gamma)} + \frac{1}{2} \bar{d}_1^{(p+1)} U_1^{(2\beta\gamma)} + \frac{1}{2} \bar{d}_1^{(p+1)} U_2^{(2\beta\gamma)} \right]^\mathcal{P} = \left[ U_0^{(2\beta\gamma)} + \frac{1}{2} \bar{d}_1^{(p+1)} U_1^{(2\beta\gamma)} + \frac{1}{2} \bar{d}_1^{(p+1)} U_2^{(2\beta\gamma)} \right]^\mathcal{P}
\]
(25)

**Boundary conditions**

The displacement boundary condition (top or bottom surface),
\[
U_0^{(2\beta\gamma)} + (-1)^{\alpha} \bar{d}_1^{(p+1)} U_1^{(2\beta\gamma)} + \frac{1}{2} \bar{d}_1^{(p+1)} U_2^{(2\beta\gamma)} = U_0^{(2\beta\gamma)}
\]
(26)
Table I. The number of equations contributed to the system by equations (18)–(29)

<table>
<thead>
<tr>
<th>Equation number</th>
<th>18</th>
<th>19</th>
<th>20,21</th>
<th>22,23</th>
<th>24</th>
<th>25</th>
<th>26,27</th>
<th>28,29</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contributed equations</td>
<td>$8M$</td>
<td>$4(4M - 1)$</td>
<td>$8M$</td>
<td>$20M$</td>
<td>$4(4M - 1)$</td>
<td>$4$</td>
<td>$1$</td>
<td></td>
</tr>
</tbody>
</table>

in which for the upper surface $p = 1$ and $\alpha = 1$, while for the bottom surface $p = M$ and $\alpha = 2$, and $\sigma_{(\alpha;ext)}^{(p)}$ is the applied displacement in the $x_1$ direction.

Alternatively, for an applied traction boundary condition (top or bottom),

\[
C_{11}^{(1\beta_1)}[U_{1}^{(1\beta_1)} - \frac{3}{2}d_{1}^{(1\beta_1)}U_{2}^{(2\beta_1)}] + C_{12}^{(1\beta_1)}V_{1}^{(1\beta_1)} + C_{13}^{(1\beta_1)}W_{1}^{(1\beta_1)}
- \Gamma_{1}^{(1\beta_1)}[T_{0}^{(1\beta_1)} - \frac{d_{1}^{(1\beta_1)}}{T_{1}^{(1\beta_1)}} + \frac{d_{1}^{(2\beta_1)}}{4T_{2}^{(1\beta_1)}}] = \sigma_{1(\text{ext})}^{(1)}
\]

\[
C_{11}^{(2\beta_1)}[U_{1}^{(2\beta_1)} + \frac{3}{2}d_{1}^{(2\beta_1)}U_{2}^{(2\beta_1)}] + C_{12}^{(2\beta_1)}V_{1}^{(1\beta_1)} + C_{13}^{(2\beta_1)}W_{1}^{(2\beta_1)}
- \Gamma_{1}^{(2\beta_1)}[T_{0}^{(2\beta_1)} + \frac{d_{1}^{(2\beta_1)}}{2T_{1}^{(2\beta_1)}} + \frac{d_{1}^{(2\beta_2)}}{4T_{2}^{(2\beta_1)}}] = \sigma_{1(\text{ext})}^{(M)}
\]

(27)

in which for the upper surface $p = 1$ and $\alpha = 1$ while for the bottom surface $p = M$ and $\alpha = 2$, $\sigma_{(\alpha;ext)}^{(p)}$ is the component in the $x_1$ direction of the applied traction in the $x_1$ direction.

Equations (18)–(27) contribute $56M$ linear algebraic equations as shown in Table I. In addition we need two more equations for the two unknown in-plane strains in equations (24c) and (24e),

\[
\bar{N}_{22}^{(\text{ext})} = \frac{1}{V_{\text{tot}}} \sum_{p=1}^{M} \sum_{\alpha=1}^{2} \sum_{\beta=1}^{2} \sum_{\gamma=1}^{2} \left[ d_{\alpha}^{(p)}h_{\beta}l_{\gamma} \bar{e}_{22}^{(\alpha\beta\gamma\chi\psi)} \right]
\]

(28)

\[
\bar{N}_{33}^{(\text{ext})} = \frac{1}{V_{\text{tot}}} \sum_{p=1}^{M} \sum_{\alpha=1}^{2} \sum_{\beta=1}^{2} \sum_{\gamma=1}^{2} \left[ d_{\alpha}^{(p)}h_{\beta}l_{\gamma} \bar{e}_{33}^{(\alpha\beta\gamma\chi\psi)} \right]
\]

(29)

in which $\bar{N}_{22}^{(\text{ext})}$ and $\bar{N}_{33}^{(\text{ext})}$ are the external in-plane loads in the $x_2$ and $x_3$ directions respectively and $V_{\text{tot}}$ is the volume of the entire column of basic cell units along $p = 1$ to $M$.

**THE OPTICAL AND OPTO-THERMO-MECHANICAL RELATIONS**

Propagation of light through a cylindrically symmetric optical fibre is completely independent of the polarization of the input light. However, when the fibre is exposed to non-cylindrically-symmetric stresses, different input polarizations propagate with different speeds and the analysis becomes quite cumbersome. In our study of a unidirectional tape and a cross-ply laminate with the optical fibre being parallel (or perpendicular) to the carbon fibres and under homogenous stresses, which share the planar symmetry of the composites, one can model the fibre as linearly bi-refringent, namely: the two linearly polarized light waves with polarization vectors parallel to the transverse axes of Figure 1 do maintain their polarizations while propagating down the embedded fibre, but each travels at its own speed. The displacement
field vector, $D$ of a wave propagating along the fibre, can thus be expressed as: \(^{10}\)

$$D(x_3, t) = A_1 \sin(\omega t - k_1 x_3) \hat{X}_1 + A_2 \sin(\omega t - k_2 x_3) \hat{X}_2$$ (30)

where $\hat{X}_1$ and $\hat{X}_2$ are orthogonal unit-amplitude displacement vectors parallel to the cross-sectional plane axes of Figure 1, respectively, $A_1$ and $A_2$ are the relevant amplitudes, $\omega$ is the angular frequency of the optical radiation, and finally, $k_1$ and $k_2$ are the wave propagation constants (if $k_1 < k_2$ then the wave with subscript ‘1’ propagates faster than wave ‘2’). $k_1$ and $k_2$ are often expressed in terms of their corresponding effective refractive indices:

$$k_1 = \frac{2\pi n_{1\text{eff}}}{\lambda_0}$$

$$k_2 = \frac{2\pi n_{2\text{eff}}}{\lambda_0}$$ (31)

Here $\lambda_0$ is the wavelength in a vacuum and, $n_{1\text{eff}}$, $n_{2\text{eff}}$ are (when $n_{1\text{eff}} < n_{2\text{eff}}$) the ‘fast’ and ‘slow’ effective refractive indices.

Applied temperature and/or strain fields will affect both the physical length and refractive index of the embedded fibre, thereby changing the phase terms, $\omega t - k_i x_3$, in equation (30). A highly sensitive method to measure these phase changes is by interferometry. We now briefly describe two distinct types of fibre-optic interferometers, both suitable for embedding in composite materials.

The Fabry–Perot sensor

In the Fabry–Perot interferometer, multiple-reflections between two parallel mirrors give rise to interference effects for both the transmitted and reflected waves.\(^1\) In its fibre-optic implementation, the interferometer may be formed by a protected interruption in the fibre, where the two air–glass interfaces form the reflectors, Figure 3(a), or by a short piece of fibre with semi-reflective coatings on both ends, which is fusion spliced to fibre feeders, Figure 3(b). Normally, single-mode fibres are used and the interference which takes place between the mirrors depends on the optical length of the mirror separation (i.e., the product of the length and the refractive index) and affects both the reflected and transmitted intensities. Since the optical fibres in Figure 3(a) are merely used for light transport, this type of sensor is referred to as extrinsic and is predominantly used for external applications.\(^11\) The intrinsic Fabry–Perot of Figure 3(b), whose construction does not involve any increase of diameter over that of the fibre itself, serves well as an embedded sensor and has proved its usefulness in quite a few demonstrations.\(^5\) When used in its reflective mode, an incident light is transmitted through the down-lead fibre and upon reaching the first mirror it will be partially reflected and partially transmitted. The transmitted wave will again be partially reflected from the other mirror so that two waves will propagate back to the transmission station where they are detected. (Since, most often, the semi-reflective coatings have fairly low reflectivities ($\approx 10\%$) multiple reflections are negligible). For a sensor of active length $L$, located a distance $x_3 = x_3^0$, the received displacement vector is the sum of the two reflected waves:

$$D_{\text{reflected}} = A_1 \left[ \sqrt{R_a} \sin(\omega t - k_1(2x_3^0)) \right] \cdot \hat{X}_1 + A_2 \left[ \sqrt{R_b} \sin(\omega t - k_2(2x_3^0)) \right] \cdot \hat{X}_2$$

(32)

where $R_a$ and $R_b$ are the (assumed polarization-independent) intensity reflection coefficients of the two partial mirrors. The intensity of the reflected light is given by\(^5\)

$$I_R = J \int_0^{L} |D|^2 \, dt$$

(33)
Figure 3. A schematic illustration of a Fabry–Perot optical fibre sensor embedded in a host structure (a) extrinsic, (b) intrinsic.

\( J \) is a proportionality constant. Thus,

\[
I_R = I_1 \left[ R_a + R_b + 2 \sqrt{R_a R_b} \cos \left( \frac{4 \pi n_1^{\text{eff}} L}{\lambda_0} \right) \right] \\
+ I_2 \left[ R_a + R_b + 2 \sqrt{R_a R_b} \cos \left( \frac{4 \pi n_2^{\text{eff}} L}{\lambda_0} \right) \right]
\]

(34)

where \( I_1 \) and \( I_2 \) are the intensities of the transmitted waves in the two polarizations, respectively. Mechanical and/or thermal fields affect the reflected intensities through their influence on \( n_1^{\text{eff}}, n_2^{\text{eff}} \) and \( L \). Following References 4 and 5 we define:

\[
n_{\text{ave}} = \frac{n_1^{\text{eff}} + n_2^{\text{eff}}}{2}
\]

\[
n_{\text{diff}} = \frac{n_1^{\text{eff}} - n_2^{\text{eff}}}{2}
\]

(35)

and find:

\[
\Delta n_{\text{ave}} = -\frac{n_0^3}{2} \left( \epsilon_h + \epsilon_3 - \left( \epsilon + \frac{2 \epsilon_{\text{eff}}}{n_0^2} \frac{dn_0}{dT} \right) \Delta T \right)
\]

(36a)

\[
\Delta n_{\text{diff}} = -\frac{n_0^3}{4} \left( \epsilon_3 \right)
\]

(36b)
where $\varepsilon_{33}$ is the strain component in the direction of the optical fibre ($x_3$), $\varepsilon_h$ and $\varepsilon_s$ are the hydrostatic and maximum shear strains respectively,

$$
\varepsilon_h = \frac{\varepsilon_{11} + \varepsilon_{22}}{2} \tag{37a}
$$

$$
\varepsilon_s = \sqrt{(\varepsilon_{22} - \varepsilon_{11})^2 + \varepsilon_{23}^2} \tag{37b}
$$

The strains used in equations (37a) and (37b) are the relevant strains generated in the subcell ($\alpha\beta\gamma$) which is occupied by the optical fibre.

The parameters $p_{32}$, $p_{33}$ are the Pockel photoelastic constants of the substance between the two mirrors, $\alpha$ is the thermal expansion coefficient, $\Delta T$ is temperature change from a reference value, and $n_0$ and $dn_0/dT$ are the refractive index of the free fibre and its thermal coefficient.

The intensities of the launched waves in the two polarizations determine the magnitude of the reflected light and its dependence on strain and temperature. If $I_1$ or $I_2$ are zero, only one polarization exists (say $I_1 \neq 0$, $I_2 = 0$) and the reflected intensity is a simple cosinusoidal function of $4\pi n_1^{eff}L/\lambda_0$,

$$
\Re = R_a + R_b + 2\sqrt{R_a R_b} \cos\left(\frac{4\pi n_1^{eff}L}{\lambda_0}\right) \tag{38}
$$

where $\Re$ is the ratio between the reflected and launched intensities ($\Re = R/R_0$). When $I_1 \neq 0$, $I_1 = 0$, the response is again cosinusoidal but with a different argument: $4\pi n_2^{eff}L/\lambda_0$. When both polarizations exist the dependence is more complicated. However, in the special case where the two polarizations are equally excited ($I_1 = I_2$)

$$
\Re = R_a + R_b + 2\sqrt{R_a R_b} \cos\left(\frac{4\pi n_{\alpha\beta\gamma} L}{\lambda_0}\right) \cos\left(\frac{4\pi n_{\alpha\beta\gamma} L}{\lambda_0}\right) \tag{39}
$$

Combining equations (34)-(39) we finally find

$$
\Re = R_a + R_b + 2\sqrt{R_a R_b} \cos(K_1 + K_2 \varepsilon_{33} + K_3 \varepsilon_h + K_4 \Delta T) \cos(K_5 \varepsilon_s) \tag{40}
$$

where

$$
K_1 = \frac{4\pi n_0 L_0}{\lambda_0}
$$

$$
K_2 = \frac{2\pi n_0 L_0}{\lambda_0} (2 - n_0^2 p_{32})
$$

$$
K_3 = -\frac{2\pi n_0^3 L_0}{\lambda_0} (p_{33} + p_{32})
$$

$$
K_4 = \frac{2\pi L_0}{\lambda_0} \left[ n_0^3 (p_{33} + 2p_{32}) \alpha + 2 \frac{dn_0}{dT} \right] \tag{41}
$$

$$
K_5 = -\frac{\pi n_0^3 L_0}{\lambda_0} (p_{33} - p_{32})
$$

Equation (40) provides the relation between the total reflectivity and the mechanical strains ($\varepsilon_{33}$, $\varepsilon_h$, and $\varepsilon_s$), the thermal strains, as well as the change of the distance between mirrors.

In order to serve as a local sensor, the fibre section between the mirrors is chosen to be short, only a few millimetres in length. Since the optical wavelength $\lambda_0$ is very small ($\approx 1 \mu m$),
strains as low as 50 μs are enough (for L = 2 mm) to move the first cosine of equation (40) through a complete cycle (or fringe). Strains of interest (i.e., from zero up to a few thousand microstrains) are normally measured by fringe counting, which is not a trivial task. A much less sensitive sensor which is also non-local is the radio-frequency interferometer to be introduced next.

Optical radio-frequency interferometer

To reduce the sensitivity to strains and temperature one only needs to work at much longer wavelengths. However, a given single-mode silica fibre can work only in a rather restricted range of wavelengths. One way to work around this problem is to build a radio-frequency electrical interferometer, where the coaxial cable of one arm is replaced by a combination of an electrical-to-optical converter, an embedded optical fibre together with two pigtails and an optical-to-electrical converter, see Figure 4 also presented by Rausch and Ruffin. In the electrical-to-optical converter, a radio-frequency sine wave of frequency f (of the order of a few GHz) amplitude modulate the optical wave to produce;

$$D(x, t) = \left[1 + \beta \sin(2\pi ft - \tilde{k}_1 x)\right] A_1 \sin(\omega t - k_1 x) \tilde{X}_1 + \left[1 + \beta \sin(2\pi ft - \tilde{k}_2 x)\right] A_2 \sin(\omega t - k_2 x) \tilde{X}_2$$  \hspace{1cm} (42)

β (≪ 1) is the amplitude modulation index. In analogy with equation (31)

$$\tilde{k}_1 = 2\pi N_1^{\text{eff}} / \Lambda_0$$

$$\tilde{k}_2 = 2\pi N_2^{\text{eff}} / \Lambda_0$$  \hspace{1cm} (43)

Here, $N_1^{\text{eff}}$ and $N_2^{\text{eff}}$ are the effective refractive indices associated with the group velocities of the two polarizations and $\Lambda_0(= 2\pi / f)$ is the corresponding wavelength. Since the optical-to-

![Diagram](image-url)

Figure 4. A schematic illustration of a radio-frequency optical fibre sensor.
electrical converter extracts the intensity of the received optical wave and since $\beta$ is small, the
voltages (with frequency $f$) at the inputs and output of the mixer are related by:

$$V_{\text{mixer}} \propto V_1 \cdot V_2 \propto \sin(2\pi f t)(I_1 \sin(2\pi f(t - \tau_1)) + I_2 \sin(2\pi f(t - \tau_2)))$$

(44)

$\tau_1$ and $\tau_2$ are the differences in time of propagation between the radio-frequency signal that
travelled through the coaxial cable and the signal that was carried by the two optical waves in
the fibre. After the low-pass filter that extinguishes oscillations at frequency $f$ and above we
obtain:

$$V_{\text{out}} \propto I_1 \cos(2\pi f \tau_1) + I_2 \cos(2\pi f \tau_2)$$

(45)

When $I_1 = I_2$, equation (45) reduces to:

$$V_{\text{out}} = \cos(2\pi f(\tau_1 + \tau_2)/2) \cos(2\pi f(\tau_1 - \tau_2)/2)$$

where for simplicity in notation we replaced the proportionality sign by equality. $\Phi_0^+$ and $\Phi_0^-$
are the arguments of the two cosines in the absence of strain and temperature fields. In their
presence and under the assumption that the phase and group refractive indices are approxi-
mately the same, the induced phase changes, $\Delta \Phi_+$ and $\Delta \Phi_-$ can be directly evaluated from
equations (40)–(41) to give:

$$V_{\text{out}} = \cos\left(\Phi_0^+ + \left(\frac{\lambda_0}{\lambda_0^c}\right)(K_1 + K_2 \epsilon_{33} + K_3 \epsilon_h + K_4 \Delta T)\right) \cos\left(\Phi_0^- + \left(\frac{\lambda_0}{\lambda_0^c}\right)K_5 \epsilon_r\right)$$

(47)

Working with 1 $\mu$m light at a radio-frequency $f$ of 3GHz, we have $\lambda_0^c/\lambda_0 = 10^{-5}$, and the
sensitivity, while being quite sufficient for practical purposes, is way below that of the
Fabry–Perot sensor. To avoid working near extrema of the cosine function, the radio-frequency
is adjusted so that $\Phi_0^+ = -\pi/2$, and since the argument of the second cosine is very close to
zero we finally obtain:

$$V_{\text{out}} = \sin\left(\left(\frac{\lambda_0}{\lambda_0^c}\right)(K_1 + K_2 \epsilon_{33} + K_3 \epsilon_h + K_4 \Delta T)\right)$$

(48)

Note that this sensor integrates the effects of the applied fields over its length.

**SOLUTION PROCEDURE, NUMERICAL EXAMPLES AND DISCUSSION**

Equations (5)–(11) provide a system of $40M$ linear algebraic equations which computes
three-dimensional temperature distribution when the structure is being subjected to a thermal
field. Once the thermal effect is estimated, use is made of equations (18–29) which provide a
system of $56M + 2$ linear algebraic equations. The two phase system models a medium that
consists of $M$ generic cells through the thickness $x_t$ (each one consists of eight subcells) as
shown in Figures 1 and 2. The choice of properties for each subcell allows one to construct a
composite laminate such that certain rows in the $x_t$ and/or $x_3$ directions are filled by
reinforcing fibres which are surrounded by subcells of epoxy matrix. In order to embed an
optical fibre sensor orientated in the $x_t$ direction, a row of subcells in this direction is filled
with optical fibre properties. Once the mechanical behaviour of the system subjected to
external applied loading is determined, the resulting field is expressed in terms of reflected
light intensity for a given incident light transmitted during the mechanical process, as may be
seen in a normalized form in equation (40). A computer code based on the aforementioned
theory was developed. The program was executed on a Digital Alpha based computer with execution time of the order of several seconds.

A set of numerical examples is presented to demonstrate the model. AS4 graphite fibres are used in conjunction with 3502 epoxy matrix. Their material properties are shown in Tables II and III. A single-mode optical fibre is chosen for the sensing element. Its mechanical, thermal and optical properties are shown in Table IV. The sensor was chosen as an intrinsic Fabry–Perot sensor which means that the substance between mirrors is glass as opposed to air in the extrinsic type.

A parametric investigation is performed to examine the influence of different terms in the phase–strain relation model. To this end, \( M \) layers of graphite–epoxy composite and one layer of a Fabry–Perot fibre optic embedded in an epoxy matrix, were represented by \( M + 1 \) generic unit cells, respectively. Each of the \( M \) generic unit cells through the thickness, consists of a pure graphite fibre surrounded by pure epoxy matrix. The \( M + 1 \) generic unit cell consists of a subcell filled with optical fibre surrounded by pure epoxy matrix. A schematic description of this representation is shown in Figure 1. For all the generic cells the following dimensions were chosen: \( d_1^{(p)} = h_1 = l_1 = 80 \, \mu \text{m} \), \( d_2^{(p)} = h_2 = l_2 = 47 \, \mu \text{m} \). It should be mentioned that the generic unit cell with the fibre optic was located at second from the top surface (\( p = 2 \)).

Two forms of composites are considered: unidirectional tape (\( M = 4 \)) and cross-ply laminate (\( M = 8 \)). Each of the examined laminates is subjected to several load cases. The influence of \( N_{22}^{\text{ext}} \) and \( N_{33}^{\text{ext}} \) is studied for several combinations of load levels in conjunction with temperature field. In each case the resulting mechanical strains in the optical fibre determine the reflected light intensities normalized into the ratio of the reflected light intensity over the

<p>| Table II. Mechanical properties of the AS4 graphite fibres, GPa. (The properties are given in the fibre local co-ordinate system) |</p>
<table>
<thead>
<tr>
<th>( E_{11} )</th>
<th>( E_{22} )</th>
<th>( E_{33} )</th>
<th>( G_{12} )</th>
<th>( G_{13} )</th>
<th>( G_{23} )</th>
<th>( v_{12} )</th>
<th>( v_{13} )</th>
<th>( v_{23} )</th>
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<td>220</td>
<td>22</td>
<td>22</td>
<td>22</td>
<td>8.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.35</td>
<td></td>
</tr>
</tbody>
</table>

| Table III. Mechanical properties of the epoxy MPa |
|---|---|---|
| \( E \) | \( \nu \) | \( G \) |
| 3516 | 0.35 | 1.301 |

| Table IV. Mechanical and optical properties of the sensor. (Note: Pockel’s constants indices coincide with the laminate global co-ordinates) |
|---|---|---|---|---|---|---|---|---|---|---|
| \( E \) (GPa) | \( \nu \) | \( G \) (GPa) | \( \alpha \) (\( 10^{-6}/^\circ \text{C} \)) | \( \frac{dn_0}{dT} \) (\( 10^{-5}/^\circ \text{C} \)) | \( n_0 \) | \( P_{33} \) | \( P_{32} \) | \( \lambda_0 \) (\( \mu \text{m} \)) | \( L_0 \) | \( R_a \) | \( R_b \) (mm) |
| 73.1 | 0.16 | 31.2 | 0.5 | 1.2 | 1456 | 0.17 | 0.36 | 1.3 | 8.5 | 0.7 | 0.3 |
incident light intensity (equation (40)). One set of examples refers to mechanical loading only, then a parallel set of examples refers to the same mechanical conditions yet temperature fields are added in two forms, a constant change of temperature with increasing applied

\[ \mathcal{R} = \frac{I_n}{I_0} \]

\[ \mu \epsilon \]

\[ \% \text{ of Ext. Load} \]

Figure 5. Fabry–Perot normalized reflected light intensity and strain components versus increasing external loading, four layers unidirectional tape, zero temperature, \( N^{0i} = 0 \) MPa, \( N^{0f} = 100 \) MPa

\[ \mathcal{R} = \frac{I_n}{I_0} \]

\[ \mu \epsilon \]

\[ \% \text{ of Ext. Load} \]

Figure 6. Fabry–Perot normalized reflected light intensity and strain components versus increasing external loading, four layers unidirectional tape, zero temperature \( N^{0i} = 100 \) MPa, \( N^{0f} = 100 \) MPa
external tractions and constant external traction with increasing temperature change.

A unidirectional tape was first loaded by an increasing load acting in the \( x_3 \) direction, which is perpendicular to the graphite fibres' longitudinal axis \( (x_3) \). This is the 'weak' direction and

\[
\mathcal{R} = \frac{I_n}{I_0}
\]

\[
\mu \varepsilon
\]

![Figure 7](image1.png)

**Figure 7.** Fabry–Perot normalized reflected light intensity and strain components versus increasing external loading, eight layers cross-ply laminate, zero temperature, \( \mathcal{N}_{23}^{ei} = 100 \text{ MPa} \), \( \mathcal{N}_{33}^{ei} = 0 \text{ MPa} \)

\[
\mathcal{R} = \frac{I_n}{I_0}
\]

\[
\mu \varepsilon
\]

![Figure 8](image2.png)

**Figure 8.** Fabry–Perot normalized reflected light intensity and strain components versus increasing external loading, eight layers cross-ply laminate, zero temperature, \( \mathcal{N}_{23}^{ei} = 0 \text{ MPa} \), \( \mathcal{N}_{33}^{ei} = 100 \text{ MPa} \)
Figure 9. Fabry–Perot normalized reflected light intensity and strain components versus increasing external loading, eight layers cross-ply laminate, $\Delta T = 100^\circ C$, $N_{22}^{ex} = 100$ MPa, $N_{33}^{ex} = 0$ MPa

Figure 10. Fabry–Perot normalized reflected light intensity and its ‘fast’ and ‘slow’ constituents versus external load, eight layers cross-ply laminate, $\Delta T = 100^\circ C$, $N_{22}^{ex} = 100$ MPa, $N_{33}^{ex} = 0$ MPa
Figure 11. Fabry–Perot normalized reflected light intensity and strain components versus increasing external loading, eight layers cross-ply laminate, $T = 100^\circ C$, $N_{33}^{ext} = 100$ MPa, $N_{33}^{ext} = 100$ MPa

Figure 12. Fabry–Perot normalized reflected light intensity and its ‘fast’ and ‘slow’ constituents versus external load, eight layers cross-ply laminate, $\Delta T = 100^\circ C$, $N_{33}^{ext} = 100$ MPa, $N_{33}^{ext} = 100$ MPa
Figure 13. Fabry–Perot normalized reflected light intensity and strain components versus increasing temperature change, eight layers cross-ply laminate, $N_{22}^{ex} = 100$ MPa, $N_{33}^{ex} = 100$ MPa

Figure 14. Fabry–Perot normalized reflected light intensity and its 'fast' and 'slow' constituents versus $N_{22}^{ex} = 100$ MPa, $N_{33}^{ex} = 100$ MPa
indeed, when this load grows, so do the strains. When the major load is in the \( x \), direction, we start with a perfect cosine curve, Figure 5, in which \( \varepsilon_{33} \) is the major component over the others. The maximum shear strain \( \varepsilon_s \) is the most significant compared with the other two, \( \varepsilon_{33} \) and \( \varepsilon_h \). Since there is a product of two cosines in the expression of the light intensity, and the argument of the first consists of \( \varepsilon_{33} \) and \( \varepsilon_h \) while the other consists of \( \varepsilon_s \), we may recognize the major influence of the second cosine function. As the load in the \( x_2 \) direction grows, the entire curve forms a global behaviour of a cosine curve which 'rides' on a local cosine as a result of the product of the two cosines. It might be dramatically seen in Figure 6, where the loads in both directions are equal and \( \varepsilon_s \) is larger than \( \varepsilon_{33} \).

The same loading conditions were applied at the same stages to the cross-ply laminate. At first, the behaviour of the light intensity curve is qualitatively similar to the previous material system, Figure 7. When the cross-ply laminate is loaded in the direction of the optical sensor, \( x_{33} \), the maximum shear strain, \( \varepsilon_s \), triggers the cosine product such that the curve amplitude grows and decays symmetrically around the value of the incident light intensity, \( \varepsilon_s \), which is when the ratio of the reflected light over the incident light intensities is equal to one. This form of the optical curve may be seen in Figure 8. In the cases in which \( \varepsilon_{33} \) is larger than the other strain components, the results are extremely different from the counterpart cases of the unidirectional system.

Figures 9 and 11 show results of the same mechanical conditions as in Figures 7 and 8 but, in addition, subjected to constant temperature change field of 100°C. For the purpose of this study, Figures 10 and 12 show how the light intensity curves in Figures 9 and 11 were superimposed by the 'fast' and 'slow' polarization directions of the wave. Next, a case in which the mechanical loading is kept constant while the temperature change increases from 0 to 100°C, is presented. Figure 13 shows such a case while Figure 14 is its counterpart showing the superimposed intensities respectively.

The last three figures, 15–17, show the same cases as presented in Figures 9 and 11 analysed for a case of radio-frequency sensor. The distance between mirrors in the case of the Fabry–Perot sensor is replaced by the entire optical fibre length transplanted within the composite plate, taken as 50 cm. The results are calibrated by the choice of the incident light wavelength, here taken as 10 cm. Both parameters, \( L_0 \) and \( \lambda_0 \), change the nature of the resulting curve such that only a small portion of the descending part of the previous cosine curve is obtained, thus it looks linear. This allows the calibration to produce a direct strain versus load curve presentation.

**SUMMARY AND CONCLUSIONS**

A mathematical model is offered which can be employed to obtain the mechanical and thermal strains in a composite system that is sensed by an optical fibre. The mechanical formulation is based on a micromechanical model which enables one to represent accurately each of the components that form the entire system. An example for such a system is graphite fibres and a surrounding epoxy resin which form a composite layer, and an optical fibre embedded between layers of the composite laminate.

The model can predict the local effects in the micromechanical level. These effects provide the geometrical changes in the shape of the cross-section of the optical fibre which affects the value of the refraction indices and enables accurate results for the change in the optical fibre length. In addition, the micromechanical approach enables one to predict the exact thermal distribution throughout the host medium thickness, thus providing exact temperature field
Figure 15. Radio-frequency normalized reflected light intensity and strain components versus increasing external loading, eight layers cross-ply laminate, $\Delta T = 100^\circ C$, $N_{11}^{in} = 100$ MPa, $N_{22}^{in} = 0$ MPa

Figure 16. Radio-frequency normalized reflected light intensity and strain components versus increasing external loading, eight layers cross-ply laminate, $\Delta T = 100^\circ C$, $N_{11}^{in} = 100$ MPa, $N_{22}^{in} = 100$ MPa
data in the optical fibre. The latter discussion gives rise to the advantages of the micromechanical approach as opposed to other approaches like classical lamination theory or treatment of the composite as an anisotropic medium. The latter approaches utilize effective mechanical quantities to compute averaged results therefore lacking the ability to pin-point local phenomena so needed in the present application. $L_0$, $\lambda_0$ and $n_0$ are all changed locally and affect the optical phase enormously, thus should be treated as accurately as possible.

Two forms of composites, unidirectional tape and a cross-ply laminate, were used, applied to certain load and temperature change levels. Major changes in the behaviour of the two material systems under similar loading conditions were observed in terms of the mechanical strains as well as the optical reaction. Two optical applications were examined. An intrinsic Fabry–Perot sensor and a radio-frequency (RF) sensor. The example figures reflect the differences in the nature of the results obtained by each of the sensor types.

REFERENCES