Decision-Directed PLL for Coherent Optical Pulse CDMA Systems in the Presence of Multiuser Interference, Laser Phase Noise, and Shot Noise

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Abstract—A digital decision-directed phase-locked loop (PLL) for use in optical pulse code-division multiple-access (CDMA) systems based on coherent correlation demodulation is proposed. PLL performance is affected by multiuser interference, laser phase noise and optical shot noise. The effect of these sources of interference and noise on PLL performance is evaluated based on a nonlinear model (the Fokker–Planck method) since a linear analysis yields large deviations between the analytical results and actual performance at low signal-to-noise ratios (SNR’s). After describing the implementation of the PLL, the steady-state probability density function (pdf) of the phase estimator is derived. Numerical evaluation of the variance of the phase estimator is given for Gold codes. The linewidth requirements of the laser for an acceptable phase estimator variance and the value of the optimal loop bandwidth minimizing the impact of the interference and noise on the PLL are discussed.

Index Terms—Carrier phase synchronization, coherent optical pulse CDMA, laser linewidth, laser phase noise, multiuser interference, phase-locked loops.

I. INTRODUCTION

O PTICAL fiber networking is one solution path to meeting the growing demand of the information society with respect to the provision of a range of telecommunication services. At the core of the development is the huge, inherent bandwidth of a single-mode optical fiber that can support up to several terabits/s transmission capacity. Since the speed at which electrical signals can modulate optical carriers is limited, optical multiplexing techniques have to be employed to exploit the full transmission capacity.

Code division multiple access (CDMA), its roots in spread-spectrum (SS) communications [1], is one possible technique that simultaneously and asynchronously multiplexes multiple users on the same frequency band and timeslot through unique signature codes. Within an optical fiber network implementation, CDMA techniques bring a number of attributes:

- asynchronous access capability;
- accurate time of arrival measurement (a means to achieve fine code synchronization);
- ultrashort optical pulse code chip duration (supporting high-transmission capacity) whilst maintaining the possibility of bit duration detection in the electrical domain [2];
- ability to support variable bit rate and bursty traffic;
- a natural increase in the security of transmission.

Due to the development of the optical amplifier [3], [4], incoherent optical fiber transmission based on intensity modulation and direct detection has important significance in current point-to-point trunk transmission harnessing wavelength division multiplexing (WDM) principles. Efficient optical amplification devices, fully compatible with optical fiber transmission, meant that the aggressive development of coherent optical communications, studied widely ten to 20 years ago [5] ceased, receiving little or no attention subsequently. Similarly, in the area optical CDMA networking, most of the research activity has centered on incoherent implementations, driving initially the development of unipolar pseudo-random codes [6]–[8]. Apart from the higher cost of coherent networking when compared to incoherent transmission, the complexity of the implementation at the receiver due to the phase stability requirements of the local oscillator and synchronization between the transmitted and local signals, was a contributing factor in the move away from this approach. However, in the evolution of high-speed and high-capacity optical fiber networks, the attraction of coherent transmission principles is still compelling and worthy of further investigation.

One implementation of a coherent optical pulse CDMA technique [2] is that, through proper code and carrier phase synchronization [9], [10], a locally generated optical pulse code sequence is correlated with the received signal. The resultant correlation signal is integrated over the bit duration to produce a discriminator value. The system employs bipolar pseudorandom codes; thus better correlation performance can be obtained and a large number of simultaneous users can be supported. Additionally with this approach, the hardware demands on the detector for incoherent systems—since decisions have to be carried out within the chip duration—are relaxed since decisions are executed within the bit duration in the electrical domain. It therefore offers one alternative to offering large transmission and network capacity.

Crucial to the successful operation of coherent reception in CDMA systems is proper code and carrier phase synchronization between the received signal and the locally generated code
It is an essential element in overall performance for coherent optical pulse CDMA systems because of the ultrashort chip duration of the optical pulses (several picoseconds or narrower). Fine synchronization is also required for multuser interference cancellation in order to improve system capacity [12]. Code acquisition and tracking for coherent optical pulse CDMA systems have been reported elsewhere [9], [10], the focus here is on the phase synchronization requirements of the system. Previous studies on phase locking for coherent reception [13]–[17] have concentrated on detailing the required laser linewidth for the necessary receiver sensitivity, obtaining the phase locking performance by minimizing the required optical power at the receiver. In contrast for optical fiber networks based on CDMA, the aim is to increase transmission and network capacity, the product of the number of users and the bit rate of each user. Discussion of the requirements of laser linewidth is, nevertheless, still important in order to determine the necessary variance of the phase estimator for successful coherent correlation demodulation.

In this paper, a digital decision-directed phase-locked loop (PLL) is proposed for optical pulse CDMA systems realized through coherent correlation demodulation. PLL performance is affected by multuser interference, laser phase noise and optical shot noise. The analysis method used here is based on a nonlinear model (the Fokker–Planck method) since a linear analysis yields large deviations between analytical results and actual performance at low signal-to-noise ratios (SNR’s). Effectively, laser phase noise is considered as additive noise and a way to derive the power spectral density (PSD) for the rate of change of phase is introduced. After describing the implementation of the PLL, the steady-state probability density function (pdf) of the phase estimator for a first-order loop is derived. Numerical evaluation of the phase estimator variance is given for Gold codes. The linewidth requirements of the laser for an acceptable phase estimator variance and the value of the optimal loop bandwidth, minimizing the impact of the interference and noise on the phase estimator are discussed. The required linewidth of the laser is specified for a certain phase estimator variance.

The remainder of the paper is organized as follows. In Section II, the PLL implementation is described. A statistical analysis of the phase estimator in the presence of multuser interference, phase noise and shot noise is developed in Section III, and its steady-state pdf by a nonlinear model is derived. The variance of phase estimator is discussed in Section IV. Section V presents the results of numerical analyzes for Gold codes. Section VI summarizes the findings of the study.

II. SYSTEM DESCRIPTION

An optical pulse CDMA system based on coherent correlation demodulation is illustrated in Fig. 1(a) [2]. A pulsed laser, emitting narrow pulses of width $T_b$ at an interval $T_b$ ($=NT_c$ with the code gain $N$), is used as the light source. The pulse sequence is phase-modulated by the information data via an electro-optical modulator (EOM) and encoded by an optical tapped delay line encoder with a predetermined phase shift on each branch, producing an optical bipolar code [see Fig. 1(b)] [18]. The encoded optical signals of all transmitters at a common optical carrier frequency and unique signature code sequences, are multiplexed onto one channel and broadcast to all receivers via a passive star coupler. At the receiver, the code of the intended user is generated by a pulsed local oscillator in tandem with the same optical encoder as that of the user transmitter. Through proper synchronization of the code [9], [10] and carrier phase [discussed here], the local code is multiplied by the received signal, chip by chip via 3 dB coupler and a dual-balanced detector. The resultant photocurrent comprising the autocorrelation and crosscorrelation signal components is integrated over one bit duration $T_b$ and then discriminated to recover the information data in the electrical domain.

The receiver configuration to be used in the above system is depicted in Fig. 2, consisting of three parts: a noncoherent delay-locked code process [10], a decision-directed phase locking process and a demodulation process. The key to simplifying the configuration is to effectively combine the PLL and the demodulator with the code tracking loop. The local code and its $T_c$ delayed version of the in-phase and the quadrature-phase components are generated by a $T_c$ optical code delay device and $\pi/2$ optical carrier phase shifters. The mixing of the received and local optical signals produces the photocurrent correlation signal in the four branches. These signals are used in the code tracking, phase locking and demodulation process. The code tracking performance has been discussed in [10], thus the focus here is on the phase locking process.

The received optical pulse CDMA signal consisting of $K$ asynchronous users can be represented as

$$r(t) = \sqrt{\frac{2P_0}{N}} \sum_{k=1}^{K} d_k(t - \tau_k) \chi_k(t - \tau_k) \cos[\omega_0 t + \phi_k(t)]$$

(1)

where $P_0$ is the average received optical pulse power, $\omega_0$ is the angular frequency of the light sources, $\tau_k$ is the code delay
Fig. 2. Receiver structure for a coherent optical pulse CDMA system using noncoherent code delay-locked loop and decision-directed phase locked loop.

of the \(k\)th user uniformly distributed over \([0, T_b]\), and \(\varphi_k(t)\) is the optical carrier phase to be locked with an one-sided PSD represented generally by [19]

\[
S_c(\omega) = \sum_{n=-1}^{0} \zeta_n \omega_n^2 / \omega^2 \quad \omega_n \leq \omega \leq \omega_h
\]  

(2)

where \(\{\zeta_n\}\) are constants. Defining a rectangular pulse by \(p_T(t) = 1\) for \(0 \leq t < T\) and \(p_T(t) = 0\) otherwise, the information data of the \(k\)th user, \(d_k(t)\), can be written as

\[
d_k(t) = \sum_{j=-\infty}^{\infty} d_j^{(k)} p_T(t - jT_b)
\]  

(3)

with consecutive data bits, \(\{d_j^{(k)}\}\), taking on a value of \pm 1 with equal probability. The code sequence of the \(k\)th user \(c_k(t)\) can be written as

\[
c_k(t) = \sum_{n=-\infty}^{\infty} c_n^{(k)} p_{T_c}(t - nT_c)
\]  

(4)

with the periodic train chips \(\{c_n^{(k)}\} \ (n = 0, \ldots, N - 1)\) taking on values of \pm 1. It is assumed that, without loss of generality, the receiver needs to recover the data of user one \((k = 1)\). Hence, the locally generated optical code sequence of user one for phase locking and demodulation can be represented by

\[
L_{\alpha}(t) = \sqrt{2P_L/N} c_1(t - \tau) \cos \left[ \omega_0 t + \phi_1(t) - p_{\alpha} \frac{\pi}{2} \right]
\]  

(5)

where \(P_L\) is the average optical power of the local oscillator; \(\phi_1(t)\) is the carrier phase of the local oscillator, determined by the PLL discriminator signal; \(\tau\) is the code phase of the local oscillator assumed to be locked to user one when \(\tau = \tau_1\); and \(p_{\alpha} \ (\alpha = I, Q)\), defined as \(p_I = 0\) and \(p_Q = 1\), indicates the in-phase and the quadrature-phase components of the local oscillator signal.

The received signal correlates to the local code by using a 3-dB coupler and a dual-balanced detector, and the resultant photocurrents for phase locking and demodulation are

\[
i_{\alpha}(t) = 2\eta \Re \left\{ \frac{r(t) L_{\alpha}(t)}{M} \right\} + n_{\alpha}(t)
\]

\[
= A \sum_{k=1}^{K} d_k(t - \tau_k) e(t - \tau_k) \cos \left[ \varphi_k(t) + p_{\alpha} \frac{\pi}{2} \right] + n_{\alpha}(t) \quad (\alpha = I, Q)
\]  

(6)

where \(M = 4\) is the number of the optical correlators and \(\varphi_k(t) = \varphi_k(t) - \phi_1(t)\) are defined as \(\theta\) at \(k = 1\)

\[
A = \frac{2\eta \sqrt{P_L T_f}}{MN}
\]  

(7a)

and \(n_{\alpha}(t)\) is the optical shot noise with zero-mean and one-sided PSD given by

\[
\mu_{\text{op}} = 2e\sigma_0 \left( \frac{P_0 + P_L}{MN} \right)
\]  

(7b)

with \(e\) the electronic charge and \(\eta\) the responsivity of the photodiode. The resultant correlated signal is integrated over \([0, T_b]\). Sampling at \(t = T_b\) gives

\[
Z_\alpha = A T_b d_0^{(k)} \cos \left[ \varphi_1(t) + p_{\alpha} \frac{\pi}{2} \right]
\]

\[
+ A \sum_{k=2}^{K} \cos \left[ \varphi_k(t) + p_{\alpha} \frac{\pi}{2} \right] Y_{k,1}(\tau_k, 0) + N_\alpha
\]  

(8)

where it is assumed that at \(\tau_1 = 0\)

\[
Y_{k,1}(\tau_k, 0) = \int_0^{T_b} d_k(t - \tau_k) c_k(t - \tau_k) c_1(t) dt,
\]  

(9a)

\[
N_\alpha = \int_0^{T_b} n_{\alpha}(t) dt.
\]  

(9b)

In a decision-directed PLL, the information data recovered by the demodulator is used to multiply a one-bit delayed output of the integrator for the phase locking operation, removing the information data component

\[
\Psi(\theta) = Z_\alpha d_0^{(k)}
\]  

(10)

where \(d_0^{(k)}\) is the demodulated data, \(K_m\) the value of the multiplication gain, identical for all branches, consolidated by the gain of the following voltage-controlled clock (VCC) \(K_{\text{VCC}}\), giving an overall closed-loop gain \(K_p\). Thus \(K_m\) can be assumed to be unity.

The discriminator output in (10) is fed into a loop filter, followed by a VCC. The VCC is used to drive a continuously variable optical phase modulator in order to align the carrier phase. Denoting the transfer functions of the loop filter and the VCC as \(F_p(S)\) and \(K_{\text{VCC}} / S\) respectively, the instantaneous code phase estimate \(\phi_1(t)\) for the local oscillator is related to the phase discriminator \(\Psi(\theta)\) as

\[
\phi_1(t) = K_p F_p(S) \Psi(\theta)
\]  

(11)
where $K_P$ is the product of the VCC gain $K_vcc$ and the multiplier gain $K_m$, and $S$ is the Heaviside operator $S = d/dt$. Expression (11) then can be rewritten as

$$
\frac{d[\varphi_1(t) - \varphi(t)]}{dt} = \frac{d\theta}{dt} = \varphi_1(t) - K_P[\Psi(\theta) \ast f_p(t)]
$$

(12)

where $\varphi_1(t)$ represents the rate of change of the phase; $f_p(t)$ is the transfer function of the loop filter in the time domain; and $\ast$ denotes the convolution operation. Expression (12) is the general derivative for the phase estimator error $\theta$ of a decision-directed PLL for coherent optical pulse CDMA systems. The integration of the first-order PLL, i.e., $F_p(S) = 1$, can be represented as

$$
\Delta \theta = -K_P A T_b g_0 \Phi_{0}^{(i)} \sin \theta + \Delta \varphi_1 - K_P \Phi_{0}^{(i)}
$$

$$
\times \int_{t}^{t+\Delta t} \left[ A \sum_{k=1}^{K} Y_{k,1}(\tau_k, 0) \sin \varphi_k(t) + N_{\alpha} \right] dt.
$$

(13)

III. PDF OF PHASE ESTIMATOR IN A FIRST-ORDER LOOP

In this section, a statistical analysis of the effect of the phase noise, multiuser interference and shot noise on the phase estimator of the decision-directed PLL for coherent optical pulse CDMA systems is carried out. In a multiuser channel, the SNR reduces significantly with increasing number of simultaneous users. A linear analysis for low SNR's yields a large deviation between the analytical results and actual performance of the phase locking. Thus a nonlinear model based on a Fokker–Planck method [19] is used to derive the steady-state pdf $p(\theta)$ of the phase estimator $\theta$. The steady-state pdf of a first-order loop is considered in which the stationary equation is [10]

$$
\frac{\partial}{\partial \theta}[\gamma_n(\theta)p(\theta)] = \frac{1}{2 \Delta \theta^2} \gamma_2(\theta)p(\theta)
$$

(14)

where $\gamma_n(\theta) (n = 1, 2)$, defined by

$$
\gamma_n(\theta) = \lim_{\Delta \theta \to 0} \frac{1}{\Delta \theta^n} \left[ \gamma(\theta) \right]^{n-1} \theta
$$

(15)

can be obtained by the use of the integration expression (13). The overhead bar in (15) is used to define the statistical expectation.

Hence, in order to obtain the steady-state pdf of the phase estimator error $p(\theta)$, the primary objective is to determine the quantities $\gamma_n(\theta) (n = 1, 2)$ through (13). Substituting (13) into (15) at $n = 1$, yields

$$
\gamma_1(\theta) = -K_P A T_b g_0 \Phi_{0}^{(i)} \sin \theta.
$$

(16)

From (13), $\gamma_2(\theta)$ is the PSD’s of the phase noise, multiuser interference and shot noise. The three interference and noise terms, having a zero-mean, are time dependent and pairwise uncorrelated with respect to one another. Hence their PSD’s can be derived separately.

First, the one-sided PSD of the rate of change of the phase is derived in Appendix A and summarized briefly as follows. The rate of change of the phase $\varphi(t)$ can be represented by defining a measure of the fractional frequency stability [19], [20] by

$$
\varphi(t) = \frac{\varphi(t) - \varphi[(i-1)t_s]}{t_s}
$$

(18)

where $\varphi(x)$ is the phase value at sampling time point $x$, and $t_s$ is the time duration during which the two phases are sampled, taking a value of the bit duration $t_s = T_b$.

From (18), the autocorrelation function of the rate of change of the phase can be obtained by using the autocorrelation function of the phase noise $G_\varphi(v)$

$$
\varphi(t)\varphi(t + v) = \frac{1}{t_s^2} [2G_\varphi(v)G_\varphi(t_s + v) - G_\varphi(t_s - v)].
$$

(19)

The autocorrelation function is not dependent on the sampling point index $n$. Substituting the expression for the autocorrelation function of the phase noise

$$
G_\varphi(v) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} S_\varphi(\omega) \exp(-j\omega v)
$$

into (19), yields the autocorrelation function of the rate of change of the phase

$$
\varphi(t)\varphi(t + v) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi t_s^2} S_\varphi(\omega) \{2\cos(\omega v)
$$

$$
- \cos[\omega(t_s + v)] - \cos[\omega(t_s - v)]\}.
$$

(20)

Noticing that the autocorrelation function is zero at $|v| > T_b$, its one-sided PSD can be obtained by

$$
\mu_p = \int_{-T_b}^{T_b} \varphi(t)\varphi(t + v) dv.
$$

(22)

In this paper, the expression of the one-sided PSD of phase noise [13]

$$
S_\varphi(\omega) = \frac{4\pi T_b \Delta \nu}{\omega^2} \quad (0 < \omega < \infty)
$$

(23)

is employed where $\Delta \nu$ indicates the laser linewidth. Using (23) and (21), the one-sided PSD of $\varphi(t)$ can be obtained as

$$
\mu_p = \frac{8\pi T_b \Delta \nu}{t_s} = 8\pi \Delta \nu.
$$

(24)

Defining

$$
Y_{k,1}(x, v) = \int_{|\xi|}^{T_b + |\xi|} d\xi(t - \xi)\varphi_1(t) dt
$$

(25)

its autocorrelation function can be developed

$$
Y_{k,1}(x, 0)Y_{k,1}(x, v) = \left\{ \hat{R}_{k,1}(x) + \hat{R}_{k,1}(x)\hat{R}_{k,1}(x) \right\}
$$

$$
+ \left[ B[|v|, x]R_{k,1}(x) - \hat{R}_{k,1}(x) \right] \cdot Q_{k,1}(\min\{|v|, x\}, \max\{|v|, x\}, x)
$$

(26)

where $R_{k,1}(\tau_k)$ and $\hat{R}_{k,1}(\tau_k)$ are the continuous-time partial cross-correlation functions defined in [21]; $\max\{\alpha, \beta\}$ and
\( \min \{\alpha, \beta\} \) are defined as the largest and smallest values between \( \alpha \) and \( \beta \), respectively; \( B(\cdot) \) is the function \( B(\alpha, \beta) = 1 \) for \( 0 \leq \alpha \leq \beta \) and \( B(\alpha, \beta) = 0 \) otherwise; and
\[
Q_{k_A}(\alpha, \beta, \gamma) = \int_\alpha^\beta c_k(t-\gamma)c_\beta(t)\,dt. \tag{27}
\]

From (26), the effect of the multiuser interference on \( \gamma_2(\theta) \) can be calculated by using (13) and (15), and taking a statistical average over \( \tau_k \) [24]
\[
\mu_I = \frac{\langle AK\rangle^2 T_b^3 I_k}{N^4} \tag{28}
\]

where
\[
I_K = \frac{1}{3} \sum_{l=0}^{K} \sum_{I=0}^{N-1} \Psi[2C_{k_A}(l), NC_{k_A}(l-N)-D_{k_A}(l-N)]
\]
\[
\Psi[2C_{k_A}(l), D_{k_A}(l)] + \Psi[2C_{k_A}(l-N), D_{k_A}(l-N)]
\]
\[
\Theta[C_{k_A}(l), C_{k_A}(l)] + \Theta[C_{k_A}(l), C_{k_A}(l-N)] - \Theta[C_{k_A}(l), C_{k_A}(l-N)] \tag{29a}
\]

with
\[
\Psi[A(l), B(l)] = A(l)B(l) + \frac{1}{2} A(l+1)B(l+1)
\]
\[
+ \frac{1}{2} A(l+1)B(l) + A(l+1)B(l+1)
\]
\[
\Theta[A(l), B(l)] = \frac{5}{4} A(l)B(l)
\]
\[
+ \frac{3}{4} A(l+1)B(l+1)
\]
\[
+ \frac{1}{4} A(l+1)B(l+1) + \frac{1}{4} A(l)B(l+1). \tag{29b}
\]

In (29a) \( C_{k_A}(\cdot) \) is the discrete aperiodic correlation function defined in [21], and \( D_{k_A}(\cdot) \) is defined as
\[
D_{k_A}(l) = \begin{cases} 
\sum_{j=0}^{N-1-l} \left( \sum_{j=0}^{N-l-1} (j+l) c_j^{(k)} c_{j+l}^{(1)} \right), & 0 \leq l \leq N-1 \\
\sum_{j=0}^{N-l-1} \left( \sum_{j=0}^{N-1-l} j c_j^{(k)} c_{j+l}^{(1)} \right), & 1 \leq N \leq l \leq 0 \\
0, & |l| \geq N.
\end{cases} \tag{30}
\]

The effect of the shot noise on \( \gamma_2(\theta) \) is
\[
\mu_G = (K_b T_b)^2 \mu_{\text{exp}}. \tag{31}
\]

Solving (14) and using the one-sided PSD of all noise sources, the pdf of the phase estimator can be obtained as [22]
\[
p(\theta) = C \exp(\rho \cos \theta) \quad (-\pi \leq \theta < \pi) \tag{32}
\]
where \( C \) is a constant
\[
C = \frac{1}{\int_{-\pi}^{\pi} \exp(\rho \cos \theta)\,d\theta} = \frac{1}{2\pi I_0(\rho)} \tag{33}
\]
and \( \rho \) is an equivalent loop SNR
\[
\rho = \frac{4d_0^{(1)} d_0^{(3)} K_p A T_b}{\mu_p + \mu_m + \mu_G}. \tag{34}
\]

For a given bit error rate (BER)
\[
d_0^{(1)} = \begin{cases} 
d_0^{(1)} & \text{with probability } (1-\text{BER}) \\
-d_0^{(1)} & \text{with probability BER}
\end{cases} \tag{35}
\]

The average equivalent loop SNR is thus
\[
\bar{\rho} = \frac{4AT_b K_p (1-2\text{BER})}{\mu_p + \mu_m + \mu_G}. \tag{36}
\]

### IV. VARIANCE OF PHASE ESTIMATOR

The variance of the phase estimator can be calculated by using the pdf of (32)
\[
\sigma^2 = \int_{-\pi}^{\pi} \theta^2 p(\theta)\,d\theta. \tag{37}
\]

Furthermore, at large equivalent loop SNR’s (\( \rho > 5 \)) [19, Figs. 12–18], the pdf of the phase estimator can be approximated by a Gaussian distribution and the variance of the phase estimator can be obtained by
\[
\sigma^2 = \frac{1}{\bar{\rho}}
\]
\[
= \left( \frac{\pi \Delta \gamma}{2 B_p} + \frac{B_p I_K}{\bar{\rho} N^4} + \frac{B_p \mu_{\text{exp}}}{\bar{\rho} A^2} \right) / (1-2\text{BER}) \tag{38}
\]
where the bit rate \( R_b \) is \( 1/T_b \) and the loop bandwidth \( B_p \) is
\[
B_p = AT_b K_p/4. \tag{39}
\]

As shown in (38), a tradeoff exists between the noise due to the interference and shot noise as a function of bandwidth and the magnitude of the phase noise. The optimum value of the loop bandwidth is
\[
B_p = \sqrt{\frac{\pi \Delta \gamma}{2 (I_K / N^4 + \mu_{\text{exp}} / A^2)}} \tag{40}
\]
and the optimal variance is
\[
\sigma^2 = \sqrt{2 \pi \Delta \gamma (I_K / N^4 + \mu_{\text{exp}} / A^2)} / (1-2\text{BER}). \tag{41}
\]

### V. NUMERICAL EVALUATION

Using the results of the previous sections, numerical evaluations of PLL performance for Gold code sequences are given in this section. A pulsed laser source with chip duration \( T_c = 10 \) ps is assumed; the optical power of the local oscillator is set at 0 dBm and the responsivity of the photodiode is 0.85 A/W.

In Figs. 3–8, the Gold codes with a code length \( N = 127 \) are employed. The codes are generated by the modulo-2 addition of the pair of maximal length sequences \( (211, 217) \) in octal, by adding one maximal length sequence to the phase-shifted version of the other, chip by chip, by synchronous clocking. Since a maximal length sequence of length \( N \) can be shifted \( N \) times, a pair of maximal length sequences can generate \( N \) codes plus the two base sequences [23]. The first maximal length sequence is assigned to the intended user in the analysis. Fig. 3 illustrates the behavior of the phase estimator pdf, \( p(\theta) \), as a function of the number of users, received optical
power, loop bandwidth and laser linewidth. The peak of the pdf clearly increases with a reduction of the laser linewidth and decreases as the number of simultaneous users. The pdf changes largely with the number of simultaneous users. The existence of multiuser interference limits the selection of the loop bandwidth, as shown in (38): a tradeoff exists between the effect due to this interference and the magnitude of the phase noise as a function of bandwidth. The limits imposed by multiuser interference and phase noise cannot be overcome through increasing the input optical power.

Fig. 4 shows the variance of the phase estimator as a function of the received optical power for different number of users and laser linewidths when the loop bandwidth to bit-rate ratio is 0.1. The variance becomes larger with an increase of the number of the simultaneous users and a decrease of the laser linewidth. The limit due to shot noise is overcome by proper increase of the optical power of the received signal. A small reduction in the variance results when the received optical power is $P_o = 25$ dBm, the variance being defined by the phase noise and multiuser interference. Fig. 5 depicts the variance of the phase estimator as a function of the number of simultaneous users for different laser linewidths at a loop bandwidth to bit-rate ratio of 0.1 and a received optical power of $30$ dBm. The linear approximation is used to make a comparison with the exact numerical results; good agreements can be observed for small values of the variance. It is well known that the linear approximation agrees well with the numerical results when the variance of the phase estimator is less than 0.2 [19].

Since the effect of multiuser interference and phase noise cannot be reduced by control of the optical power of the received signal, the practical approach to obtaining the required variance is through the use of a laser with a predefined linewidth in addition to an optimization of the loop bandwidth. Fig. 6 gives the requirements of the laser linewidth as a function of the number of simultaneous users for the required variance when the loop bandwidth is optimized. The required laser linewidth to bit-rate ratio is about $10^{-4}$ at a variance of 0.03 and about $10^{-5}$ at 0.01, respectively. A smaller laser linewidth is necessary to reduce the effect due to the multiuser interference. Fig. 7 shows the optimal loop bandwidth as a function of the number of the simultaneous users for a required variance. The optimal loop bandwidth depends on the number of simultaneous users and the received optical power. In a practical operational system, it is desirable to keep the laser linewidth and the loop bandwidth constant. Therefore, in order
to obtain a required variance of the phase estimator, the values of the laser linewidth and the loop bandwidth which limit the largest effect due to multiuser interference has to be employed. Fig. 8 gives the variance of the phase estimator as a function of the number of simultaneous users with the values of the optimal loop bandwidth and the required laser linewidth equal to 0.01 and 0.03 for 120 users. It can be seen that all values for the variance are not larger than the required one (0.01 or 0.03) when the number of simultaneous users is less than 120.

For a system design with code length \( N = 31 \) or \( N = 63 \), Fig. 9 shows the required laser linewidth for different code lengths. The pair of the two preferred maximal length sequences in octal [23] used to generate the codes are (45, 75) and (103, 147), respectively. A narrower laser linewidth is necessary for the shorter length code.

In the discussions of Figs. 3–9, the detection error of the decision-directed PLL was ignored. The difference of the variance with and without detection error is shown in Fig. 10 for different bit error rates (BER’s); the difference is small when the BER is less than 10^{-2}, especially for small variance values.

VI. CONCLUSION

A digital decision-directed PLL for use in optical pulse CDMA systems based on coherent correlation demodulation has been investigated. PLL performance is affected by laser phase noise, multiuser interference and optical shot noise. The implementation of the PLL has been described and its performance has been analyzed through the Fokker–Planck method.

It is shown that PLL performance is sensitive to the multiuser interference and laser phase noise although the effect of shot noise can be overcome by increasing the received optical power. A tradeoff exists between the limits imposed by phase noise and the magnitude of the multiuser interference and the shot noise, as a function of bandwidth. The laser linewidth
requirements and the value of the optimal loop bandwidth for a required variance of the phase estimator have been detailed.

APPENDIX A

In this Appendix, the autocorrelation function and one-sided PSD for the rate of change of phase noise \( \dot{\phi}(t) \) is derived. The rate of change of phase noise can be represented through defining a measure of the fractional frequency stability \([19],[20]\) as

\[
\dot{\phi}(t) = \phi(t) - \phi[i(t - 1)] \quad \text{(A1)}
\]

where \( \phi(x) \) is defined as the phase value sampled at time point \( x \) and \( t_s \) is the time interval in which the two phases are sampled, considered here to be the bit duration \( t_s = T_b \).

The autocorrelation function of the rate of change of the phase can be obtained by using the autocorrelation function of the phase noise \( G_{\phi}(v) \)

\[
\int_{0}^{t} \phi(t) \phi(t + v) = \frac{1}{t_s} \phi'[i(t - 1)] \phi'[i(t - 1) + v] - \phi'[i(t - 1)] \phi'[i(t - 1) + v]
\]

\[
= \frac{1}{t_s} (2G_{\phi}(v) - G_{\phi}(v) + G_{\phi}(v) - G_{\phi}(v)) \quad \text{(A2)}
\]

The autocorrelation function of the rate of change of the phase does not depend on the interval index \( i \). Substituting the expression of the autocorrelation function of the phase noise \( G_{\phi}(v) \)

\[
G_{\phi}(v) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} S_{\phi}(\omega) \exp(-j\omega v) \quad \text{(A3)}
\]

into (A2) yields the autocorrelation function of the rate of change of the phase

\[
\dot{\phi}(t) \dot{\phi}(t + v) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi t_s} S_{\phi}(\omega)
\]

\[
\cdot \{2 \cos(\omega v) - \cos[\omega(t_s + v)] - \cos[\omega(t_s - v)]\} \quad \text{(A4)}
\]

and its one-sided PSD

\[
\mu_p = 2 \int_{-T_b}^{T_b} \dot{\phi}(t) \dot{\phi}(t + v) dv, \quad \text{(A5)}
\]

In this paper, the expression of the PSD of phase noise \([13]\)

\[
S_{\phi}(\omega) = \frac{4\pi \Delta \nu}{\omega^2} \quad (0 < \omega < \infty) \quad \text{(A6)}
\]

is employed where \( \Delta \nu \) indicates the laser linewidth. Substituting (A6) into (A5) and noting that

\[
\int_{0}^{\infty} \frac{\sin^2(x)}{x^2} dx = \frac{\pi}{2} \quad \text{(A7)}
\]

the one-sided PSD of \( \dot{\phi}(t) \) can be obtained as

\[
\mu_p = \frac{8\pi T_b \Delta \nu}{t_s} \quad \text{(A8)}
\]

Using \( t_s = T_b \) yields

\[
\mu_p = 8\pi \Delta \nu \quad \text{(A9)}
\]


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