Lab 3

1. The Properties of the DFT transform

1.1 Theoretical background
This part consists of a sequence of short demos. The purpose of these demos is to demonstrate several important properties of the DFT transform with the emphasis on the relation between images in the spatial domain and their DFT transforms.

Note. For this lab all workspaces are ready. You are supposed to analyze the results and write the report.

1.2 The Project – The Properties of DFT Transform
a. Images and their spectra:
   Load the workspace 1_A.wksp. In this workspace we define and display an image of a square, calculate its DFT and display its spectral magnitude.

b. Size modification in spatial domain:
   Load the workspace 1_B.wksp. Change the size of the box (using “Box Y Height” and “Box X Width” parameters in the Box Projection glyph), calculate the DFT of the modified box and display its magnitude. When you expand or shrink the box, what happens to the spectrum of the modified image? Explain this property.

c. Multiplication by complex exponent in spatial domain:
   Load the workspace 1_C.wksp. Modify the image by multiplying it by a complex sinusoid. Change the frequency of the sinusoid (using the parameters “Num. along Width” and “Num. along Height”), calculate the DFT of the modified image and display its magnitude. What happens to the spectrum of the modified image? Explain this property.

d. Multiplication by complex exponent in frequency domain:
   Load the workspace 1_D.wksp. Calculate the DFT of the original box image and display its magnitude. Modify the resulting DFT by multiplying it by a complex sinusoid. Change the frequency of the complex exponent (using the parameters “Num. along Width” and “Num. along Height”), calculate the inverse DFT of the modified spectrum and display its magnitude. What happens to the modified image in the spatial domain? Explain this property.

e. Rotation in the spatial domain:
   Load the workspace 1_E.wksp. Calculate the DFT of the original box image and display its magnitude. Modify the image by rotating it by 45 degrees (inside the box glyph). Calculate the DFT of the rotated image and display its magnitude. What happens to the spectrum of the rotated image? Explain this property.

f. Two horizontal boxes:
   Load the workspace 1_F.wksp. Create new image by addition of two
horizontally shifted boxes. Calculate the DFT of the new image and display its magnitude. What happens to the spectrum of the new image? Explain this property.

g. Two diagonal boxes:
Load the workspace 1_G.wksp. Create new image by addition of two diagonally shifted boxes. Calculate the DFT of the new image and display its magnitude. What happens to the spectrum of the new image? Explain this property.

h. Four boxes:
Load the workspace 1_H.wksp. Create new image by addition of four shifted boxes. Calculate the DFT of the new image and display its magnitude. What happens to the spectrum of the new image? Explain this property.

2. Inverse Filtering

2.1 Theoretical background
Inverse filtering is the process of recovering the input of a system from its output. For example, in the absence of noise the inverse filter would be a system that recovers the input of a system from its output correctly. Suppose that the frequency response of the system is \( H(\omega_1, \omega_2) \). The inverse filter is the reciprocal of \( H(\omega_1, \omega_2) \):
\[
H^I(\omega_1, \omega_2) = 1 / H(\omega_1, \omega_2).
\]
However, \( H^I(\omega_1, \omega_2) \) does not exist if \( H(\omega_1, \omega_2) \) has any zeros. In order to stabilize the inverse filter, we introduce the pseudoinverse filter:
\[
H^{-1}(\omega_1, \omega_2) = 1 / H(\omega_1, \omega_2) \text{ if } H \neq 0,
\]
\[
H^{-1}(\omega_1, \omega_2) = 0 \text{ if } H = 0.
\]

2.2 The Project – Part 2 (Inverse Filtering)
Load the workspace 2.wksp. Use both of the input images lena.jpg and dip.jpg. Blur the image using the Low-Pass filter. Display the original image, the spectrum of the original image, the spectrum of the Low-Pass filter, the spectrum of the blurred image and the blurred image in the spatial domain. Restore the image from its blurred modification using Inverse Filtering. Display the spectrum of the resulting image and the resulting image itself. How well you restore the original image? Explain the restoration error.

Notes.
Submission is in hardcopy, 2 weeks after the lab, including images and explanations.