Comparison Of Two-Dimensional Rebinning Algorithms For Image Reconstruction From Projections

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Summary

CT-scanners evolved from the simple parallel projection scheme into more complex divergent (fan) projection schemes. The fan projection geometry requires an adaptation of the original Radon solution which was derived in the parallel geometry. There are two approaches by which the problem can be solved. The first is to literally adopt the inverse integral Radon transform using the mathematical relation between the different scanning geometries. The second approach, which may be found to be more attractive, is to reorder, or *rebinn*, the fan projections into a set of parallel projections and then simply apply for the reconstruction available well-defined parallel projection reconstruction methods. The issue that is needed to be dealt with in this approach concerns the reconstruction of a uniformly sampled discrete parallel projections from a uniformly sampled discrete fan projections. This requires resampling the projections through interpolation.

In this work, different projection rebinning methods for image reconstruction from fan projections were compared. To this goal, a geometrical relation between parallel and fan beam projections was first formulated. This geometrical relation provided the basis for the rebinning approach to the reconstruction of an image from its fan projection using conventional reconstruction methods, such as filtered back-projection. A computer model was developed which simulates the entire process through the following steps: a) The acquisition of fan/parallel sinogram (Radon transform) from two-dimensional images, b) Rebinning from fan to parallel projection, c) Image reconstruction from the rebinned parallel projection (inverse Radon transform).

The following signal interpolation methods for the rebinning process were compared: a) conventional interpolation methods such as nearest neighbor, bilinear, Keys bi-cubic and spline which were implemented in the signal domain and are provided by the built-in Matlab functions. b) the discrete sinc-interpolation method implemented through sampled data processing in the Fourier domain.

Two approaches for applying the discrete sinc-interpolation method were studied: a global approach and a local approach. In the global approach, all samples of the fan sinogram are used for estimating required samples of parallel projections, and the process is implemented through a oversampling (magnification) method. In the local approach, a local sliding window was used, and the required sample is obtained by discrete sinc interpolation of neighboring samples enclosed by the window.

The rebinning methods were tested using two test images, the Shepp-Logan Head Phantom and a test CT brain image. For quantitative comparison of the methods, standard deviation of the reconstruction error was measured between the original and reconstructed test images. The simulation results show that the discrete sinc- interpolation method provides smaller reconstruction error than the other methods. Reconstruction by rebinning/denoising was demonstrated using local discrete sinc interpolation in DFT domain.

1. Parallel and fan projection geometry

The parallel projection $p(s,\theta)$ of the function $f(x,y)$ is denoted as the line integral along a path L and is described by the following relation (see Fig 1):

$$\boldsymbol{p(s,\theta) = \int_L f(x,y)du = \iint f(x,y)\delta(x\cos\theta + y\sin\theta - s)dxdy,}$$

where $\boldsymbol{s}$ and $\boldsymbol{\theta}$ are respectively the perpendicular distance from the origin and the rotation angle of the line integration ***L***.

The fan projection $h(\alpha,\beta)$ of the function $f(x,y)$ is denoted as the line integral along a path L and is described by the following relation (see Fig 1):

$$\boldsymbol{h(\alpha,\beta) = \int_L f(x,y)du = \iint f(x,y)\delta(x\cos(\alpha+\beta-\pi/2) + y\sin(\alpha+\beta-\pi/2) + R\sin\beta)dxdy,}$$

where α is the angle of rotation, β the opening angle of the fan and R is the radius of rotation.

In order to enable the rebinning process for constructing a parallel projection (sinogram) from fan projection, we first need to establish a parallel-fan geometry relation which is denoted as follows and can be derived from Fig 2.

$$\boldsymbol{s = R\sin(\beta),}$$
$$\boldsymbol{\theta = \alpha + \beta.}$$

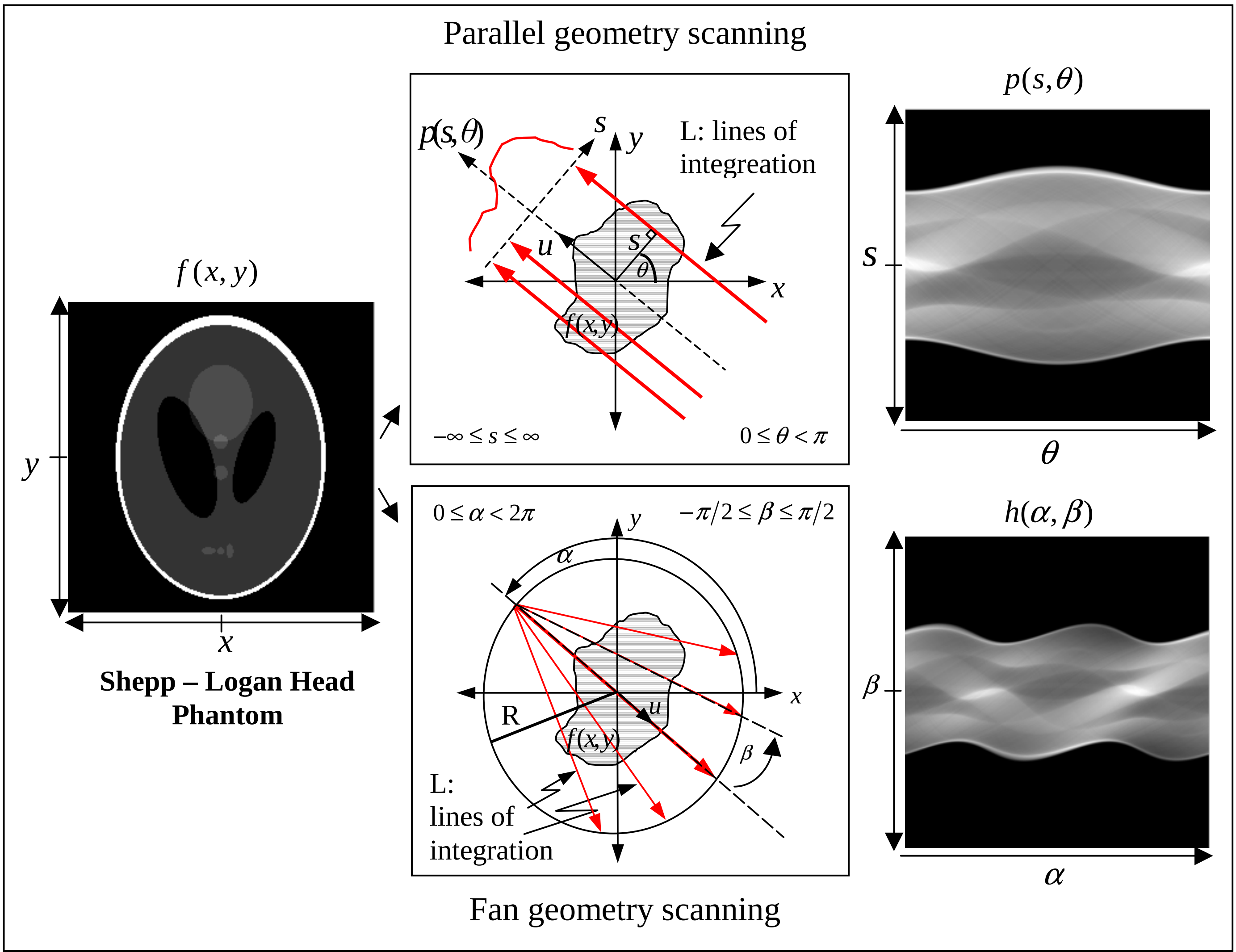


Fig.1 : Parallel and Fan scan geometry

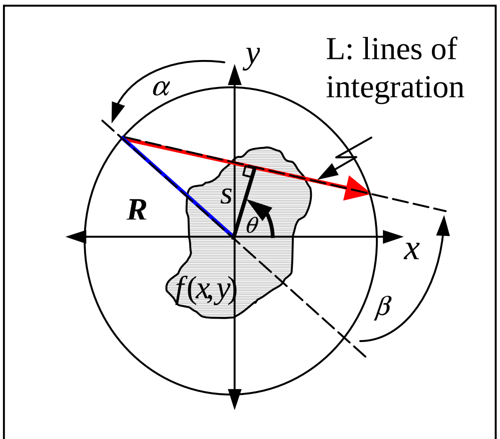


Fig.2 : Parallel Fan geometry relation geometry

2. Rebinning through the discrete sinc-interpolation method

Since projections (fan/parallel) are given in discrete domain upon a finite uniform grid and not in a continuous form, interpolation is required in order to estimate the samples value of the required rebinned parallel projection. It is known from the continuos sampling theorem that the ideal interpolation kernel for a perfect reconstruction of a band-limited function is the sinc function [1] :

$$\mathbf{sinc}(x) = \frac{\sin(x)}{x}.$$

The discrete sinc function is the discrete counterpart of the continuos sinc function in the discrete domain and will be referred as - sincd and is defined as follows:

$$\mathbf{sincd}(M;N;x) = \frac{\sin(\pi M x/N)}{N\sin(\pi x/N)}.$$

The interpolation in this case is carried out as:

$$f(x) = \sum_{k=0}^{N-1} f_k \,\mathbf{sincd}(M;N;\frac{x}{\Delta x} - k),$$

where, M is a parameter equal to $N \pm 1$ (N is even) or N (N is odd) and Δx is the sampling interval. The discrete sinc interpolation approximates the continuos sinc interpolation and converges to it as N goes to infinity. The discrete sinc interpolation method was applied for our purposes using two approaches: global and local ones.

2.1 Global discrete sinc-interpolation rebinning

The global sincd-interpolation rebinning approach is as follows (Fig 4):

- Magnify, using discrete-interpolation, the fan sinogram given on uniform grid in both axis by a magnification factor L, which yields a magnified sinogram.
- For each required sample needed for the construction of the rebinned parallel sinogram find its exact location in the magnified fan sinogram, from the fan-parallel geometry relation, and obtain its value from the nearest pixel.

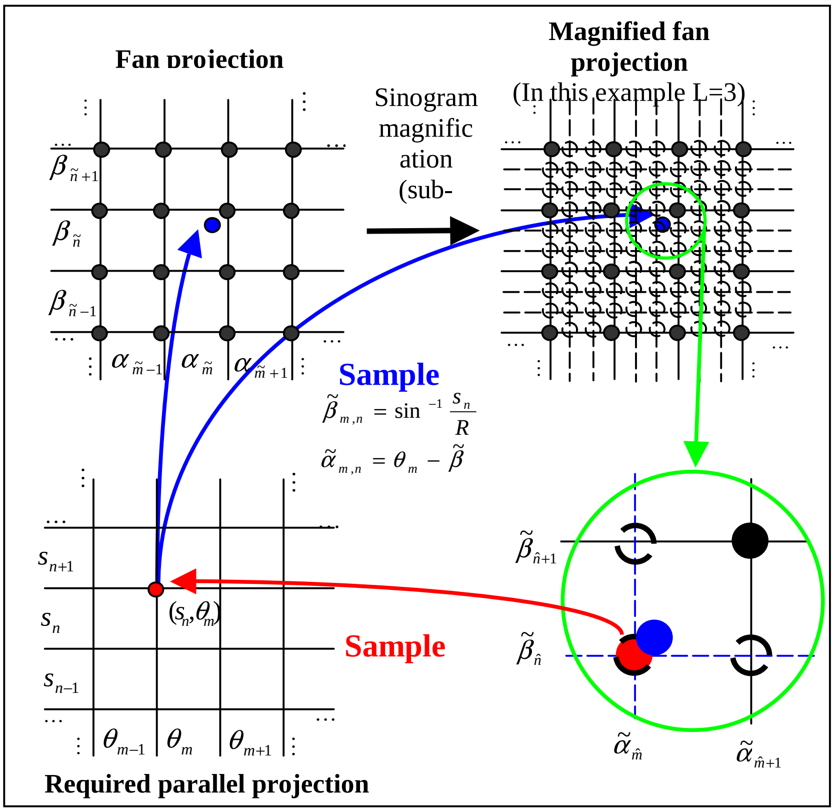


Fig. 4

2.2 Local discrete sinc-interpolation rebinning

The local sincd-interpolation rebinning approach is defined as follows (Fig 5):

- For each required sample needed for the construction of the rebinned parallel sinogram calculate its exact location in the fan geometry using the fan-parallel geometry relation..
- Find the nearest coordinate in the fan sinogram and make it the center of the 2-dimensional window.
- With the calculation of the p-shift value in each direction (α, β) obtain the required interpolated centered sample using the discrete sinc-interpolation method.
- Repeat the process in the same manner for every required sample through sliding the window across the original signal.

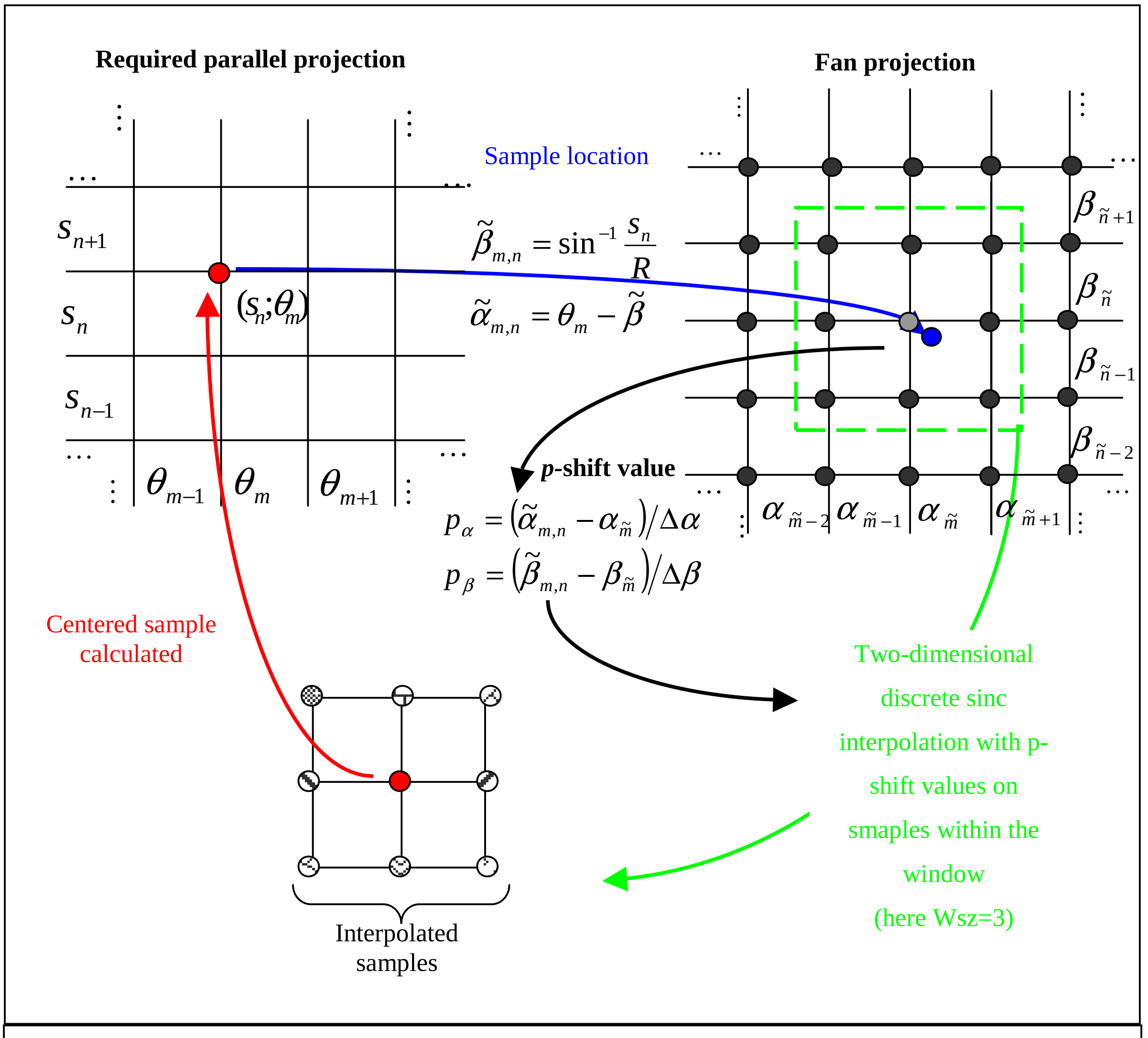


Fig. 5 : Local sliding approach of Discrete sinc interpolation

2.3 Quantitative comparison

The two approaches were investigated using fan projection $[\beta_{256} \times \alpha_{256}]$ obtained from the Shepp-Logan Head phantom image [256x256] and rebinned to parallel projection $[s_{256} \times \theta_{256}]$. The filtered back-projection method was used for image reconstruction. For quantitative comparison of the methods, normalized standard deviation of the reconstruction error was measured between the original and reconstructed test images.

In the global approach, the rebinning process was tested with several magnification factors L=(2,4,..,10) and was compared to rebinning performed by conventional interpolation method such as nearest-neighbor, bi-linear,bi-cubic (Keys), cubic spline which are provided by the Matlab program.

In the local approach different sizes of sliding windows ([3X3], [5X5], [7X7],.., [21X21]) were implemented, and compared to the results obtained by the global approach.

The obtained results are summarized in Table 1 and illustrated in Fig. 6. Note that process of generating projection and the reconstruction algorithm also contribute to the reconstruction error therefore creating a certain bias to comparison of contribution of the rebinning process only.

Table 1. Reconstruction errors

Method	Description	Normalized error standard deviation
Conventional interpolation	Nearest Neighbor	0.4239
	Bi-linear	0.3485
	Bi-cubic (Keys)	0.3263
	Spline	0.31995
Global discrete-sinc interpolation	Magnification Factor : 2	0.3562
	Magnification Factor : 4	0.3253
	Magnification Factor : 6	0.3189
	Magnification Factor : 8	0.3191
	Magnification Factor : 10	0.3187
Local discrete-sinc interpolation	Window size : [3x3]	0.3769
	Window size : [5x5]	0.3362
	Window size : [7x7]	0.3263
	Window size : [9x9]	0.3225
Local discrete-sinc interpolation	Window size : [11x11]	0.3207
	Window size : [13x13]	0.3198
	Window size : [15x15]	0.3191
	Window size : [17x17]	0.3187
	Window size : [19x19]	0.3183
	Window size : [21x21]	0.3181

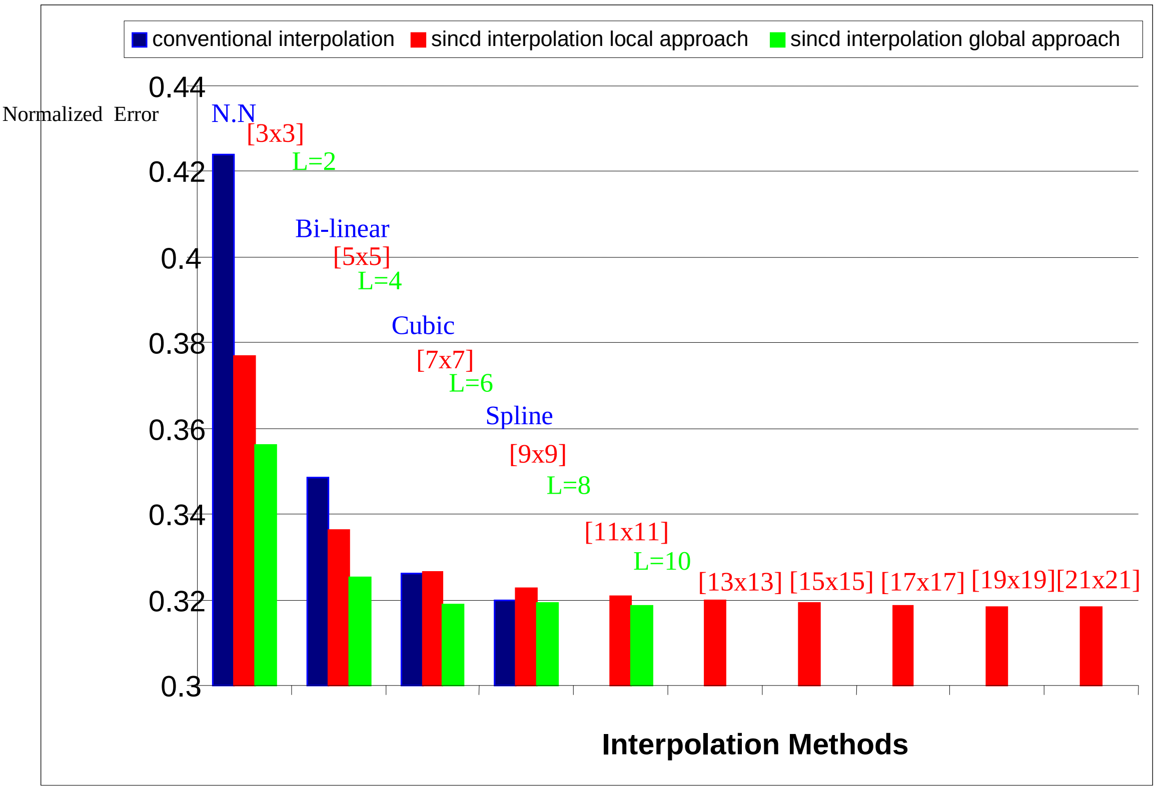


Fig. 6 : Normalized standard error deviation for different interpolation methods

It can be seen that the discrete sinc- interpolation method implemented in the global approach can provide smaller reconstruction error than the other conventional methods. In the case of discrete sinc-interpolation implemented in the local approach it can be seen that the larger the window size is the smaller quantitative error is.

Moreover, the computational complexity per sample of the sincd interpolation method implemented in the global approach is of $O(L^2 \log(N))$ which is lower than the computational complexity of high-order interpolation such as spline which is $O(N^2)$ where N is the number of signal samples.

3. Denoising through local sliding window approach

Local adaptive filtering performed in the spatial (time) frequency domain (such as DFT or DCT) enables image denosing, debluring and enhancement. The filtering principle is illustrated in Fig. 7

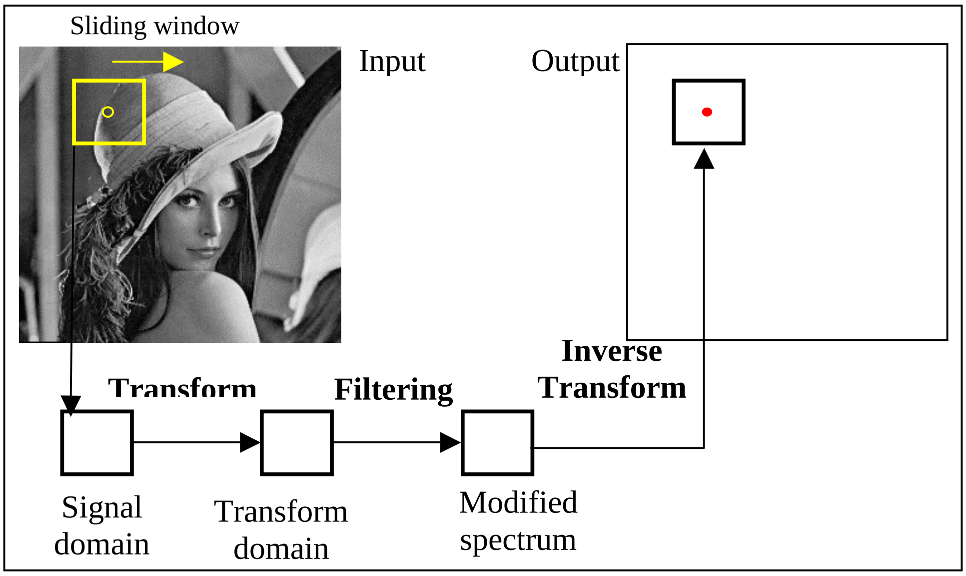


Fig. 7 : Local sliding window transform domain processing model

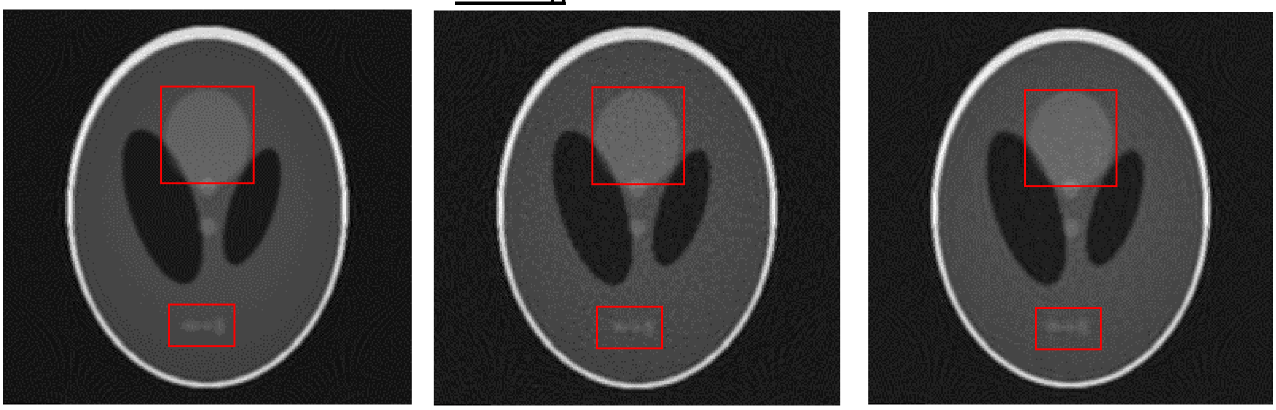
As discrete sinc interpolation is implemented in the Fourier domain we can use the advantage of local adaptive filtering, and perform denosing simultaneously with rebinning using the local approach. In the experiments, we implemented the simplest rejecting filter that, with a pre-given threshold, maintains the transform coefficients that are greater than the threshold and zeros the coefficients that are smaller than the threshold.

An illustrated example of reconstructed images (Shepp-Logan Head phantom [256x256]) obtained from rebinned parallel projection ($[s_{256} \times \theta_{256}]$, fan projection $[\beta_{256} \times \alpha_{256}]$ through the rebinning using a window size [9x9]) with and without noise and with the application of the rejecting filter (with an optimal threshold) is given in Fig 8. The reconstruction method used was the filtered back-projection one.

Reconstructed image from rebinned parallel projection with **no additive noise** added to the fan sinogram

Reconstructed image from rebinned parallel projection with **additive noise** added to the fan sinogram - **no filtering**

Reconstructed image from rebinned parallel projection with **additive noise** added to the fan sinogram - **with filtering**



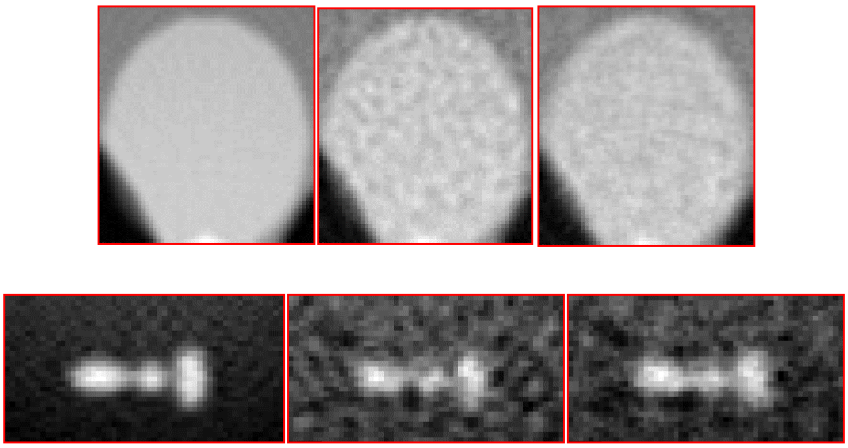


Fig. 8 : Local sliding window transform domain processing model

4. Description of the computer model

In order to test, compare and investigate different approaches to rebinning, simulations were conducted within the Matlab 5.2 software framework. The Matlab programs were written so that they implement the whole process, beginning from acquiring fan/parallel sinogram (Radon transform) from two-dimensional images to performing the rebinning process from fan to parallel projection to reconstruction the images back (inverse Radon transform) and to finally perform quantitative error estimation. Flow chart of the computer model is illustrated in the following diagram (Fig 9):

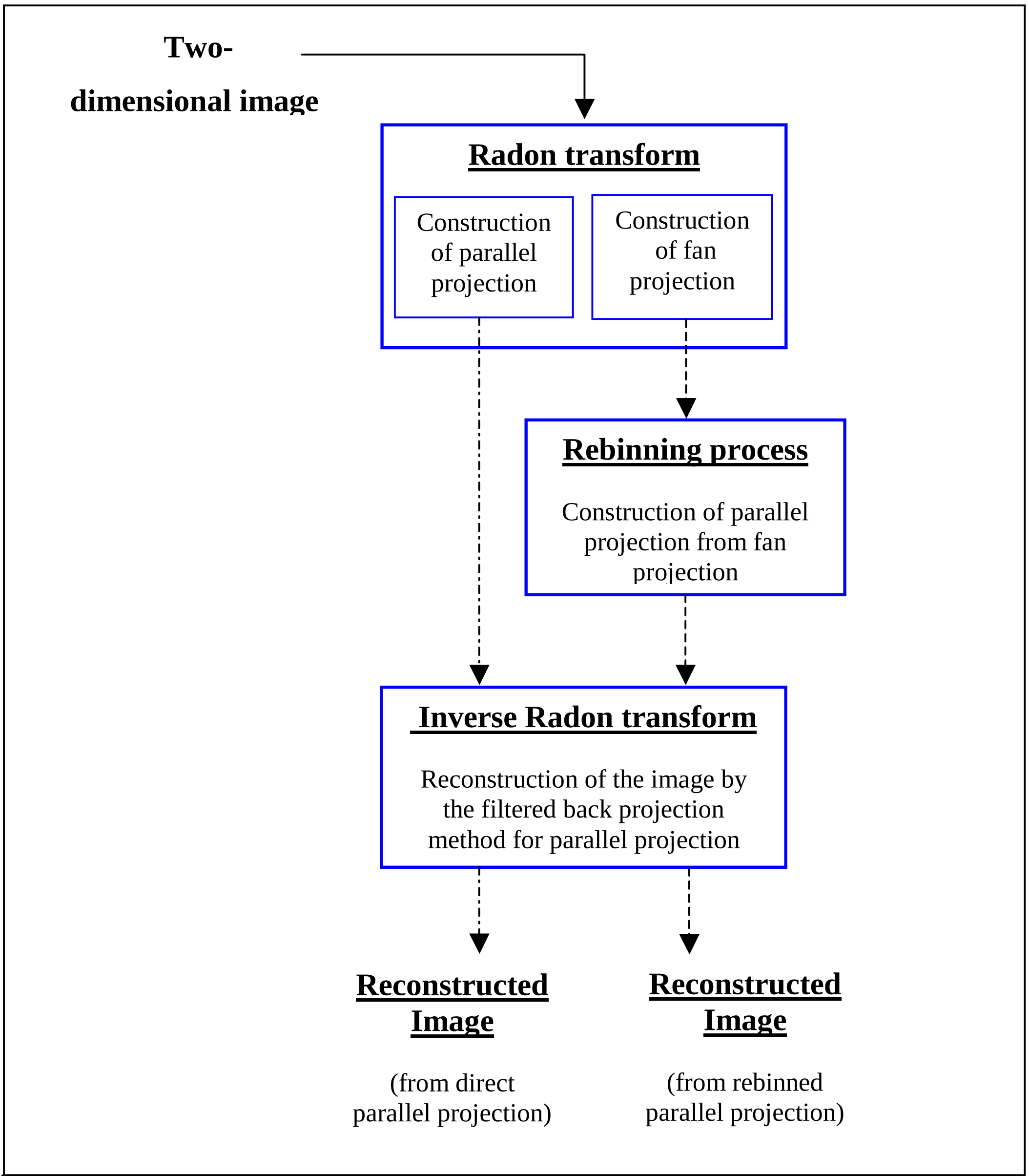


Fig. 9 : A general diagram of the simulation model

For each scanning geometry i.e., parallel and fan projection, a Matlab program was written with the use of the built-in Matlab 5.2 Radon transform program. (The parallel projection was written to provide an additional reference to the rebinning performance). In additional to the output sinogram, the program provides the corresponding axis of the projections in the required format needed for the written rebinning program (see Fig 10).

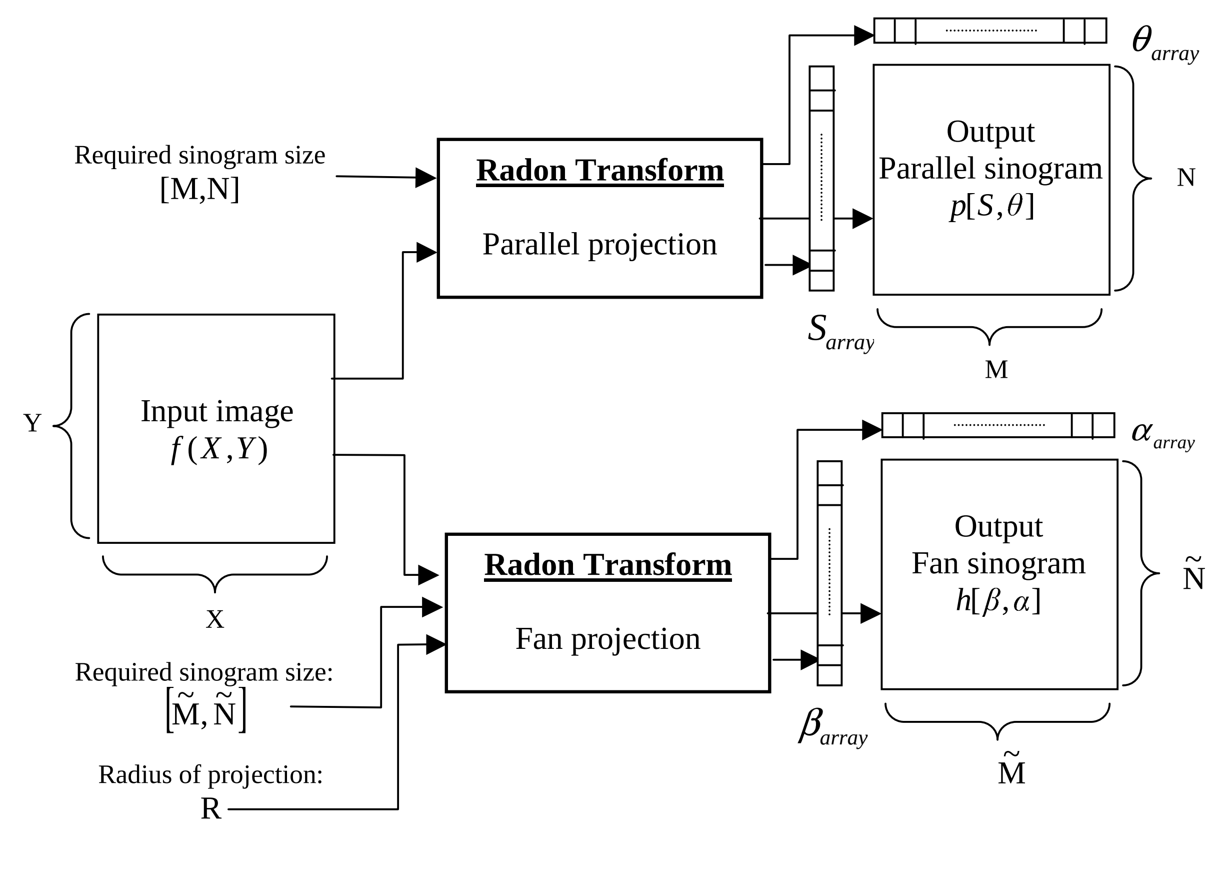


Fig. 10 : Schematic diagram of parallel projection program and fan projection program

The program for implementing rebinning through theglobal approach receives as input the magnification factor *L*, by which the discrete sinc interpolation is implemented. The program enables choosing different interpolation methods such as nearest-neighbor, linear, Keys bi-cubic and spline, all of which are built-in functions of Matlab 5.2.

The program, that implements rebinning through local sliding window approach receives as an input the window size parameters $\boldsymbol{W_\beta, W_\alpha}$ (odd numbers). An additional threshold parameter *Thr*. defines the rejecting-filter threshold that is needed to perform denoising of additive noise. Both programs output is a set of rebinned parallel projection and a corresponding array of the parallel projection geometry grid. A schematic diagram of the programs is given in Fig 11.

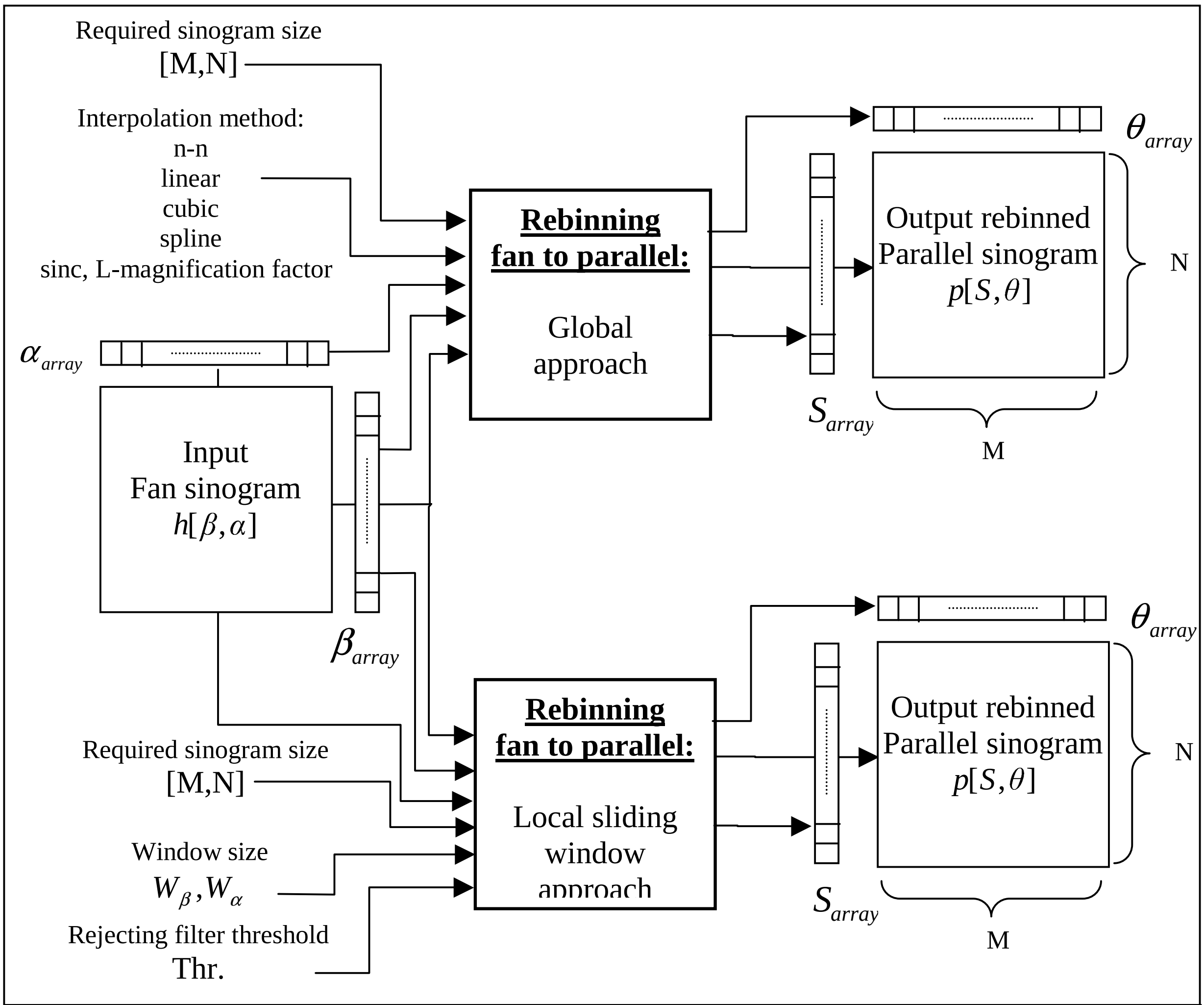


Fig. 11 : Schematic diagram of Global and Local rebinning programs

The image reconstruction method chosen to implement the inverse Radon transform is the parallel filtered back-projection model. The program is based on the Matlab 5.2 inverse Radon program. It receives as an input set of parallel projections and its corresponding grid array. (see Fig 12).

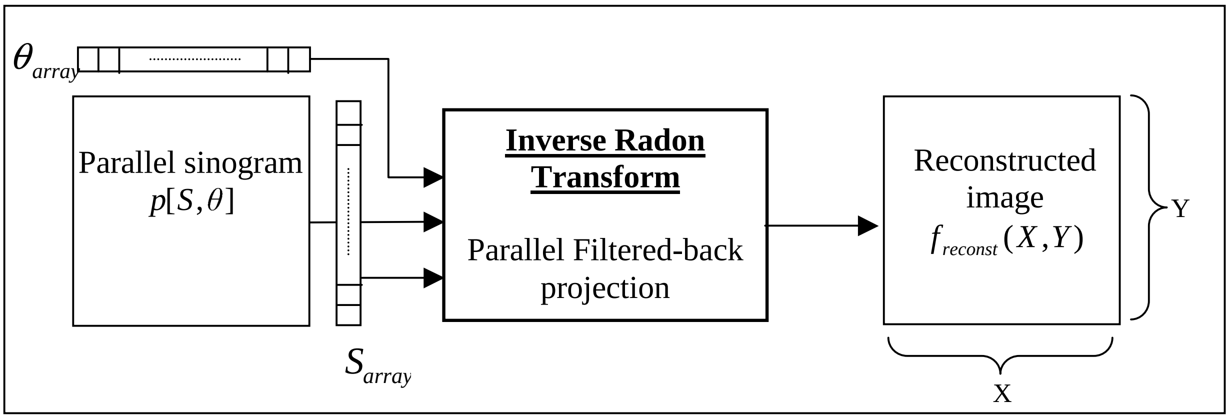


Fig. 12 : Schematic diagram of inverse Radon Transform programs