Color component transformations for optical pattern recognition

V. Kober,* V. Lashin,* I. Moreno, and J. Campos

Department of Physics, Autonomous University of Barcelona, E-08193 Bellaterra, Spain

L. P. Yaroslavsky

Department of Interdisciplinary Studies, Tel Aviv University, Tel Aviv 69978, Israel

M. J. Yzuel

Department of Physics, Autonomous University of Barcelona, E-08193 Bellaterra, Spain

Received January 2, 1997; accepted February 24, 1997

Several elementwise component transformations performed over primary color image components (RGB) before optical multichannel correlations are proposed to improve real-time multispectral pattern recognition. The first transformation is deduced from the theory of the optimal filter for object location and recognition extended to multispectral images. Several modifications of this transformation are studied. We investigate these transformations in terms of noise robustness and discrimination capability. Computer simulation with noisy input images for various kinds of correlation filter are presented to illustrate improvement of color pattern recognition by using the proposed transformations. Experimental results are also presented. © 1997 Optical Society of America [S0740-3232(97)01510-X]

1. INTRODUCTION

Optical correlation methods have been widely used in polychromatic pattern recognition in recent years.1–11 The purpose of introducing color in optical pattern recognition is twofold: improvement of the recognition process and an increase in processing capacity. The capacity is increased because different wavelengths of light do not interfere with each other and can process different information. For instance, Case12 and Yu13 used wavelength-multiplexed matched spatial filters. Color-coded correlation responses are produced that recognize several objects simultaneously.

The improvement of the recognition process when color information is added is based on the fact that for a color object both shape and intensity distribution may depend strongly on the illuminating wavelength. That is, color may introduce additional information for recognition. Various approaches have been suggested for improving color pattern recognition by involving color information in the pattern recognition process. One of the simplest ways to take into account the color content of objects is to carry out a correlation filtering in three red–green–blue (RGB) channels independently, then to make either arithmetical or logical elementwise operations over the correlation outputs, and finally to find signal maximums on the resulting correlation plane. In Refs. 5–8 it has been shown that using a phase-only filter14 in the multichannel correlation procedure produces a high level of recognition of targets with a given shape and color.

The color information of a signal can be involved in the recognition process more effectively if preprocessed color components are introduced in the optical multichannel correlator. Badique et al.3,4 have developed a projection of color images on a generalized color plane. This approach reduces the number of components used in the correlation process and enhances the ability of the correlator to discriminate among images of different colors. Another approach involves preprocessing of RGB components on the basis of human perception.9,11

In this paper we suggest an approach to the synthesis of color component transformations that is directly connected to the correlators’ discrimination capability, and we investigate several simple transformations that can be regarded as simplified implementations of an optimal process that guarantees the highest discrimination capability of the correlators in color pattern recognition.15

We consider optoelectronic systems as shown in Fig. 1, in which pattern recognition is carried out in four time-sequential steps at a high frame rate:

1. Elementwise component transformation of RGB components in such a manner that the subsequent channel correlations can be performed independently.
2. Correlation filtering in each channel.
3. Arithmetic operations over the channel correlation planes with the unified correlation plane as their output.
4. Localization of the signal maxima in the unified correlation plane.

Two methods of forming the unified correlation plane will be compared. The first method, shown in Fig. 1(a), is based on the addition of the intensities of the correlations in the different channels (addition of intensities). The second method, shown in Fig. 1(b), is based on the addition of amplitudes. The first method can be optically

0740-3232/97/1002656-14$10.00 © 1997 Optical Society of America
2. ELEMENTWISE COMPONENT TRANSFORMATIONS

Let \( \{s_k(x, y)\} \) be a color real-valued image and \( \{o_k(x, y)\} \) be a color real-valued object to be recognized in the image. Here \( x \) and \( y \) are the spatial coordinates, and \( k \) denotes the index of the primary color components \( (k = 0, 1, 2) \); that is, \( s_0(x, y) \), \( s_1(x, y) \), and \( s_2(x, y) \) are R, G, and B components of the image (the correspondence between the index \( k \) and the components (R, G, B) is, of course, arbitrary). By elementwise color component transformation we mean the transformation \( T \) applied to each pixel \((x, y)\) of the color image \( \{s_k(x, y)\}\) that produces a transformed image in the form

\[
\{t_k(x, y)\} = T\{s_k(x, y)\}.
\]

In the design of elementwise color component transformations, which improve the discrimination capability of the correlators, we base our approach on the theory\(^\text{15}\) of optimal correlators that consists of a linear filter followed by a device for localizing the highest signal maximum at the filter output. From this theory it follows that the optimal filter for object localization and recognition, which secures the highest ratio of the filter output at the point of the object location to the standard deviation of the filter output computed over the entire output plane (with the exception of the area occupied by the object), can be approximated in the Fourier transformation domain by the following equation:

\[
H_{\text{opt}}(f_x, f_y) = \frac{O^*(f_x, f_y)}{AV(|S(f_x, f_y)|^2)},
\]

where \((f_x)\) and \((f_y)\) are componentwise frequencies and spatial frequencies, respectively; \(O(f_x, f_y)\) and \(S(f_x, f_y)\) are the Fourier transforms of the signals \(O_k(x, y)\) and \(S_k(x, y)\) over \(k\), \(x\) and \(y\), respectively; the superscript asterisk means complex conjugation; and AV represents spectrum averaging over unknown object signal coordinates (regarded as random) and noise. This representation assumes that the image filtering is performed in the Fourier domain of the componentwise and the coordinate-wise Fourier transformation as follows: first, the image’s three-dimensional (3D) spectrum is multiplied by the filter’s frequency response; next, the inverse Fourier transformation of the spectrum component indexed by \(f_x\) over spatial frequencies \(f_y\) and \(f_y\) is performed for all spatial coordinates \(x\) and \(y\); and finally, an elementwise summation of the obtained components is performed. This sum represents, in fact, the inverse Fourier transform over color coordinates.

Such processing is hard to implement in the optoelectronic system that we are considering, and it needs to be simplified. The first simplification is to remove the spectrum averaging operation AV and regard the filter as

\[
H(f_x, f_y) = \frac{O^*(f_x, f_y)}{|S(f_x, f_y)|^2},
\]

implemented in an optical correlator more easily because realistic detectors work with intensities, whereas the second method corresponds better to the theory of optimal correlators.\(^\text{15}\) In such systems, a computer with a frame grabber may be used to perform the transformations and to take correlation snapshots, and spatial light modulators (SLMs) can be used to input the transformed scene and the filter into an optical setup.

Our goal is to synthesize the elementwise component transformations that improve the discrimination capability of the system in color pattern recognition. We also investigate the robustness of the systems to the additive noise that is always present in image sensors.

In Section 2 we introduce our approach, provide a review of the theory and suggest several elementwise color component transformations. In Section 3 we examine and compare the transformations in terms of noise robustness. The behaviors of the expected value and the standard deviation of the transformed noisy signal are studied. A noise analysis in the correlation plane for the two optoelectronic systems of Fig. 1 is presented in Section 4. In Section 5 we present computer simulation results that test the transformations involved in color pattern recognition in terms of discrimination capability for various kinds of noisy input images and correlation filters. Optical experimental results obtained in an optoelectronic system with the addition of intensities complement our computer simulation results.
This filter can be considered as two filters in cascade:

\[ H(f_c, f_x, f_y) = H_1(f_c, f_x, f_y)H_2(f_c, f_x, f_y), \] (4)

where

\[ H_1(f_c, f_x, f_y) = \frac{1}{|S(f_c, f_x, f_y)|}, \] (5)

\[ H_2(f_c, f_x, f_y) = \frac{O^*(f_c, f_x, f_y)}{|S(f_c, f_x, f_y)|}. \] (6)

The filter \( H_1(f_c, f_x, f_y) \) whitens the 3D spectrum of the input image, and the filter \( H_2(f_c, f_x, f_y) \) carries out the corresponding transformation of the object. The filter given in Eq. (4) still requires working in the 3D spectral domain, where the spectra, as functions of color and spatial frequencies, are not separable.

The next assumption is that the 3D spectral powers of the object \( \bar{O}(f_c, f_x, f_y) \) and of the image \( S(f_c, f_x, f_y) \) are similar. This assumption is realistic in applications in which an object is to be recognized among similar objects. Note that this assumption justifies the use of phase-only correlators.\(^{16}\) With this assumption we arrive at two whitening filters,

\[ H_3(f_c, f_x, f_y) = \frac{1}{|S(f_c, f_x, f_y)|}, \] (7)

\[ H_0(f_c, f_x, f_y) = \frac{1}{|O(f_c, f_x, f_y)|}, \] (8)

which should be applied to the color input image and the color target object image before multiplication of their color spectra and 3D inverse Fourier transform.

One can try to implement these whitening procedures as preprocessing of the input and object images. To simplify this preprocessing further, one can separate whitening over color frequencies and whitening over spatial frequencies such that whitening over color frequencies can be carried out elementwise over the initial image color components. In this way we arrive at the elementwise color component transformation that we shall call elementwise circular component normal whitening (CW-N).

### A. Elementwise Circular Component Normal Whitening

Let us briefly summarize the CW-N method. The transformed color components \( \{d_m(x, y)\} \) can be written by using discrete Fourier transforms as follows:

\[ d_m(x, y) = \frac{1}{3} \sum_{m=0}^{2} \tilde{S}_m(x, y)\exp(i2\pi km/3), \] (9)

where \( \{\tilde{S}_m(x, y)\} \) is the discrete spectrum of the transformed color image and \( m \) is the index of the spectrum components \( (m = 0, 1, 2) \). This spectrum can be expressed as

\[ \tilde{S}_m(x, y) = S_m(x, y)/|S_m(x, y)|, \] (10)

where \( \{S_m(x, y)\} \) is the discrete spectrum of the input color image along the color axis defined as

\[ S_m(x, y) = \sum_{k=0}^{2} s_k(x, y)\exp(-i2\pi mk/3). \] (11)

Equation (10) means that the nonzero spectrum's amplitudes are equal to 1. Finally, the explicit formulas for the transformed signal \( \{d_m(x, y)\} \), following directly from Eqs. (9)–(11), are (to the accuracy of irrelevant constant factor 1/3):

\[ d_0(x, y) = 1 + 2 \left\{ \sum_{k=0}^{2} s_k^2(x, y) - s_0(x, y)s_1(x, y) - s_0(x, y)s_2(x, y) - s_1(x, y)s_2(x, y) \right\}^{1/2}, \]

\[ d_1(x, y) = 1 + 2 \left\{ \sum_{k=0}^{2} s_k^2(x, y) - s_0(x, y)s_1(x, y) - s_0(x, y)s_2(x, y) - s_1(x, y)s_2(x, y) \right\}^{1/2}, \]

\[ d_2(x, y) = 1 + 2 \left\{ \sum_{k=0}^{2} s_k^2(x, y) - s_0(x, y)s_1(x, y) - s_0(x, y)s_2(x, y) - s_1(x, y)s_2(x, y) \right\}^{1/2}, \] (12)

We refer to the above whitening operation as circular whitening because it is based on the discrete Fourier transform. In this case the sequence of color components \( \{s_k(x, y)\}, (k = 0, 1, 2) \) are assumed to be periodic along the color axis. Recently we demonstrated by both com-
puter simulation and optical experiment that an improvement of color pattern recognition can be obtained with this operation for real-life images.\(^{17}\)

Note that the transformed components \(\{d_k(x, y)\}\) have the following features:

1. Vectors obtained from the transformed components by the circular shift are orthonormal; that is,

\[
\sum_{k=0}^{2} [d_k(x, y)]^2 = 1,
\]

\[
d_0(x, y)d_1(x, y) + d_0(x, y)d_2(x, y) + d_1(x, y)d_2(x, y) = 0. \tag{13}
\]

2. The transformed components are directly associated with the corresponding circular Laplacian of the components

\[
\begin{align*}
   s_0(x, y) &= \frac{s_1(x, y) + s_2(x, y)}{2}, \\
   s_1(x, y) &= \frac{s_0(x, y) + s_2(x, y)}{2}, \\
   s_2(x, y) &= \frac{s_0(x, y) + s_1(x, y)}{2}.
\end{align*}
\]

This feature clarifies the physical meaning of the transformation: the transformation computes elementwise deviations of the color components from the average value of the color components.

3. The transformation degenerates when all components are zero (monochrome image). In this case the transformed components are equal to a constant that does not depend on image coordinates, because the transformation is elementwise and is separated from whitening over spatial frequencies.

Note that the proposed method normalizes the energy of all the pixels along the color axis. In the following sections we suggest some modifications of the CW–N method.

**B. Elementwise Component Centering, or Circular Laplacian**

As noted above, the CW–N method is directly related to the circular Laplacian of the components; therefore we will study this transformation as a simplification of the previous one. We will refer to this method as elementwise component centering (CNT) or circular Laplacian. The transformed components \(\{s_k(x, y)\}\) are given by

\[
\begin{align*}
   z_0(x, y) &= s_0(x, y) - \frac{s_1(x, y) + s_2(x, y)}{2}, \\
   z_1(x, y) &= s_1(x, y) - \frac{s_0(x, y) + s_2(x, y)}{2}, \\
   z_2(x, y) &= s_2(x, y) - \frac{s_0(x, y) + s_1(x, y)}{2}. \tag{14}
\end{align*}
\]

**C. Elementwise Component Centering with Normalization**

As stated above, the CW–N method performs a normalization of the energy along the color axis. Consequently, a better approximation of the CW–N method is given by a normalization of the previous transformation, which we call elementwise component centering with normalization (CNT–N). The expressions \(\{q_k(x, y)\}\) for this transformation are

\[
\begin{align*}
   q_0(x, y) &= \frac{s_0(x, y) - s_1(x, y) - s_2(x, y)}{2} \\
   &= \frac{\sum_{k=0}^{2} s_k^2(x, y) - s_0(x, y)s_1(x, y) - s_0(x, y)s_2(x, y) - s_1(x, y)s_2(x, y)}{\left(\sum_{k=0}^{2} s_k^2(x, y) - s_0(x, y)s_1(x, y) - s_0(x, y)s_2(x, y) - s_1(x, y)s_2(x, y)\right)^{1/2}}, \\
   q_1(x, y) &= \frac{s_1(x, y) - s_0(x, y) - s_2(x, y)}{2} \\
   &= \frac{\sum_{k=0}^{2} s_k^2(x, y) - s_0(x, y)s_1(x, y) - s_0(x, y)s_2(x, y) - s_1(x, y)s_2(x, y)}{\left(\sum_{k=0}^{2} s_k^2(x, y) - s_0(x, y)s_1(x, y) - s_0(x, y)s_2(x, y) - s_1(x, y)s_2(x, y)\right)^{1/2}}, \\
   q_2(x, y) &= \frac{s_2(x, y) - s_0(x, y) - s_1(x, y)}{2} \\
   &= \frac{\sum_{k=0}^{2} s_k^2(x, y) - s_0(x, y)s_1(x, y) - s_0(x, y)s_2(x, y) - s_1(x, y)s_2(x, y)}{\left(\sum_{k=0}^{2} s_k^2(x, y) - s_0(x, y)s_1(x, y) - s_0(x, y)s_2(x, y) - s_1(x, y)s_2(x, y)\right)^{1/2}}. \tag{15}
\end{align*}
\]
D. Elementwise Circular Whitening

To obtain the elementwise circular whitening (CW) transformation, we slightly modify the condition given in Eqs. (13) in order to maintain the energy of the centered signal \{c_k(x, y)\} [Eqs. (14)] at the coordinates (x, y); that is,
\[
\sum_{x=0}^{2} [c_k(x, y)]^2 = C_E \sum_{x=0}^{2} [c_k(x, y)]^2,
\]
\[
e_0(x, y)e_1(x, y) + e_0(x, y)e_2(x, y)
+ e_1(x, y)e_3(x, y) = 0,
\]
where \(e_k(x, y)\) is a new transformed signal obtained from \(c_k(x, y)\) and \(C_E\) is a constant. The transformed color components \(e_k(x, y)\) can be written as follows:
\[
e_0(x, y) = [s_0(x, y) - s_1(x, y)] + [s_0(x, y) - s_2(x, y)]
+ \sum_{x=0}^{2} s_k(x, y) - s_0(x, y)s_1(x, y)
- s_0(x, y)s_2(x, y) - s_1(x, y)s_2(x, y)]^{1/2},
\]
\[
e_1(x, y) = [s_1(x, y) - s_0(x, y)] + [s_1(x, y) - s_2(x, y)]
+ \sum_{x=0}^{2} s_k(x, y) - s_0(x, y)s_1(x, y)
- s_0(x, y)s_2(x, y) - s_1(x, y)s_2(x, y)]^{1/2},
\]
\[
e_2(x, y) = [s_2(x, y) - s_0(x, y)] + [s_2(x, y) - s_1(x, y)]
+ \sum_{x=0}^{2} s_k(x, y) - s_0(x, y)s_1(x, y)
- s_0(x, y)s_2(x, y) - s_1(x, y)s_2(x, y)]^{1/2}.
\]

This transformation does not degenerate when two of three components are equal to zero; in this case it is reduced to an identical transformation.

E. Elementwise Quasi-Singular-Value Decomposition

The next elementwise transformation, which is referred to as quasi-singular-value decomposition (QSVD), is based on the idea of the singular-value-decomposition method. We consider the color signal \(s_k(x, y), k = (0, 1, 2)\) as a column vector \(s\) in a 3D vector space at each pair of coordinates (x, y). The matrices \(ss^T\) (non-negative, symmetric) and \(ss\) (single number) have the following nonzero eigenvalue:
\[
\lambda(x, y) = s^T s = \sum_{x=0}^{2} [s_k(x, y)]^2.
\]

According to singular-value decomposition, the column vector \(s\) has the representation
\[
s = \psi v = \sum_{n=0}^{\infty} u_n(x, y)\psi_n(x, y),
\]
where \(\Psi = \{\psi_n(x, y), (n = 0, 1, 2)\}\) is a unitary matrix of \(3 \times 3\) elements whose columns are the eigenvectors of \(ss^T\), and \(v = \{v_n(x, y)\}\) is a column vector whose elements are the eigenvalues of \(ss^T\) (the spectral representation of \(s\)). We note that the spectral components can be computed as
\[
v^T = \{\sqrt{\lambda(x, y), 0, 0}\}.
\]

The position and the sign of the nonzero spectral component in Eq. (20) are undefined by the singular-value-decomposition method. We suggest transformation of the components by using the following procedure performed at each pair of coordinates (x, y):

- The nonzero magnitude \([\sum_{x=0}^{2} s^2_k(x, y)]^{1/2}\) is assigned to the channel having the maximum of the absolute value of the centered signal,
- The sign of the nonzero spectral component is determined by the sign of the relevant centered component.
- All other components are set to zero.

Consequently, QSVD is a transformation that exploits differences among image components in a nonlinear way, thus producing a transformed signal in which only one component is nonzero. The name of the transformation is chosen by analogy with the method of singular-value-decomposition, which provides a single number for a vector.

F. Elementwise Quasi-Singular-Value Decomposition with Normalization

Elementwise quasi-singular-value decomposition with normalization (QSVD–N) is a simplified version of the QSVD transformation in which a normalization of the channel energy is performed, in analogy with the previous transformations. Consequently, the output nonzero component is determined only by the sign of the relevant centered component. The ternary output of the transformation is attractive for optical implementation, because there exist very fast SLM’s that work in this ternary mode.

3. NOISE ROBUSTNESS OF COMPONENT TRANSFORMATIONS

In this section we investigate the behavior of the expected value and the standard deviation of the transformed noisy signal (the input signal is corrupted by additive noise) versus the standard deviation of input noise.

Let \(n_k(x, y), (k = 0, 1, 2)\) denote zero-mean, additive noise. It is assumed that noise is statistically independent in different channels and, with no loss of generality, that noise has the same variance in different channels. The noisy input color image is given by
\[
\hat{s}_k(x, y) = s_k(x, y) + n_k(x, y).
\]
Since we consider real-time pattern recognition, an element-wise transformation $\mathbf{T}$ is directly applied to the noisy color image $\{\hat{s}_k(x, y)\}$. The transformed noisy image is given by

$$\{\hat{t}_k(x, y)\} = \mathbf{T}\{\hat{s}_k(x, y)\}.$$  

(22)

A number of tests are performed to investigate the behavior of the expected value and the standard deviation for the transformed noisy signal as a function of the standard deviation of the input noise. These tests are based on computer simulation, because the statistical estimations for some elementwise transformations are difficult to treat analytically. We observed a similarity in the behavior of the expected value and the standard deviation for the different channels. The input color signal $\{s_k(x, y)\}, \ (k = 0, 1, 2)$ was varied in the interval $[0,1]$. Figure 2 illustrates typical distributions of the expected value and the standard deviation. These results are obtained when input noise is white, zero-mean, and Gaussian distributed; and the input color signal, as an example, is given by $\{0.1, 0.3, 0.7\}$. The statistical measurements are obtained over 65,536 trials. The expected value and the standard deviation are shown only for the second output channel, because these magnitudes in the other output channels have similar distributions. From Fig. 2 we can see that for the transformations CNT, CW, CNT–N, CW–N, and QSVD–N, the expected value tends to be a constant, while the standard deviation of input noise increases. The absolute magnitude of the expected value for the QSVD transformation is an increasing function when the input standard deviation is large. We note that for the transformations CNT, CW, and QSVD, the standard deviation of the transformed noisy signal increases versus the standard deviation of the input noise. On the other hand, for transformations with normalization (CNT–N, CW–N, and QSVD–N), the output standard deviation yields a constant if the input standard deviation increases.

To evaluate the robustness to noise of a transformation $\mathbf{T}$, we use the signal-to-noise ratio (SNR) measure defined for the $k$th transformed channel at the coordinates $(x, y)$ as follows:

$$\text{SNR}_T[\hat{t}_k(x, y)] = \frac{|E[\hat{t}_k(x, y)]|^2}{\text{var}[\hat{t}_k(x, y)]},$$  

(23)

where $E(\cdot)$ and $\text{var}(\cdot)$ denote the statistical expected value and the variance, respectively.

Since a correlation filter is computed by using the transformed noiseless target, it is useful to characterize target distortions that are due to noise by the normalized mean-square-error (NMSE) measure defined for the $k$th transformed channel at the coordinates $(x, y)$ as

$$\text{NMSE}_T[\hat{t}_k(x, y)] = \frac{E[|t_k(x, y) - \hat{t}_k(x, y)|^2]}{[t_k(x, y)]^2}.$$  

(24)

It is clear that if $E[\hat{t}_k(x, y)] = t_k(x, y)$, then $\text{NMSE}_T = 1/\text{SNR}_T$.

The centering (or circular Laplacian) describes well the differences (dissimilarities) among channel signals, and, moreover, it is a basic operation for the proposed transformations. We investigate the elementwise component transformations in terms of SNR and NMSE as a function of SNR$_Z$ (SNR for the centered signal). If SNR$_Z$ is close to zero, then channel signal differences are small compared with noise fluctuations. This means that the input channel signals may be regarded as indistinguishable, and therefore elementwise transformations are senseless. The performances of elementwise component transformations in terms of SNR and NMSE are shown in Fig. 3. These results are obtained by computer simulation in the second output channel under test circumstances described above. It can be seen that all transformations have the desirable properties; that is, SNR$_T$ is an increasing function versus SNR$_Z$, whereas NMSE$_T$ decreases while SNR$_Z$ increases.
In this section we analyze the correlation outputs of optoelectronic systems shown in Fig. 1. These outputs are denoted as OUT. In Section 5 we verify the results presented in this section by computer simulation. Let \( c_k(x, y) \) be the correlation outputs of noisy color image components in Eq. (21), then the correlation outputs are stochastic processes given by

\[
\hat{c}_k(x, y) = c_k(x, y) + b_k(x, y),
\]

where

\[
c_k(x, y) = \iint s_k(\xi, \eta)h_k(x - \xi, y - \eta)d\xi d\eta
\]

are the deterministic correlation outputs and

\[
b_k(x, y) = \iint n_k(\xi, \eta)h_k(x - \xi, y - \eta)d\xi d\eta
\]

are the random correlation outputs. A random correlation output is a stationary random process with zero mean and with autocorrelation function expressed as follows:

\[
B_k(x, y) = R_k(x, y) * h_k(x, y) * h_k(-x, -y).
\]

where the asterisk denotes convolution.

## 4. NOISE ANALYSIS IN THE CORRELATION PLANE

In this section we analyze the correlation outputs of optoelectronic systems shown in Fig. 1. These outputs are denoted as OUT. In Section 5 we verify the results presented in this section by computer simulation. Let \( \{h_k(x, y)\}, (k = 0, 1, 2) \) be the impulse responses of filters used in the channels of the linear system. To simplify the following analysis, we assume that

- The \( k \)th channel input noise \( n_k(x, y) \) is a realization from a zero-mean, additive, stationary process with autocorrelation functions \( R_k(x, y) \).
- Noise in one channel is statistically independent of noise in every other channel;
- Noise has the same variance in different channels.
- The impulse responses of the filters do not depend on parameters of the input noise (it is difficult to take into account the statistical parameters of the transformed noise for filter design, because, in general, the transformed noise is signal dependent).

If the inputs to the linear systems are the noisy color image components in Eq. (21), then the correlation outputs are stochastic processes given by

\[
\hat{c}_k(x, y) = c_k(x, y) + b_k(x, y),
\]

where

\[
c_k(x, y) = \iint s_k(\xi, \eta)h_k(x - \xi, y - \eta)d\xi d\eta
\]

are the deterministic correlation outputs and

\[
b_k(x, y) = \iint n_k(\xi, \eta)h_k(x - \xi, y - \eta)d\xi d\eta
\]

are the random correlation outputs. A random correlation output is a stationary random process with zero mean and with autocorrelation function expressed as follows:

\[
B_k(x, y) = R_k(x, y) * h_k(x, y) * h_k(-x, -y).
\]

where the asterisk denotes convolution.

### A. System with Addition of Intensities

The expected value of the system output OUT in Fig. 1(a) can be written as

\[
E[OUT(x, y)] = E\left[ \sum_k |\hat{c}_k(x, y)|^2 \right]
\]

\[
= \sum_k |c_k(x, y)|^2 + \sum_k B_k(0, 0).
\]

It is of interest to note the following properties of \( E[OUT(x, y)] \):

1. Since \( \{B_k(0, 0)\}, (k = 0, 1, 2) \) are the variances of \( \{b_k(x, y)\} \), then \( E[OUT(x, y)] \) is linearly increasing versus the variance of the channel input noise at the coordinates \((x, y)\).

2. \( E[OUT(x, y)] \) for noisy and noiseless inputs yields the same positions of the peaks.

3. The above conclusions are correct for colored and white input noise.

If the centering is performed before channel correlations, then the expected value of the system output can be rewritten as follows:

\[
E[OUT(x, y)] = E\left[ \sum_k |\hat{c}_k(x, y) * h_k(x, y)|^2 \right]
\]

\[
= \sum_k |z_k(x, y) * h_k(x, y)|^2 + \sum_k B^*_k(0, 0),
\]

where

\[
B^*_k(x, y) = \frac{1}{9} [4R_k(x, y) + R_{(k-1)mod 3}(x, y)
\]

\[
+ R_{(k+1)mod 3}(x, y)] * h_k(x, y) * h_k(-x, -y).
\]
It is easy to show that $E[\text{OUT}_T^2(x, y)]$ has the same properties as $E[\text{OUT}(x, y)]$ in Eq. (29).

Let us now evaluate the system output when other transformations (CW, QSVD, CNT–N, CW–N, and QSVD–N) are used. To simplify the analysis, we represent transformed noisy color image components in Eq. (22) as a sum of deterministic and zero-mean random terms; that is

$$\hat{t}_k(x, y) = t^d_k(x, y) + t^n_k(x, y), \quad (32)$$

where $t^d_k(x, y) = E[\hat{t}_k(x, y)]$ and $t^n_k(x, y) = \hat{t}_k(x, y) - E[\hat{t}_k(x, y)]$.

In general, both terms in Eq. (32) are signal dependent. This means that $\{\hat{t}_k(x, y)\}$, $(k = 0, 1, 2)$ are no longer stationary processes. We can rewrite the expected value of the system output as

$$E[\text{OUT}_T(x, y)] = E\left[\sum_k |\hat{t}_k(x, y) * h_k(x, y)|^2\right]$$

$$= \sum_k |t^d_k(x, y) * h_k(x, y)|^2$$

$$+ \sum_k B'_k(x, y), \quad (33)$$

where

$$B'_k(x, y) = \int \int \int E[t^n_k(\xi, \eta)t^n_k(\alpha, \beta)]$$

$$\times h_k(x - \xi, y - \eta)$$

$$\times h_k(x - \alpha, y - \beta) d\xi d\eta d\alpha d\beta \quad (34)$$

is the variance of correlation output in the $k$th channel at the coordinates $(x, y)$. We note that both terms in Eq. (33) are functions of the input noise variance. The first term depends on the input noise variance by means of the expected value of the transformed signal. The second term is related to the input noise variance through the variance of the transformed signal. Taking into account typical distributions of the expected value and the standard deviation of the transformed signal shown in Fig. 2, we can state for $E[\text{OUT}_T(x, y)]$ the following properties:

1. If the input noise variance is small, then $E[\text{OUT}_T(x, y)]$ for the transformations CW, QSVD, CNT–N, CW–N, and QSVD–N is determined mainly by the first term in Eq. (33). Furthermore, since for the transformations with normalization and for the QSVD transformation the absolute values of $|t^d_k(x, y)|$ decrease as a function of the input noise variance, then $E[\text{OUT}_T(x, y)]$ also decreases (see Fig. 2(a), the standard deviation of input noise in the interval $[0,1]$). This may be shown by applying the Schwarz inequality to the first term in Eq. (30).

2. In contrast, if the input noise variance is large, then the first term in Eq. (33) for the transformations CW, CNT–N, CW–N, and QSVD–N tends to be a constant [Fig. 2(a)]. This means that $E[\text{OUT}_T(x, y)]$ depends mainly on the second term. We see that for the transformations with normalization, the function $E[\text{OUT}_T(x, y)]$ yields a constant when the input noise variance increases, because the second term is also a constant [Fig. 2(b)]. On the other hand, for the CW transformation, $E[\text{OUT}_T(x, y)]$ is increasing versus the input noise variance because the relevant variance of the transformed signal is an increasing function. For the case of the QSVD transformation, the function $E[\text{OUT}_T(x, y)]$ is always increasing, because the first and the second terms in Eq. (30) increase when the input noise variance increases.

B. System with Addition of Amplitudes

Taking into account the assumptions about the input noise, we can write the expected value of the system output OUT in Fig. 1(b) for the correlation outputs given by Eq. (25) as

$$E[\text{OUT}(x, y)] = E\left[\sum_k \hat{c}_k(x, y)\right]^2$$

$$= \sum_k c_k(x, y)^2 + \sum_k B_k(0, 0). \quad (35)$$

It is clear that the function $E[\text{OUT}(x, y)]$ has the same properties as the preceding system output given in Eq. (29). In a similar manner, when the centering is carried out before correlations we can express the expected value of the system output as

$$E[\text{OUT}_C(x, y)] = E\left[\sum_k \hat{z}_k(x, y) * h_k(x, y)\right]^2$$

$$= \sum_k z_k(x, y) * h_k(x, y)^2$$

$$+ \sum_k B^c_k(0, 0). \quad (36)$$

Obviously, the behavior of this function versus the input noise variance is identical to that of the function given in Eq. (30).

Finally, if other transformations (CW, QSVD, CNT–N, CW–N, and QSVD–N) are performed as elementwise pre-processing, we can write the expected value of the system output as follows:

$$E[\text{OUT}_T(x, y)] = E\left[\sum_k \hat{t}_k(x, y) * h_k(x, y)\right]^2$$

$$= \sum_k t^d_k(x, y) * h_k(x, y)^2 + B'(x, y), \quad (37)$$

where

$$B'(x, y) = \sum_k \sum_m \int \int \int E[t^n_k(\xi, \eta)t^n_m(\alpha, \beta)]$$

$$\times h_k(x - \xi, y - \eta)$$

$$\times h_m(x - \alpha, y - \beta) d\xi d\eta d\alpha d\beta \quad (38)$$
is the variance of the system output at the coordinates \((x, y)\).

By comparing Eqs. (33) and (34) with Eqs. (37) and (38) and by taking into account the typical distributions of the expected value and the standard deviation of the transformed signal presented in Fig. 2, we can conclude that the function \(E[\text{OUT}_T(x, y)]\) in Eq. (33) versus the input noise variance has properties similar to those of the function in Eq. (37).

5. COMPUTER SIMULATIONS AND OPTICAL EXPERIMENT

In this section we demonstrate the performance of color pattern recognition with all the proposed elementwise preprocessings in terms of discrimination capability (DC) and its sensitivity to the additive noise in a scene signal. Figure 4 is an example of the input color scene. The test color image contains \(256 \times 256\) pixels. Following the convention shown in Fig. 4(a), there are one target butterfly (B4) and three nontarget butterflies (B1, B2 and B3) in the test image. Since we use SLM’s in an optical setup, the input signal range (in each channel) is normalized by the same factor to the modulation range \([0,1]\). The averages over target areas in all channels are equal to 0.25. In the following computer simulation and optical experiment, we normalize the maximum amplitude of channel filter transfer functions to unity to achieve the highest realizable optical efficiency. To investigate the

![Fig. 4. Test RGB color scene components: (a) Red, (b) Green, (c) Blue. The target is the object B4.](image_url)
system outputs for a colored additive noise model, we use colored noise as a realization of a zero-mean, Gaussian stationary process with the exponential correlation function; that is,

\[ R_k(x, y) = \sigma^2 \rho_k |x^s y^s|, \quad (39) \]

where \( \rho_k \) is the correlation coefficient \([0,1]\) in the \( k \)th channel \((k = 0, 1, 2)\) and \( \sigma^2 \) is the input noise variance.

Two types of block diagram for pattern recognition are considered, as shown in Fig. 1. In the first method [Fig. 1(a)] the addition of the intensities is performed. In the second method [Fig. 1(b)] the amplitudes are summed up. Despite the fact that the second method in general provides better performance in terms of DC compared with that of the first one, the second method, which is based on the addition of channel intensities, can be optically implemented in the optical correlators much more easily, because realistic detectors work with intensities. We could, however, use more-sophisticated techniques, such as interferometry, to detect both modulus and phase.

A. Noise Robustness Tests for the Autocorrelation Peak
In addition to the analysis given in Section 4, we illustrate the behavior of the expected value of system outputs in Eqs. (33) and (37) by computer simulation. The classical matched filter \(19\) (CMF) is used in the experiment. Figure 5 shows the autocorrelation peaks versus the standard deviation of the input noise for the proposed transformations. These results are given with 95% confidence; that is, we assert that the true mean of the autocorrelation peak \( E[\text{OUT}_T(x, y)] \) lies within the following interval:

\[ \frac{\text{OUT}_T(x, y) - t_u(N)\bar{\sigma}}{\sqrt{N}} \leq N, \text{OUT}_T(x, y) + t_u(N)\bar{\sigma} \leq \sqrt{N}, \quad (40) \]

where \( \text{OUT}_T(x, y) = \frac{1}{N} \Sigma_k \text{OUT}^k_T(x, y) \) is the sample mean of \( \text{OUT}_T(x, y) \). Here \( \text{OUT}^k_T(x, y) \) is the system output in the \( k \)th statistical trial of input noise, \( \bar{\sigma}^2 = \frac{1}{N} \Sigma_k (\text{OUT}^k_T(x, y) - \text{OUT}^k_T(x, y))^2 \) is the sample variance, and \( t_u(N) \) is the Student percentile with \( N - 1 \) degrees of freedom. \(20\)

The statistical measurements are obtained over 30 statistical trials of input noise. From the graphs in Fig. 5 we observe a good accordance with the properties described in Section 4. Moreover, even computer simulation with 30 statistical trials of input noise provides unbiased positions of peaks in the unified correlation plane for all elementwise transformations. These results are obtained when the zero-mean, white, Gaussian input noise is used. Computer simulation results with the colored-noise model for different values of \( \{\rho_k\}, (k = 0, 1, 2) \) ranging from zero to one are similar. Therefore they also coincide with the conclusions in Section 4.

B. Discrimination Study
To illustrate the performance of color pattern recognition based on the proposed elementwise transformations in terms of DC, we carried out numerous computer simulation tests. DC is defined as

\[ \text{DC} = 1 - \frac{|C^B|^2}{|C^O|^2}, \quad (41) \]

where \( C^O \) and \( C^B \) are the target and the nontarget peak, respectively; that is, \( C^B \) is the maximum in the correlation plane over the background area to be rejected, and \( C^O \) is the maximum in the correlation plane over the area of object to be recognized. The area of the object to be recognized is determined in the close vicinity of the target location (the size of the area is similar to the size of the target). The background area is complementary to the object area. A negative value of DC means that the filter fails to recognize the target, and a false alarm appears.

In our discrimination study, correlation filters \( \text{CMF}, \text{phase-only filter}^{14} \) (POF), and optimal correlator \( 15 \) (OC) \( ) \) are applied in transformed channels to recognize a color target among objects of similar shape but with different color contents. We designed the transfer function of the OC for real-time pattern recognition in the \( k \)th channel as follows:

\[ \text{Amplitude addition} \]

\[ \text{Intensity addition} \]

\[ \text{Lower confidence limit} \]

\[ \text{Upper confidence limit} \]
\[ H_k^{OC}(f_1, f_2) = \frac{O_k^*(f_1, f_2)}{|S_k(f_1, f_2)|^2 + |O_k(f_1, f_2)|^2}, \quad (42) \]

where \( O_k(f_1, f_2), S_k(f_1, f_2) \) are the spectra of the target and of the input signal, respectively, in the \( k \)th channel. Here the asterisk denotes complex conjugate. The theory of the optimal filter in terms of probability at the peak location leads to Eq. (42), which depends on the input scene.\(^{15}\) This implies a real-time filter design that could be more difficult in OC’s. For this reason several approximations of Eq. (42) have been proposed,\(^{21}\) which depend on the available information about the input scene. In these approximations a real-time generation of the filter is not needed.

First, the elementwise transformations are tested with the noiseless input color image. We perform independently three correlations between the transformed scenes and the filters (CMF, POF, OC) matched to the transformed butterfly B4. Summing either intensity or amplitude correlation outputs according to block diagrams shown in Fig. 1, we obtain the resulting correlation plane \( OUT_f(x, y) \). The correlation peaks and DC computed over \( OUT_f(x, y) \) with the various filters (CMF, POF, OC) are presented in Tables 1–3. These results demonstrate the following conclusions:

1. The transformations provide a significant improvement in the recognition process compared with correlation results obtained without elementwise preprocessing (RGB case).
2. The method in Fig. 1(b) (addition of amplitudes) yields better performance in terms of DC than that of the method in Fig. 1(a) (addition of intensities).

### Table 1. Correlation Peaks and DC Obtained with the CMF for the Cases of Addition of Amplitudes and Addition of Intensities

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Type of Correlation</th>
<th>Test Color Image</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B1</td>
<td>B2</td>
</tr>
<tr>
<td>RGB</td>
<td>A(^{a})</td>
<td>1.97</td>
</tr>
<tr>
<td></td>
<td>I(^{b})</td>
<td>0.66</td>
</tr>
<tr>
<td>CNT</td>
<td>A</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>0.03</td>
</tr>
<tr>
<td>CNT–N</td>
<td>A</td>
<td>2.08</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>0.91</td>
</tr>
<tr>
<td>CW</td>
<td>A</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>0.04</td>
</tr>
<tr>
<td>CW–N</td>
<td>A</td>
<td>3.14</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>1.07</td>
</tr>
<tr>
<td>QSVD</td>
<td>A</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>0.66</td>
</tr>
<tr>
<td>QSVD–N</td>
<td>A</td>
<td>2.40</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>1.40</td>
</tr>
</tbody>
</table>

\(^{a}\) Amplitude correlation.  
\(^{b}\) Intensity correlation.  
\(^{f}\) Target image.

### Table 2. Correlation Peaks and DC Obtained with the POF for the Cases of Addition of Amplitudes and Addition of Intensities

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Type of Correlation</th>
<th>Test Color Image</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B1</td>
<td>B2</td>
</tr>
<tr>
<td>RGB</td>
<td>A(^{a})</td>
<td>688</td>
</tr>
<tr>
<td></td>
<td>I(^{b})</td>
<td>230</td>
</tr>
<tr>
<td>CNT</td>
<td>A</td>
<td>4.65</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>2.39</td>
</tr>
<tr>
<td>CNT–N</td>
<td>A</td>
<td>225</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>84.4</td>
</tr>
<tr>
<td>CW</td>
<td>A</td>
<td>8.55</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>3.04</td>
</tr>
<tr>
<td>CW–N</td>
<td>A</td>
<td>410</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>151</td>
</tr>
<tr>
<td>QSVD</td>
<td>A</td>
<td>113</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>81.6</td>
</tr>
<tr>
<td>QSVD–N</td>
<td>A</td>
<td>125</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>82.2</td>
</tr>
</tbody>
</table>

\(^{a}\) Amplitude correlation.  
\(^{b}\) Intensity correlation.  
\(^{f}\) Target image.

### Table 3. Correlation Peaks and DC Obtained with the OC for the Cases of Addition of Amplitudes and Addition of Intensities

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Type of Correlation</th>
<th>Test Color Image</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B1</td>
<td>B2</td>
</tr>
<tr>
<td>RGB</td>
<td>A(^{a})</td>
<td>0.0225</td>
</tr>
<tr>
<td></td>
<td>I(^{b})</td>
<td>0.0106</td>
</tr>
<tr>
<td>CNT</td>
<td>A</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>0.0001</td>
</tr>
<tr>
<td>CNT–N</td>
<td>A</td>
<td>0.0220</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>0.0092</td>
</tr>
<tr>
<td>CW</td>
<td>A</td>
<td>0.0006</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>0.0003</td>
</tr>
<tr>
<td>CW–N</td>
<td>A</td>
<td>0.0538</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>0.0248</td>
</tr>
<tr>
<td>QSVD</td>
<td>A</td>
<td>0.0061</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>0.0047</td>
</tr>
<tr>
<td>QSVD–N</td>
<td>A</td>
<td>0.0144</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>0.0121</td>
</tr>
</tbody>
</table>

\(^{a}\) Amplitude correlation.  
\(^{b}\) Intensity correlation.  
\(^{f}\) Target image.

3. Comparing the three filters (CMF, POF, OC), we see that the OC is the best one with respect to DC. As one would expect, POF yields the highest magnitudes in the unified correlation plane.

4. For the test color input scene, the best results in terms of DC are obtained by applying the QSVD–N preprocessing for all filters (CMF, POF, OC).

As an example, optical experiments are performed with the QSVD–N preprocessed input scene and with the POF.
These optical experiments were performed with a convergent correlator that uses two SLM's, one to introduce the input scene and the other to introduce the POF, as sketched in Fig. 6. These SLM’s are twisted nematic liquid crystal panels from an Epson videoprojector, model VP-100PS. Under proper conditions of applied voltage and input polarization, these kinds of modulator can produce amplitude-only or phase-mostly modulations. Two half-wave plates, HWP1 and HWP2, are placed in front of each SLM for convenient rotation of the plane of vibration of the linearly polarized incident light. Phase-only distribution of the POF’s is displayed by SLM2 working in the phase-mostly configuration. The result of applying the proposed transformations to the RGB signal is real-valued images with positive and negative values. The optical implementation of these real-valued images requires a SLM that produces real-valued modulation. To introduce this kind of input scene, we used two approximations of the real-valued signal. The first one is the absolute value of the input signal. The result is real positive values that can be introduced by SLM1 working in the amplitude-only configuration. A linear polarizer, POL, is needed behind SLM1 to produce the different lev-

Fig. 6. Scheme of the optical convergent correlator with two SLM’s: HWP, half-wave plate; POL, polarizer; L, lens.

Fig. 7. Correlation intensity distributions obtained with a QSVD–N preprocessed color scene and with a POF. (a) Absolute-valued input signal, computer simulation; (b) absolute-valued input signal, optical experiment; (c) phase-encoded input signal, computer simulation; (d) phase-encoded input signal, optical experiment.
els of transmission. The second approximation consists of performing a transformation from the real-valued input signal to a phase-only input signal with phase range from 0 to $\pi$. The result is a phase-only distribution that can be introduced by SLM1 working in the phase-mostly configuration. We have done computer simulations for the case of a real-valued input signal and for these two approximations. The POF displayed by SLM2 is matched to the target image transformed with the same transformations (absolute value or phase value) as the input scene. We have observed that both approximations maintain the improvement in recognition, owing to the proposed transformations with respect to the original RGB components. Computer simulation and experimental results corresponding to the absolute-valued input signal are shown in Figs. 7(a) and 7(b), respectively. A very good accordance between experimental and simulated results is obtained. Figures 7(c) and 7(d) show computer simulation and the optical results for the case of a phase-only-valued input signal. Again a very good accordance is obtained. In both cases, a good recognition of the correct target is obtained.

Next, the robustness of DC for the systems shown in Fig. 1 is investigated. In the same way as in Subsection 5.A, we corrupt the input color image in Fig. 4 with both white and colored Gaussian noise with zero mean and different variances. For each input noise parameter used here, 30 statistical trials are conducted. The magnitudes of DC computed over the resulting correlation plane $\text{OUT}_T(x, y)$ after each statistical trial are averaged. The transformations are tested with the correlation filters (CMF, POF, OC). Discrimination capability versus the standard deviation of input noise for various transformations is presented in Fig. 8. Only the example with white noise is presented. In Fig. 8(a) the curve with the CMF and with the intensity addition is not presented because DC for this case is always negative, which indicates a poor recognition capability. The results in Fig. 8 show that

1. DC for all transformations and for all filters decreases when SNR$_T < 1$ and the input noise variance increases.
2. As one would expect, the CMF is the best filter in terms of noise robustness compared with the POF and the OC. The curves of DC with the CMF tend to zero much more slowly than those with the POF and the OC.
3. DC’s both with addition of amplitudes and with addition of intensities have identical behavior with respect to noise robustness.
4. It is interesting to note that for the transformations with normalization (CNT–N, CW–N), DC increases when the input noise variance is small. It appears that with these transformations a low-level input noise helps to enhance dissimilarities between the objects to be rejected and the target while preserving low dissimilarities between noiseless and noisy targets.
5. Table 4 presents an example of color pattern recognition with a threshold for DC of 0.5 and with SNR of the input scene (SNR is defined as the ratio between the

<table>
<thead>
<tr>
<th>Filter</th>
<th>CMF</th>
<th>POF</th>
<th>OC</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>I</td>
<td>A</td>
<td>I</td>
</tr>
<tr>
<td>RGB</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>CNT</td>
<td>$+$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
<tr>
<td>CNT–N</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
<tr>
<td>CW</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
<tr>
<td>CW–N</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
<tr>
<td>QSVD</td>
<td>$+$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
<tr>
<td>QSVD–N</td>
<td>$+$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
</tbody>
</table>

*A plus sign indicates a correct recognition; a minus sign indicates a false alarm. A stands for amplitude, I for intensity.
ACKNOWLEDGMENTS

This research was financed by Comision Interministerial de Ciencia y Tecnología project TAP96-1015-C03-01 and Comissió Interministerial Recerca i Tecnologia project 1995SGR-00587. V. Kober acknowledges financial support from Generalitat de Catalunya. V. Lashin and I. Moreno acknowledge financial support from the Ministry of Education and Science of Spain.

The authors’ e-mail address is ifop1@cc.uab.es.

"Permanent address, Institute of Information Transmission Problems of the Russian Academy of Science, 19 Yermolovoy Street Moscow 101447, Russia.

REFERENCES