NONLINEAR OPTICAL CORRELATORS

WITH IMPROVED DISCRIMINATION CAPABILITY

FOR OBJECT LOCATION AND RECOGNITION

L. P. Yaroslavsky$^1$
Department Of Interdisciplinary Studies
Faculty of Engineering, Tel Aviv University
Tel Aviv, 69978,
Israel

$^1$ On leave from the Institute of Information Transmission Problems, Russian Academy of Sciences, Moscow, Bolshoy Karetny, 19, Russia
1. INTRODUCTION. A REVIEW OF THE THEORY

In this paper, we discuss the synthesis of optical correlators for object location and recognition. In object (target) location, it is required to determine the position (coordinates) of a small target object on an observed image that, generally, contains this object surrounded by a clutter of background objects and image details that may camouflage the target object. The localization device has to locate the target object as accurately as possible with the lowest possible probability of false identification of the target object with one of the background objects. In object recognition, the observed image of an object has to be identified with a certain image from a set of template images. The recognition device has to be able to perform the identification with the lowest possible probability of misrecognition.

The capability of the localization and recognition devices to discriminate between the target object and false background objects (in target localization) or wrong template images (in object recognition) is called their discrimination capability. It is the discrimination capability of the localization and recognition devices that we will be concerned with in this paper.

As it is well known, the “gold standard” for the devices for localization or recognition of objects observed in a mixture with additive Gaussian noise is the combination of a matched filter and a unit for localizing the signal maximum at its output. However, in target location in images with a cluttered background or in recognition of images that can not be regarded as copies of one and the same images with a Gaussian noise added, this device is far from being optimal and has low discrimination capability.

This classical localization scheme with matched filtering can be regarded as a special case of localization and recognition devices which use, instead of the matched
filter, a linear filter whose frequency response is optimized for a particular task. The function of the linear filter in localization (recognition) devices is to transform the signal space in a way which enables decision making on the base of individual signal samples only rather than using the entire signal. Due to the separation into independent linear and point-wise nonlinear units, data analysis and implementation, both digital and optical, of such devices is much simplified. This motivates the use of this scheme in image processing and, in particular, for target location provided the filter is optimized to guarantee the highest possible discrimination capability.

The synthesis of such an optimal filter was outlined in [1]. It was shown that the requirement for the highest discrimination capability (or low probability of misrecognition) is practically equivalent to the requirement that the filter has to provide the maximal ratio of the signal value in the point of the output plane, where the target object is located, to the standard deviation of the signal in the background part of the output plane or, in the object recognition, to the standard deviation of the filter output for the entire set of objects to be rejected. Since a number of random factors, such as sensor’s noise, unknown reference object position, variability of its orientation, size, etc., are involved in the localization (recognition), this requirement should be satisfied on average over these factors. For the object location task, the frequency response of the optimal filter was found in [1] as

\[ H_{\text{opt}}(\bar{f}) = \frac{AV_y(\mathbf{T}^\ast(\bar{f}))}{AV_{\text{imsys}}AV_{x_0}\left(\mathbf{B}(\bar{f})\right)^2} \]  

(1a)
where \( \text{TO}^* (\hat{f}) \) is the complex conjugate of the target object Fourier spectrum, \( |B(\hat{f})|^2 \) is the power spectrum of the background component of the input image, and \( \text{AV}_o \), \( \text{AV}_{\text{imsys}} \), and \( \text{AV}_{\text{x}} \) denote, respectively, averaging of the corresponding variables over unknown parameters defining the variability of the target object signal, averaging over realizations of imaging system noise and averaging over unknown coordinates of the target. By the background image component we mean the part of the observed image outside the area occupied by the target object. The filter (1a) if realizable is adaptive and provides the best (in the class of linear filters) possible performance for the given individual input image. We will refer to it as the Optimal Adaptive Correlator (OPAC).

One can show that, for object recognition, a similar set of filters

\[
H_{\text{opt}}^{(i)} (\hat{f}) = \frac{\text{AV}_o \left( \text{TIM}_i^* (\hat{f}) \right)}{\text{AV}_{\text{set}} \left( \left| \text{TIM}_{k \neq i} (\hat{f}) \right|^2 \right)}
\]

(1b)
is optimal, where \( H_{\text{opt}}^{(i)} (\hat{f}) \) is the optimal filter for \( i \)-th template image, \( \text{TIM}_i^* (\hat{f}) \) is the complex conjugate of the \( i \)-th template image, and \( \text{AV}_{\text{set}} \left( \left| \text{TIM}_{k \neq i} (\hat{f}) \right|^2 \right) \) denotes averaging spectra of the set of the template images excluding \( i \)-th one. It is assumed here that the input image is subjected to filtering with the above filters, and the image identification is done by the selection of the template image whose produces the highest output. Because of the obvious similarity between optimal filters for object location and recognition, we will confine ourselves with the discussion of former. The extension of the results to the latter is straightforward.
The design of the OPAC requires knowledge of the power spectrum of the background objects averaged over (random) coordinates of the target object. These data are not known before the target is located and have to be determined from the observed input image.

As a zero order approximation of the background power spectrum one may use squared module of the entire observed image spectrum:

\[ AV_{\text{sys}} AV_{\text{sys}} |B(\tilde{f})|^2 \approx |IM(\tilde{f})|^2 \]  

(2)

This approximation is based on an assumption that the object size is much smaller than the size of the entire image (area of search).

For more accurate estimation of the background power spectrum, one can use the following two models for the representation of the image background component:

\[ b(\tilde{x}) = w(\tilde{x} - \tilde{x}_0)im(\tilde{x}) \],  

(3)

and

\[ b(\tilde{x}) = im(\tilde{x}) - to(\tilde{x} - \tilde{x}_0) \]  

(4)

where \( im(\tilde{x}) \) is the observed image, \( b(\tilde{x}) \) is its background component, \( w(\tilde{x} - \tilde{x}_0) \) is a window function:

\[ w(\tilde{x} - \tilde{x}_0) = \begin{cases} 0, & \text{within the target object} \\ 1, & \text{elsewhere} \end{cases} \],  

(5)

\( \tilde{x}_0 \) is the target coordinate, and \( to(\tilde{x} - \tilde{x}_0) \) is the target object.

For the model (3), one can show that, in the assumption of uniform a priori distribution of the object coordinates \( \tilde{x}_0 \) over the picture area \( S \),
\[ AV_{imsy} AV_{x_0} |B(\tilde{f})|^2 \approx AV_{imsy} \left\{ |IM(\tilde{f})|^2 \cdot |W(\tilde{f})|^2 / S \right\} \]  

(6)

where \( \cdot \) denotes convolution and \( |W(\tilde{f})|^2 \) is squared magnitude of the window function Fourier spectrum.

For the model (4), one can show that, in the same assumption of the uniform distribution of the object coordinates \( \tilde{x}_0 \) over the picture area \( S \),

\[ AV_{imsy} AV_{x_0} |B(\tilde{f})|^2 \approx AV_{imsy} \left\{ |IM(\tilde{f})|^2 + |TO(\tilde{f})|^2 \right\}. \]  

(7)

As for the variability of the target object, there are two options to account for it. First, one can subdivide the range of the variations into segments small enough to secure that the variations within each segment are negligible and then design the filter for each individual version of the object. The price of this is the increase of computational costs.

Second, one can perform object averaging over the range of its variations and design a single filter for such an averaged object. Localization and recognition in this case is computationally much less costly but the process obviously might have a lower discrimination capability (higher probability of misrecognition).

In the object localization, there might be also one more cause for the object variability that has to be taken into account. This is the variability due to the background component of the image. In some applications, one may regard the object as being "cut in" into the background such that the target object signal component in the observed image can be described as

\[ to(x - x_0) = to_0(x - x_0) - \overline{w}(x - \tilde{x}_0)im(x), \]  

(8)

where \( \overline{w}(x - x_0) \) is a window function complement to the above function \( w(x - x_0) \):
\[ w(x - x_0) = \begin{cases} 1, & \text{within the target object} \\ 0, & \text{elsewhere} \end{cases} \]  \quad (9)

and \( t_{o_0}(x) \) is the “pure” target object signal that is “cut in” in the image (it is assumed zero outside the target object). In this representation, the variations of the observed target object are defined by the term \( w(x - x_0) im(\bar{x}) \). Therefore,

\[ AV_{ob}[t_0(x)] = AV_{x_0}(o(x)) = t_{o_0}(x) - AV_{x_0}(w(x)im(x - x_0)), \quad (10) \]

or, in the spectral domain:

\[ AV_{x_0}(TO(f)) = TO_0(f) - AV_{x_0}\left([IM(f)\exp(i2\pi fx_0)]\ast w(f)\right) = \]

\[ = TO_0(\hat{f}) - \hat{W}(\hat{f})\ast\left(IM(\hat{f})\cdot CF(\hat{f})\right), \quad (11) \]

where \( CF(f) \) is the characteristic function (Fourier transform) of the distribution density of the object coordinate \( x_0 \) and \( \ast \) designates convolution. If \( x_0 \) is uniformly distributed over an area of search \( S \) that is much larger than the object size, its distribution characteristic function \( CF(f) \) can be approximated by the delta-function, divided by \( S \), and therefore

\[ AV_{x_0}(TO(f)) \approx TO_0(f) - \hat{W}(f)\cdot IM(0)/S = TO_0(f) - \hat{W}(f)\cdot \bar{im}, \quad (12) \]

where \( \bar{im} \) is the arithmetic mean over the background image component.

With estimations (6,7) of the background power spectrum, the optimal filter can be implemented either as

\[ H_{opt}(\hat{f}) \propto \frac{AV_{ob}(TO^*(\hat{f}))}{AV_{imsys}[IM(\hat{f})]\cdot |\hat{W}(\hat{f})|^2} \quad (13) \]

or as
\[
H_{app}(\hat{j}) \propto \frac{AV_{ab}(TO^*(\hat{j}))}{AV_{imsys}|IM(\hat{j})|^2 + |TO(\hat{j})|^2}
\]  

(14)

with the appropriate selection for \( AV_{ab}(TO^*(\hat{j})) \) as was described above.

2. NONLINEAR OPTICAL CORRELATORS

One can implement described adaptive filters in nonlinear optical correlators. Two types of nonlinear optical correlators are regarded now most feasible: coherent optical correlators with a non-linear light sensitive optical light modulator installed in the correlator’s Fourier plane ([1], Fig. 1) and Joint Transform correlators (JTC, [2-6]) with input images joint spectrum non-linearly transformed in a computer or in an electronic amplifier before modulating the output spatial light modulator (Fig. 2). In the setup of Fig. 1, a spatial light sensitive nonlinear light modulator plays a role of the non-linear element. Its transparency in each point is controlled by and, therefore, depends on the input image power spectrum energy in this point.

In the setup of Fig. 2, an input image and a target object put side by side at the input plane are jointly Fourier transformed and their joint power spectrum \( |IM(\hat{j}) + TO(\hat{j})|^2 \) is read out by a video-camera and nonlinearly transformed point-wise in a nonlinear amplifier. The amplifier output is recorded on a spatial light modulator and Fourier transformed by the second lens to produce an output signal representing the input image-to-reference object correlation signal in each one of its side bands. The position of the reference object in the input image is indicated by the position of the correlation signal highest peak.
The design and implementation of such correlators require answering a number of practical questions such as

- what type of the nonlinear transformation one should use in the above nonlinear correlators to ensure the highest discrimination capability;
- how sensitive is the correlator’s discrimination capability to such design factors as the accuracy of realization of the nonlinear transformation, the limitation of the dynamic range of the nonlinear optical media and electronic components used, the accuracy of optical alignment.

Several other modifications of the nonlinear optical are also known. Among them, Phase-only filters (POF) and Phase-only correlators (POC) are most popular due to mainly their high light efficiency and relatively simple implementation ([6-9]). It is very instructive to compare them with the above mentioned implementations of the optimal correlator.

In the present paper, we will review recent experimental results that address the above mentioned issues. In Sect. 3, we present the results for correlators of Fig. 1 with \((-k)\)th law nonlinearity in their Fourier plane and compare their discrimination capability with that of the conventional matched filter, the Phase-only filter and the Phase-only correlator. In Sect. 4, we justify using a logarithmic or \((1/k)\)th law nonlinearity in JTCs and present corresponding experimental results. In conclusion, we summarize the discussion.

2. NONLINEAR OPTICAL CORRELATORS WITH \((-k)\)th LAW NONLINEARITY IN THE FOURIER PLANE ([11])
Optical correlators with a nonlinearity applied to the input image spectrum and/or to the target object spectrum have attracted considerable attention of researchers looking for ways to improve the performance of optical correlators in pattern recognition and target location. An important subclass of such correlators is optical correlators with \((-k)\)th law nonlinearity, which is described by the relationship:

\[
Out = \left(\left|In\right|^2\right)^{-k}, \quad (15)
\]

where \(In\) and \(Out\) are input and output signals respectively of the nonlinear transformation. According to the theory, one may expect that such correlators with the nonlinearity in their Fourier plane as shown in Fig. 1 and \(k = 1\) will approximate the OPAC (1). Computer and optical experiments ([11-13]) have shown that such correlators do considerably outperform the classical matched filter in terms of the discrimination capability. Here, we present the results of an investigation into the correlator’s sensitivity to the nonlinearity index \(k\) and the limitation of the nonlinearity’s dinamic range, and into to the method of estimation of the power spectrum of the background component of the input image. We also compare the correlator’s discrimination capability to that of the Phase-only filters and Phase-only correlators.

The simulation was carried out in the computer model that has been designed to implement the following signal processing scheme:

\[
c = \left|IFT\left(IM \cdot LDR\left(CONV(h^{[m]},\left[|I|^2 + e|TO|^2\right])\right)^{-k}\right)\left(|TO|^2\right)^{-n} TO^*\right|^2 \quad (16)
\]

where \(c\) is the correlator output signal, \(IM\) is the input image Fourier spectrum, \(TO^*\) is the complex conjugate of the target object Fourier spectrum, \(IFT\) is the inverse Fourier
transform operator, $CONV_{h_{bl}}(\cdot)$ is a $bl$-fold linear convolution operator with point spread function $h$, and $LDR$ is a point-wise nonlinear operator:

$$LDR(x) = \begin{cases} x, & x < \text{Lim} \\ \text{Lim}, & \text{otherwise} \end{cases} \tag{17}$$

The model parameters $bl, e, k, n,$ and $\text{Lim}$ define different processing modes. The parameter $bl$ defines the degree of spectrum linear smoothing, or blur that is assumed in the method of Eq.(7) of background image component spectrum estimation. When $bl=0$, no signal smoothing is performed. In the setup of Fig. 1, the convolution (6) can be associated with placement of the nonlinear spatial light modulator slightly out of the lens focus or with its low resolution power. The parameter $e$ takes one of two values, 0 or 1, and determines whether the target object's power spectrum is used to modify the observed image power spectrum (Eq.(8)). The parameter $k$ is the index of the nonlinearity. It defines the nonlinear transformation applied to the estimation of the observed input image's power spectrum. The parameter $n$ defines nonlinear transformation applied to the target object's power spectrum. It was used to simulate the phase-only filter and phase-only correlator ($n=1/2$). The parameter $\text{Lim}$ defines the degree of the dynamic range limitation.

The model covers a broad variety of nonlinear correlators. Some specific cases are shown in Table 1. By "Suboptimal Correlators" we mean in this table the family of correlators that uses an estimation of the image background component power spectrum rather than the exact power spectrum, which has to be used in the optimal correlator design.
In the experiments, a range of values for the nonlinearity index $k$ has been tested. For the exponent $n$, only those cases in which $n=0$ (matched filter) and $n=1/2$ (Phase-Only Filter) were tested. As it was mentioned above, the convolution operator was implemented as a $bl$-fold repetition ($bl=0, 1, \ldots, 7$) of the elementary blur operator with a circular symmetric frequency response:

$$H(f) = \exp\left(-f^2 / 0.1\right)$$  \hspace{1cm} (18)

where $f \in [0,1]$ is spatial frequency normalized to the correlator's bandwidth. This function approximates the power spectrum of the target window function required by formula (13) for the estimation of the image background component power spectrum according to Eq. (6).

In the experiments with limitation of the nonlinear material dynamic range, the parameter $\text{Lim}$ was selected as a $(1/4 \lim)^{th}$ fraction of the signal maximum with $\lim=0,1,\ldots,7$. Thus, the case in which $\lim=0$ corresponded to no dynamic range limitation, and the case in which $\lim=7$ corresponded to a limitation at the level of $(1/16384)^{th}$ of the signal maximal value.

The experiments were conducted using sixteen 128x128-pixel fragments of a 512x512-pixel satellite photograph of an urban area as test images (Fig. 3) and a small circular spot with a diameter of about 5 pixels as the target object embedded within the test images. The optical Fourier transform was approximated by the 2-D Discrete Fourier Transform. Two performance measures of the correlator's discrimination capability were computed in each experiment: the ratio of the object signal maximum to the signal standard deviation over the background area in the correlation plane, or the signal-to-noise ratio in terms of the background-signal variance (SNRV), and the ratio of the
highest object signal maximum to the maximum over the background area in the correlation plane, or the signal-to-noise ratio in terms of the background-signal maximum (SNRM). The former connects the experimental results with the analysis presented in [1], whereas the latter is a more adequate indicator of the correlator's ability to reliably locate a target on a cluttered background. In order to compare the discrimination capability of suboptimal correlators with the potential discrimination capability offered by the Optimal Adaptive Correlator, the latter has also been implemented in the model. In this case, to allow exact estimation of the image power spectra, the target object was not embedded in the test images.

3.1 Optimal Adaptive Correlator

The results for Optimal Adaptive Correlators designed individually for each image from the set of test images are represented in Figs. 4 a)-c), along with those for the conventional correlator (matched filter). Fig. 4 a) shows that the SNRV values at the output of the conventional correlator are almost uniformly distributed in the broad range from about 0.3 to 3 for the set of test images. This distribution allows us to suggest that the selected set of images is representative enough. Values for the SNRV and SNRM as functions of the nonlinearity index \( k \) obtained for all of the test images are plotted in Figs. 4 b) and c), respectively. The plots clearly evidence that:

- The nonlinear correlators with the matched filter and with \((-k)\)th nonlinearity in the Fourier plane can potentially outperform the conventional correlator considerably in terms of the discrimination capability. The observed gain is in the range from about 20 to more than 200 for SNRV and from about 15 to more than 100 for SNRM.
Although the optimal value of $k$ in terms of the SNRV values is equal to one, the optimal value of $k$ in terms of the SNRM tends to be slightly higher; on the average, it is approximately 10% higher.

It is remarkable that SNRM decays relatively slow for values of $k$ higher than the optimal value. Observations show that for $k > 1$ the correlator's discrimination is basically due to very few frequency components of the signals, that is, the optimal filter acts as a bandpass filter with a very narrow bandwidth.

3.2 Suboptimal correlators with $(-k)$th law nonlinearity and empirical estimation of the image power spectrum

As mentioned above, two estimation methods of estimating the background image spectrum have been studied for correlators with $(-k)$th law nonlinearity: LS-method ($e=0$ in Eq. (6)), and ATS-method ($e=1$ in Eq. (7)). Experimental SNRV and SNRM values for these methods are plotted in Figs. 5 (a) through (c) for a typical image from the test set. Averaged data for the LS method and two values of the smoothing parameter $bl$ are plotted in Fig. 6 (a) and (b) to illustrate the gain in the correlator's discrimination capability obtained by the spectrum smoothing ($bl=4$ corresponds to the highest average gain over the set of test images), compared to the direct use of the observed image spectrum as an estimation of the background image power spectrum. Averaged data for the entire set of images and for the optimal value of the smoothing parameter $bl=4$ are plotted in Fig. 7 a) and b). These results suggest the following conclusions:

- Suboptimal nonlinear correlators provide significant (on the average, about 20-fold for SNRV and eightfold for SNRM) improvement of the discrimination capability
compared with that of the conventional matched filter (a vivid comparison of output signals is presented in Fig. 8). A twofold to fivefold gap still remains between the discrimination capability of the suboptimal correlators and that potentially achievable for an exactly known power spectrum of the background-image component.

estimation improves the nonlinear correlator's discrimination capability considerably.

- The LS method outperforms the ATS-method of spectrum estimation if the degree of image spectrum smoothing is properly selected; the ATS method is much less sensitive to the degree of image spectrum smoothing, and provides considerable improvement even without the spectrum smoothing.

- The optimal value of the nonlinearity index $k$ is close to one for the correlators' performance evaluation in terms of both SNRV and SNRM; small deviations of $k$ from its optimal value are not very critical.

- Smoothing the observed image power spectrum as a method of power spectrum. A considerable degree of spectrum smoothing is required for achieving better discrimination capability; the optimal degree of smoothing corresponds to the resolution power of the nonlinear light modulator, about 5-10 times lower than that required for the image. One can interpret this fact as an indication that the resolution power (or the square root of the number of degrees of freedom) of the nonlinear media should be of the order of magnitude of $(1/\sqrt{TGTSIZE})$-th fraction of the required resolution power in the image domain, where $TGTSIZE$ is the number of pixels (resolution cells) in the target object. Obviously, this may considerably simplify the correlator’s optical design and alignment.
Along with the nonlinearity’s resolution power, the limitation of the nonlinearity's dynamic range is another important issue in designing nonlinear correlators. For the Optimal Correlator, one would expect that the limitation of the nonlinearity's dynamic range deteriorates the correlator's discrimination capability. The simulation has shown that, while this deterioration does occur, though it is not too severe (Fig. 9).

For the suboptimal correlator with the LS method of spectrum estimation, the experiments have shown that the dynamic range limitation does not necessarily deteriorate the correlator’s discrimination capability. Moreover, a sort of trade-off is possible between the degree of the dynamic range limitation and the nonlinearity index $k$: the higher the limitation, the higher nonlinearity index required for better discrimination capability. This is illustrated in Figs. 10 (a) and (b) which show that an appropriate choice of the nonlinearity index $k$ provides practically the same discrimination capability as that of the nonlinear correlator with an unlimited dynamic range. It is remarkable that the dynamic range limitation may even improve, though slightly, the discrimination capability of the correlator without the dynamic range limitation. This can be attributed to the fact that power spectrum estimation by spectrum linear smoothing is not perfect enough.

Correlators with a strongly restricted dynamic range have higher light efficiency than do correlators with no restrictions, because, according to Eq. 17, the nonlinear material can be made entirely transparent in the spectral plane wherever the signal that modulates it exceeds the limitation threshold. Therefore, it might be advisable to use the observed trade-off between the nonlinearity index and the degree of dynamic range
limitation to improve the correlator's light efficiency while preserving its discrimination capability.

3.3 Phase-Only Filter and Phase-Only Correlators

The use of phase-only filters instead of matched filters in optical correlators is advocated by the high light efficiency of phase-only filters. However, the discrimination capability of the correlators with phase-only filters is significantly lower than that of the optimal and suboptimal nonlinear correlators ([14]). The corresponding graphs for SNRV and SNRM for the nonlinear correlators with the phase-only filter representing the reference object are given in Figs. 11 for the same test image as in Fig. 5. One can see from Figs. 11 that the optimal nonlinearity index \( k \) in this case is about \( k=0.5 \), which corresponds to the phase-only-correlator. The graphs also show that spectrum smoothing performed before its nonlinear transformation considerably improves the discrimination capability of the phase-only correlators, although their discrimination capability remains lower than that of the suboptimal nonlinear correlators with a matched filter representing the target object. A comprehensive comparison of all the correlators described is presented in Fig. 12. These plots demonstrate a hierarchy of correlators with \(-k\)th low nonlinearity in terms of their discrimination capability.

4. NONLINEAR JOINT TRANSFORM CORRELATORS

Nonlinear Joint Transform correlators (NLJTCs) have been proposed as a tool for real time pattern recognition and different modifications of NLJTCs have been studied [2-6]. We will address here the issues of optimization of NLJTCs’ nonlinearity and the
sensitivity of their discrimination capability to optical misalignments and to the dynamic range and resolution power limitations of nonlinear spatial light modulators that can be used in NLJTCs. We show that NLJTCs with a logarithmic nonlinearity approximate the Optimal Adaptive Correlator. By means of computer simulation, we show also that the NLJTC’s with a \((1/k)\)th law nonlinearity within a limited dynamic range and the binary JTC may exhibit similar discrimination capability provided appropriate selection of the nonlinearity index \(k\), dynamic range limitation threshold and the binarization threshold, respectively.

4.1 Logarithmic Joint Transform Correlators

In this section, we justify the use of logarithmic nonlinearity in Nonlinear Joint Transform Correlators. With the estimate (7), signal filtering in suboptimal adaptive correlator is described by the formula:

\[
OUT(\hat{f}) = \frac{IM(\hat{f}) \cdot TO^*(\hat{f})}{AV_{imsys}\left(\left|IM(\bar{f})\right|^2 + \left|TO(\bar{f})\right|^2\right)}.
\]  

(similar filtering schemes were also discussed, from different assumptions, in [16-18]).

Let \(\Phi(\cdot)\) be a point-wise nonlinear transformation. If

\[\Phi(\cdot) = \log(\cdot),\]

the transformed joint power spectrum \(OUT_{NLJTC}\) at output of this nonlinear device can be written as

\[
OUT_{NLJTC}(\hat{f}) = \log\left|IM(\hat{f}) + TO(\hat{f})\right|^2 =
\]

\[
= \log\left(\left|IM(\hat{f})\right|^2 + \left|TO(\hat{f})\right|^2 + IM(\hat{f}) \cdot TO^*(\hat{f}) + IM^*(\hat{f}) \cdot TO(\hat{f})\right).
\]
In target location, the size of the reference object is usually much smaller than the size of the input image. Therefore, for the majority of the spectral components

$$|IM(f)|^2 + |TO(f)|^2 \gg |IM(f)| \cdot |TO(f)|.$$  \hfill (22)

With this assumption, $OUT_{NLJTC}$ is approximately equal to

$$OUT_{NLJTC}(\hat{f}) \approx \log \left( |IM(\hat{f})|^2 + |RO(\hat{f})|^2 \right)$$

$$+ \frac{IM^*(\hat{f}) \cdot TO(\hat{f})}{|IM(\hat{f})|^2 + |TO(\hat{f})|^2} + \frac{IM(\hat{f}) \cdot TO^*(\hat{f})}{|IM(\hat{f})|^2 + |TO(\hat{f})|^2}.$$  \hfill (23)

In a JTC configuration, the two last terms displaying the correlation function are readily separated. The last term of this expression reproduces the expression (19) that corresponds to the optimal adaptive correlator with estimation of the background image component power spectrum by Eq. (8) but without averaging $AV_{imsys}$. Therefore, one can conclude that the logarithmic nonlinearity in the nonlinear Joint transform correlator promises a reasonably approximation to the optimal adaptive correlator and therefore promises an improved discrimination capability. Note that the averaging $AV_{imsys}$ can be implemented by a kind of smoothing (for instance, by a linear blur) of the involved signal.

In order to verify this conclusion, computer simulation experiments were conducted using the above set of test images and the target object. Arrangement of the input images and the target object which was used as an input for the JTC is shown in Fig. 13. Pair-wise arrangement of test images allowed experiments with two images of the set made in parallel. In order to reduce boundary effects, the input images and the test image were “inscribed” into a uniform background with the gray level equal to average
gray level of the images. Experiments with this set of test images were aimed at investigation of the discrimination capability of adaptive nonlinear correlators in images with the same target and different background. In order to verify the results obtained we also performed an experiment with stereoscopic images (Fig. 14). In this case, the target object was a 21x21 pixel fragment of one image (shown in the box in Fig. 14) and the test image in which this fragment was to be located was the second image.

An important practical issue in the design of JTC’s of Fig. 2 is the required resolution power of the TV camera that reads out the joint spectrum. It is well known that the space-bandwidth product of optical system is usually much higher than that of electronic imaging devices such as TV cameras. This casts a restriction on the space-bandwidth product of NLJTCs. From the other hand, the estimation of the background image power spectrum required for the implementation of OPAC according to Eq.(7) assumes the necessity of averaging \( AV_{\text{imsys}} \) that can be implemented as smoothing by a convolution with an appropriate window function as it was mentioned above. Experiments reported in Sect. 3 evidence that the appropriate spectrum smoothing substantially improve the nonlinear correlators’ discrimination capability. In the setup of Fig. 2, a limited resolution power of the TV camera causes smoothing the joint spectrum. Averaging the joint spectrum image power spectrum \( |IM(\mathbf{f}) + TO(\mathbf{f})|^2 \) by convolution with a point spread function of the TV camera results in smoothing both \( \left( |IM(\mathbf{f})|^2 + |TO(\mathbf{f})|^2 \right) \) and \( IM(\mathbf{f})TO^*(\mathbf{f}) \) terms in Eq.(11). While the former smoothing is what one needs for the spectrum estimation and may increase the correlator’s discrimination capability, the latter may have a negative effect. Therefore,
one may expect that there exist an optimal smoothing degree. The investigation into this issue was also included in the computer simulation.

The computer model implemented the following signal transformation:

\[
\text{coroutput} = \left| \text{IFT} \left\{ \text{CONV} \left\{ h^{\text{blr}}, \text{LDR} \left[ \log \left( \text{FT} (\text{corinput}) \right) \right] \right\} \right\} \right|^2
\]  

(24).

Here, \text{coroutput} and \text{corinput} are, respectively, output and input images of the correlator, \text{FT} and \text{IFT} are direct and inverse Discrete Fourier Transforms that were used as approximations to optical Fourier Transform, \text{LDR} is the dynamic range limitation transformation (17), \text{CONV} \left\{ h^{(\text{blr})}, \cdot \right\} is a \text{blr}-fold convolution operator with a point-spread function \text{h}. Frequency responses of the convolution operator for the parameter \text{blr} = 0,1,\ldots,11 are shown in Fig. 15.

Parameter \text{Lim} of the dynamic range limitation was selected as a fraction \(0.1^{\text{lim}-1}\)th fraction of the signal maximum with \text{lim} = 1, 2, ..., 12. Thus, the case in which \text{lim}=1 corresponded to no dynamic range limitation, and the case in which \text{lim}=12 corresponded to limitation at the level of \((10^{-11})\)th of the signal maximal value.

The same two performance measures of the correlator's discrimination capability as for the above correlators with \((-k)\)th law nonlinearity, SNRV and SNRM, were computed in each experiment.

The experimental results are presented in Figs. 16 and 17. Plots in Fig. 16 represent average values of SNRV and SNRM for the logarithmic JTC as functions of the dynamic range limitation parameter \text{lim}, averaging been made over the set of test images. They show that the logarithmic JTC is not very sensitive to the limitations up to \((10^{-7})\) of
the entire dynamic range of the joint spectrum. Similarly averaged plots in Fig. 17 show how the discrimination capability of the logarithmic JTC depends on the blur of the joint spectrum. One can observe optimum in the degree of blur that tells that though the expected gain in the correlator’s discrimination capability due to the optimal joint spectrum smoothing is not very high, the discrimination capability remains high in a rather broad range of the degree of smoothing. Note that the optimum in SNRM is less pronounced than that in SNRV. In two of 16 test images no optimum was observed, and SNRM monotonically though very slowly decreased with the increase of the blur parameter \( blr \). One can conclude from these data that the requirement to the resolution power of the TV camera are not very critical: the spatial bandwidth of the camera may be 1.5-2 times less than that of the optics without noticeable losses in the correlator’s discrimination capability.

4.2 NLJTCs with \((1/k)\)th law nonlinearity

The distinctive feature of the logarithmic signal transform is that it compresses the signal's dynamic range. Similar compression can be also achieved by \((1/k)\)th law nonlinearity

\[
\Phi(\cdot) = (\cdot)^{1/k},
\]

when \( k \gg 1 \). This similarity is illustrated by Fig. 18 where logarithmic and \((1/k)\)th law nonlinearities are plotted together after a corresponding normalization by a constant. Therefore, one can expect that nonlinear JTCs with \((1/k)\)th law nonlinearity (25) and \( k \gg 1 \) will perform nearly as good as the logarithmic JTC.

The simulation experiments with NLJTCs with \((1/k)\)th law nonlinearity were carried out with the same computer model as that for the logarithmic nonlinearity except
the logarithmic transformation was substituted by the \((1/k)\)th law transformation and spectrum smoothing was not applied:

\[
\text{coroutput} = \left| \text{IFT} \left\{ \text{LDR} \left[ \left( \text{IFT}(\text{corinput}) \right)^{-k} \right] \right\} \right|^2
\]  

(26)

The corresponding averaged experimental data are plotted in Fig. 19, a) and b) for SNRV and SNRM, respectively, as functions of the nonlinearity index \(k\) for the dynamic range limitation parameter \(\text{lim} = 1, \ldots, 10\). Similar results were obtained for stereoscopic test images (Fig. 19 c). They show that

- NLJTCs with \((1/k)\)law nonlinearity may have considerably improved discrimination capability in comparison with that of JTC without nonlinear transformation of the joint spectrum (case \(k=1\)). In terms of SNRM, the averaged gain exceeds 3 times. Comparison of the corresponding data for the NLJTCs with \((1/k)\)law nonlinearity and the logarithmic JTC shows that, with an appropriate selection of the parameters \(k\) and \(\text{lim}\), the former performs slightly better.

- As for the logarithmic JTC, the discrimination capability of the NLJTC with \((1/k)\) law nonlinearity is not very sensitive to the limitation threshold provided proper selection of the nonlinearity index \(k>>1\).

- A trade-off exists between the nonlinearity index \(k\) and the dynamic range limitation parameter \(\text{lim}\): higher degree of the dynamic range limitation requires lower values of \(k\). With this trade-off, the discrimination capability remains practically the same.

- The discrimination capability of the NLJTC with \((1/k)\)th law nonlinearity does not depend noticeably on \(k\) provided \(k\) exceeds a minimal value determined by the dynamic range limitation level.
4.3. Binary JTCs

Many nonlinear media are binary, that is, they can be practically only in two states: transparent or opaque. This feature can be described as hard limiting (binarization):

\[
LDR_{\text{lim}}(x) = \begin{cases} 0, & x < \text{Lim}; \\ 1, & \text{otherwise} \end{cases}
\]  

(27)

From the experiments with the \((1/k)\)th law nonlinearity, one can conclude that even simple dynamic range limitation alone may substantially improve the NLJTC discrimination capability provided the limitation threshold is properly chosen. This fact allows us to assume that binary JTCs with the hard limitation according to Eq. (27) may also have sufficiently high discrimination capability. Experiments reported in the literature ([5, 19-21]) also support this assumption. The simulation results confirmed this conjecture. The simulation was carried out with the same set of test images according to the model:

\[
\text{coroutput} = \left| \text{IFT} \left( LDR_{\text{lim}} \left( \left| \text{FT} (\text{corinput}) \right|^2 \right) \right) \right|^2.
\]  

(28)

The simulation results are plotted in Figs. 20, a), and b) for all images of the test set as functions of the fraction of the joint spectrum energy under binarization threshold. They clearly show that, for all of the test images, there exists an optimal value of the binarization threshold for which correlator’s discrimination capability reaches the levels close to those achievable for the logarithmic and \((1/k)\)th law NLJTCs. This optimal value corresponds to the binarization threshold in the range of about \((3 \cdot 10^{-4})\)th to \((8 \cdot 10^{-4})\)th fraction of the entire energy of the joint spectrum. This range is of the same order of
magnitude as that of the ratio $7.5 \cdot 10^{-4}$ of the area occupied by the target object (5x5 pixels) to the area of the input image (128x256 pixels) which one would expect to be. One can also see that the discrimination capability of the binary JTC is relatively tolerant to reasonable deviations of the binarization threshold from its optimal value. The achievable SNRV and SNRM for binary JTCs are lower than in the optimal NLJTC but still are much higher that those for the matched filter ($lim=1, k=1$ in Fig. 19, a).

5. CONCLUSION

The discrimination capability of several types of nonlinear optical correlators for target location and recognition has been investigated by computer simulation: ideal optical adaptive correlator (OPAC), suboptimal correlators with (-$k$)th law nonlinearity and a matched filter in their Fourier domain, Phase-only-Filters and Phase-only-Correlators, nonlinear Joint Transform Correlators with logarithmic and $(1/k)$law nonlinearities applied to joint spectrum, binary Joint Transform Correlators. In the simulations, such design factors as limitation of the nonlinear media and electronic components dynamic range and resolution power have been taken into consideration. The experiments were conducted with a representative set of test images and the results were evaluated both statistically by averaging over the test set, and individually for each of the test images in the set.

The general conclusion is that, with an appropriate selection of the nonlinearity parameters, the dynamic range limitation and binarization thresholds, the nonlinear correlators exhibit the substantially improved discrimination capability and that the technical requirements to the nonlinear spatial light modulators required for the
implementation of the nonlinear correlators, to their electronic components and optical alignment are surprisingly low.

It is also worth to mention that the experiments have supported the basic assumption on which the theory of optimal adaptive correlators ([1]) was rested upon, the assumption that the discrimination capability of the correlators (defined as ratio SNRM of the correlator’s response to the target object to the highest peak of the correlator’s response to the objects to be rejected) is directly associated with the ratio SNRV of the correlator’s response to the target object to the standard deviation of the correlator’s output signal measured either over the area not occupied by the target object. In all the experiments, the values of the nonlinear correlators’ parameters optimal in terms of SNRV and in terms of SNRM were found, to all practical purposes, identical.

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REFERENCES


Table 1. A variety of correlators implemented in the computer model

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<td>∞</td>
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<tr>
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<td>0</td>
<td>&gt;0</td>
<td>&lt;∞</td>
</tr>
</tbody>
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CAPTIONS TO THE FIGURES

Fig. 1 Schematic diagram of the optical correlators with the \((-k)\)th law nonlinearity.

Fig. 2. Schematic diagram of the Joint Transform correlator with a nonlinear
transformation of the joint spectrum

Fig. 3 The image and its fragments used in the experiments.

Fig. 4 Discrimination capability of nonlinear correlators with \((-k)\)th law nonlinearity and
exactly known image power spectrum as a function of the index \(k\) \((k=1\) corresponds to
the Optimal Correlator):

a) sorted sequence of SNRVs at output of the conventional correlator \((k=0)\) for the
set of test images

b) gain in SNRV values as compared to the conventional correlator as a function
of \(k\) for the set of test images

c) gain in SNRM values as compared to the conventional correlator as a function
of \(k\) for the set of test images

Fig. 5 Discrimination capability of the suboptimal correlators with \((-k)\)th law nonlinearity
as a function of the index \(k\) and the degree of spectrum smoothing for a typical image
from the test set.

a) SNRV for the LS method of spectrum estimation;

b) SNRV for the ATS method of spectrum estimation;

c) SNRM for the LS method of spectrum estimation;

d) SNRM for the ATS method of spectrum estimation;
Fig. 6 Gain in the nonlinear correlator's discrimination capability for the LS method for estimation of the background image power spectrum, as compared to that for direct use of the observed image power spectrum:
   a) gain in SNRV with relation to the conventional correlator-matched filter; 
   b) same for SNRM.
Fig. 7 Comparison of the Optimal Correlator (curve 0) with the correlators with the LS (curve 1) and ATS (curve 2) methods for spectrum estimation with the optimized smoothing index $bl$.
   a) gain in SNRV with relation to the conventional correlator-matched filter; 
   b) same for SNRM.
Fig. 8 1-D cross sections of outputs of the conventional matched filter ($k=0$, $bl=0$) and the nonlinear correlator with LS estimation of the image power spectrum ($k=1$, $bl=4$) for one of the test images with the target object at the coordinate 65.
Fig. 9 Losses in the discrimination capability of the Optimal Correlator as a function of the dynamic range limitation degree.
Fig. 10 Gain in the nonlinear correlator's discrimination capability for the LS-method of spectrum estimation and limitation of the nonlinear media's dynamic range:
   a) gain in SNRV with relation to the conventional correlator-matched filter; 
   b) same for SNRM.
Fig. 11 Discrimination capability of the correlators with the Phase-Only-Filter and $(-k)$th law nonlinearity as a function of the index $k$ and the degree of spectrum smoothing for a typical image from the test set:
   a) SNRV for the LS method of spectrum estimation;
b) SNRM for the LS method of spectrum estimation;

Fig. 12 Comparison of the discrimination capability of the Optimal Correlator (exact opt. corr.), suboptimal nonlinear optical correlators with the LS method of power spectrum estimation (nlin. opt. corr.), Phase-Only-Correlator (POCorr), Phase-Only-Filter (POF corr), and matched filter (MF corr) for the set of test images in terms of SNRV.

Fig. 13. An example of input image of the JTC.

Fig. 14. Stereoscopic images used in the experiments.

Fig. 15. Frequency response of the blur operator used in experiments.

Fig. 16. SNRV and SNRM at the output of the logarithmic JTC versus limitation threshold.

Fig. 17. SNRV and SNRM at the output of the logarithmic JTC versus blur parameter $\text{blr}$.

Fig. 18. Illustration of the similarity between logarithmic and $(1/k)$th law nonlinearities.

Fig. 19. Average SNRV (a) and SNRM (b) at the output of JTC with $(1/k)$th law nonlinearity versus nonlinearity index $k$ for the set of test images. (c) - plot of SNRV for stereoscopic images.

Fig. 20. SNRV (a) and SNRM (b) at the output of the binary JTC versus fraction of the joint spectrum energy under the binarization threshold for the set of test images.
Fig. 1 Schematic diagram of the optical correlators with the \((-k)\)th law nonlinearity.
Fig. 2 Schematic diagram of the Joint Transform correlator with a nonlinear transformation of the joint spectrum.