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AND COMPUTATIONAL COMPLEXITY OF SIGNAL
AND IMAGE PROCESSING

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Redundancy of Signals and Transformations and Computational Complexity of Signal and Image Processing

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Abstract

We demonstrate the use of informational redundancy of signals and transforms for reducing the computational costs of signal processing. Four concrete examples of accelerated signal processing algorithms are presented to support the idea of purposeful use of signal and transform redundancy for saving the computational costs. These are an accelerated algorithm for Fourier spectral analysis, an accelerated algorithm for computing the signal local histograms, the Quantized Discrete Fourier Transforms and recursive implementation of arbitrary digital filters. The former two reduce computation time by exploiting signal redundancy. The latter two save processing time at the expense of the accuracy of representation of the corresponding signal transforms.

1. Introduction

Computational complexity is one of the most topical issues in digital signal processing. It is usually characterized by the number of arithmetic operations required for processing a signal. The theoretical lower limit of this number is of the order of the number of signal samples, \( O(N) \). Signal processing algorithms are called fast if their computational complexity is \( O(N \log N) \) or less. If the computational complexity of an algorithm is \( O(N^p) \), \( p \geq 2 \) such algorithm is usually regarded impractical. A great deal of activity of the signal processing community is directed to searching for the ways to reduce the signal processing computational complexity.

It is the fundamental fact that the processing complexity is directly connected to the quantity of information conveyed by the signal and, therefore, to the information redundancy in the signal representation. In many cases the trade-off between redundancy and computational complexity is straightforward: the fewer samples or the fewer quantization levels the signal has, the computationally simpler its processing will be. In other cases, this trade-off may not be as explicit.

Examples are the FFT-type algorithms and recursive filters which use the redundancy hidden in the structure of the transform matrices and filters' impulse responses.

One can illustrate a relationship between the informational redundancy of the signals to be processed and the processing computational complexity by an estimation of the complexity of a general computer performing signal transformations in a signal space by means of a look-up-table.

The informational redundancy of signals is defined by their \( \varepsilon \)-entropy, or rate distortion function [1, 2] Let \( H_\varepsilon(a) \) be the \( \varepsilon \)-entropy of an ensemble of signals to be processed, as defined by a required accuracy of their representation, \( H_\varepsilon(b) \) the \( \varepsilon \)-entropy of an ensemble of the processed signals, and \( H_\varepsilon(b/a) \) the conditional \( \varepsilon \)-entropy of the processed signals given the initial ones. Here, \( H_\varepsilon(a) \) is associated with input signal redundancy and \( H_\varepsilon(b/a) \) with redundancy of the signal transformation. They are linked by the relationship:

\[
H_\varepsilon(b) = H_\varepsilon(a) + H_\varepsilon(b/a)
\]

The total number \( Q \) of the resulting signals in the signal space is \( 2^{H_\varepsilon} \). This number determines the minimal size, or complexity \( CC \), of the look-up-table for the given signal processing task:

\[
CC \geq \log Q = H_\varepsilon(b)
\]

In solving practical problems of reducing the computational complexity of digital signal processing on a specific computer, we can rely upon a signal compression approach developed in communication theory. The objective of this article is to demonstrate this idea by some of its practical implementations. In Sects. 2 an accelerated algorithm for spectral analysis of signals with highly correlated samples and in Sect. 3 an accelerated algorithm for the computation of histograms in a moving window are presented as an illustration of the use of signal redundancy for reducing the computational complexity of spectral and statistical signal analysis.

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The Quantized Discrete Fourier Transform and the recursive representation of arbitrary digital filters described in Sect. 4 and 5 show the use, for the same purpose, of the redundancy of the signal transforms associated with admissible accuracy of their implementation.


Discrete Fourier Transform is one of the basic operations in digital signal processing. One can accelerate calculation of the DFT by using a signal compression coding prior to carrying out DFT and a corresponding decoding of the obtained spectrum. We describe an algorithm in which the well known DPCM coding (see, for instance, [3, 4]) is used with the aim of obtaining a coded signal with as many zero-valued samples as possible. As a result, computational time for DFT can be reduced by eliminating from the calculations those with zero operand. Since signal coding is used prior DFT, a corresponding correction of the obtained spectrum is required.

Reduction of computational expenditures by this algorithm is determined by the number of zero-valued elements in the coded signal which in turn is determined by the informational redundancy of the signal and the efficiency of its coding. Implementation of the algorithm on an IBM-PC compatible computer has shown that the gain in computational time reaches 60% for 1-D video signals of 512 samples.

3. An accelerated algorithm for recursive calculation of the signal local histograms

The local, or short time, histograms of signals carry important signal features and are used for implementation of rank order filters in signal and image processing ([5, 6, 7, 8]). In the conventional 2-D recursive algorithm ([4, 6]) for the calculation of histograms of quantized 2-D signals in a moving window the local histogram over the window for the given pixel is updated from the local histogram for the preceding pixel by adding to it difference between histograms over incoming and outcoming columns of the window. This requires storage of column-wise histograms for every pixel in the given row. We suggest to modify this algorithm and to compute and store the corresponding difference column-wise histograms rather then the histograms themselves. Since the local histograms for the adjacent pixels of real life pictures are very similar to each other, their differences contain many zero valued elements. One can eliminate operations with zero operand when updating local histogram and in this way save computer time. Testing this method on IBM PC-compatible computer has shown that for real life images a considerable gain is obtained for the window size higher than 25*25 pixels. For the window of 127*127 pixels the speed of computation was 2-5 times higher than that of the conventional algorithm, the higher figure corresponding to a more redundant image.

4. The quantized discrete Fourier transform

The Quantized Discrete Fourier Transform (QDFT) was introduced in [9] for accelerating calculations of computer generated holograms. In the QDFT, the multipliers \( \cos \phi' \) and \( \sin \phi' \) in every \( r \)-th step of the conventional FFT algorithms are replaced with the multipliers \( q \cos \phi' \) and \( q \sin \phi' \) whose values are obtained through coarse quantization of the values of \( \cos \phi' \) and \( \sin \phi' \), respectively (Fig.1). Thus, with three levels of quantization, \( q \cos \phi' \) and \( q \sin \phi' \) take up the values -1, 0 and 1; with five levels of quantization, the values -1, -1/2, 0, 1/2, 1, and so on. The resulting error in representation of the exponential bases functions of the DFT by the bases functions of the QDFT may be minimized by optimal allocation of the quantization moments for the values of the sine and cosine functions.

With three quantization levels, the QDFT requires no multiplication operations at all, and a lesser number of addition-subtraction operations. With five levels of quantization, multiplications can be replaced with the shift operations. Therefore, in its computational complexity, the QDFT is somewhere between the Haar and the Walsh-Hadamard fast transforms, that is, it may require considerably less operations than FFT, while it can be used as a reasonable approximation of the DFT whenever the speed of calculation is more important than the accuracy.

5. Recursive representation of digital filters

Many signal and image processing applications allow relatively coarse approximation to the required impulse responses of linear filters. One can use this fact for approximate representation of arbitrary digital filters in form of a parallel set of recursive filters (Fig.2) and for reducing in this way computational complexity of filtering.

It was shown in [6, 10] that such representation corresponds to expansion of the filter impulse response by a set of basis functions, such that the filters in parallel branches of the Fig. 2 perform local spectral analysis in the corresponding basis in the filter's aperture and multiply the signal spectral coefficients by the corresponding coefficients of the filter impulse response. The adder in Fig. 2 performs inverse transform for the central element of the filter's aperture. To make this scheme computationally
computationally efficient, one should select basis functions that allow recursive local spectral analysis. In the fastest possible first-order recursive filter every given sample of the output signal depends only on one sample that was computed at the preceding step. The per input sample number of operations for them is three multiplications and two additions, whatever the extent of the basis functions. It was shown in [6, 10, 11] that the following two family of bases satisfy the condition of the first order recursivity: (i) the basis of discrete exponential functions (DFT basis) its derivative bases such as that of the Discrete cosine transform, and the linearly independent basis of rectangular functions. The former corresponds to local Fourier analysis and filtering in frequency domain and is very suitable for local adaptive linear filtering; the latter is computationally much simpler and assumes recursive computation and weighted summation of signal local means over different windows within the filter’s aperture.

References


Fig. 1 FFT and QDFT transforms

DFT: \( w = \cos(2\pi N) + i\sin(2\pi N) \);
QDFT: \( w = \cos(2\pi N) + i\cos(2\pi N) \).

Fig. 2 Recursive and parallel representation of digital filters.

input signal
\[\rightarrow\] recursive filter 1
\[\rightarrow\] recursive filter 2
\[\rightarrow\] ... 
\[\rightarrow\] recursive filter R
\[\rightarrow\] adder
\[\rightarrow\] output signal