LECTURE 2

Statistical Noise Models and Diagnostics

2.1 Statistical models of random interferences:

(i) Additive signal independent noise model:

$$r = s + n . \tag{2.1}$$

Examples: Image sensor thermal white Gaussian noise; narrow band (moire) noise

(ii) Multiplicative noise model

$$r = ns . \tag{2.2}$$

Examples: Photographic graininess noise, "speckle" noise in imaging systems that use coherent irradiation (synthetic aperture radar, ultrasound imaging, holography)

(iii) Impulse noise model:

$$r = (1 - e)s + en; \qquad e = \begin{cases} 1, & \text{with certain probability } P_e \\ 0, & \text{otherwise} \end{cases} . \tag{2.3}$$

Examples: Noise in digital image transmission and storage; anomalous noise in analogue image transmission systems with nonlinear modulation.

(iv) Composite noise model:

$$r = n_m s + n_a . \tag{2.4}$$

(v) Signal dependent noise.

Example: quantization noise.

2.2 Basic statistical characteristics of random interferences:

- Probability distribution and density:

$$P_n(\mathrm{N}) = Probability((n = n) < \mathrm{N}); \tag{2.5}$$

$$p_n(n) = \lim_{\Delta n \to 0} \frac{P_n(n + \Delta n/2) - P_n(n - \Delta n/2)}{\Delta n} . \tag{2.6}$$

- Probability density moments:

  Mean value:

$$\bar{n} = \mathrm{AV}_n(n) = \int_{-\infty}^{\infty} n p_n(n) dn . \tag{2.7}$$

Variance:

$$\sigma_\nu^2 = \mathrm{AV}_\nu\left((\nu - \bar{\nu})^2\right) = \int_{-\infty}^{\infty} (\nu - \bar{\nu})^2 p_\nu(\nu) d\nu . \qquad (2.8)$$

Higher order moments:

$$\overline{\nu^{(m)}} = \mathrm{AV}_\nu\left((\nu - \bar{\nu})^m\right) = \int_{-\infty}^{\infty} (\nu - \bar{\nu})^m p_\nu(\nu) d\nu . \qquad (2.9)$$

- Autocorrelation functions:

$$CF_\nu(x_1, x_2) = \mathrm{AV}_\nu\{\nu(x_1)\nu(x_2)\} . \qquad (2.10)$$

In image processing, statistical (ensemble) averaging frequently does not make sense and should be replaced by averaging over certain limited set of image samples. Accordingly, definitions of statistical characteristics are modified:

- Distribution histogram (for discrete and quantized signals of ***N*** samples):

$$h_\nu(q) = \frac{1}{N}\sum_{k=0}^{N-1} \delta(q - \nu_k), \qquad (2.11)$$

where $\delta(x) = \begin{cases} 1, & x = 0 \\ 0, & otherwise \end{cases}$

- Distribution histogram moments:

Mean:

$$\bar{\nu} = \sum_{i=0}^{Q-1} q_i h_\nu(q_i) = \frac{1}{N}\sum_{k=0}^{N-1} \nu_k , \qquad (2.12)$$

where Q is number of quantization leveles, and $\{q_i\}$ are quantized signal values.

Variance:

$$\sigma_\nu^2 = \sum_{i=0}^{Q-1} (q_i - \bar{\nu})^2 h_\nu(q_i) = \frac{1}{N}\sum_{k=0}^{N-1} (\nu_k - \bar{\nu})^2 . \qquad (2.13)$$

Higher order moments:

$$\overline{\nu^{[m]}} = \sum_{i=0}^{Q-1} (q_i - \bar{\nu})^m h_\nu(q_i) = \frac{1}{N}\sum_{k=0}^{N-1} (\nu_k - \bar{\nu})^m . \qquad (2.14)$$

- Correlation functions and spectra

$$CF_\nu(t) = \frac{1}{N}\sum_{k_1=0}^{N-1} \nu_{k_1}\nu_{k_1+t} . \qquad (2.15)$$

$$S_\nu(r) = \frac{1}{\sqrt{N}}\sum_{t=0}^{N-1} CF_\nu(t) \exp\left(i2\pi\frac{tr}{N}\right) . \qquad (2.16)$$

2.3 Noise visibility in images

Noise visibility in images depends on:

- Type on the noise (additive, multiplication, impulse, quantization, signal dependent noise)
- Intensity of noise (variance, dynamic range, distribution histogram)
- Noise correlation function ad spectra.
- Background image

Generally:

- Correlated noise has higher visibility than uncorrelated noise of the same variance.
- Vision is less sensitive to noise in the vicinity of object boundaries.
- Visibility of noise with heavy tails of the distribution histogram (such as, for instance, impulse noise with uniform distribution) is higher then that of noise with Gaussian distribution.
- Signal dependent noise (such as quantization noise) may confuse vision by producing image-alike artefacts and hiding image details. Examples: false contours and disappearing of low contrast image details as consequences of quantization noise.

Figure 2.1 provides illustrations of these properties of vision.

2.4 Diagnostics of random interferences in images

2.4.1 Basic principle.

The type and statistical properties of noise in images may be known from certification of imaging systems. However, in many applications these data may be unavailable and should be obtained directly for available images. Such an empirical noise diagnostics in images assumes:

- Determination of noise model
- Measuring appropriate statistical parameters of noise.

Determination of noise model is usually done on the base of image visual analysis and a priori knowledge regarding the type of imaging system that generated images under question. In measuring noise statistical parameters requires breaking a vicious circle: in order to measure noise parameters one needs to separate noise from the useful signal that can be done only if the noise parameters are known. The solution is not to separate noise from the signal but instead separate their statistical characteristics.

Basic principle in empirical noise diagnostics from noisy images is: select for estimation the statistical characteristic of the noisy signal in which the presence of noise exhibits itself in easily detectable characteristic anomalies.

2.4.2 Measurement of of location and energy of "moire" noise spectral peaks.

An immediate example is measuring intensity of "moire" noise components. The characteristic property of narrow band "moire" noise is that its energy is concentrated in a few of very narrow band components. Therefore its spectrum has a few concentrated peaks that are very easily detectable on the background of the useful signal spectrum that is much wider and homogeneous. A practical algorithm for determination of frequency and energy of "moire" noise components in images is illustrated in Fig. 2.2.

2.4.3 Measuring variance of zero-mean additive white noise in images.

Following the above principle of separating statistical characteristics of noise and useful signal , one can measure variance of additive white noise in images by detecting noise correlation function peak in correlation function of the noisy image. For additive signal independent white noise, correlation function of noisy image is:

$$\begin{aligned} &\boldsymbol{CF}(\underline{s}\boldsymbol{ignal} + \underline{n}\boldsymbol{oise}) = \mathbf{AV}_n\left\{(\boldsymbol{s}+\boldsymbol{n})(\boldsymbol{s}+\boldsymbol{n})^*\right\} = \\ &\boldsymbol{CF}(\boldsymbol{s}) + \boldsymbol{CF}(\boldsymbol{n}) + \mathbf{AV}_n\left(\boldsymbol{s} \times \boldsymbol{n}^*\right) + \mathbf{AV}_n\left\{\boldsymbol{s}^* \times \boldsymbol{n}\right\} = \boldsymbol{CF}(\boldsymbol{s}) + \boldsymbol{CF}(\boldsymbol{n}) \end{aligned} \qquad (2.17)$$

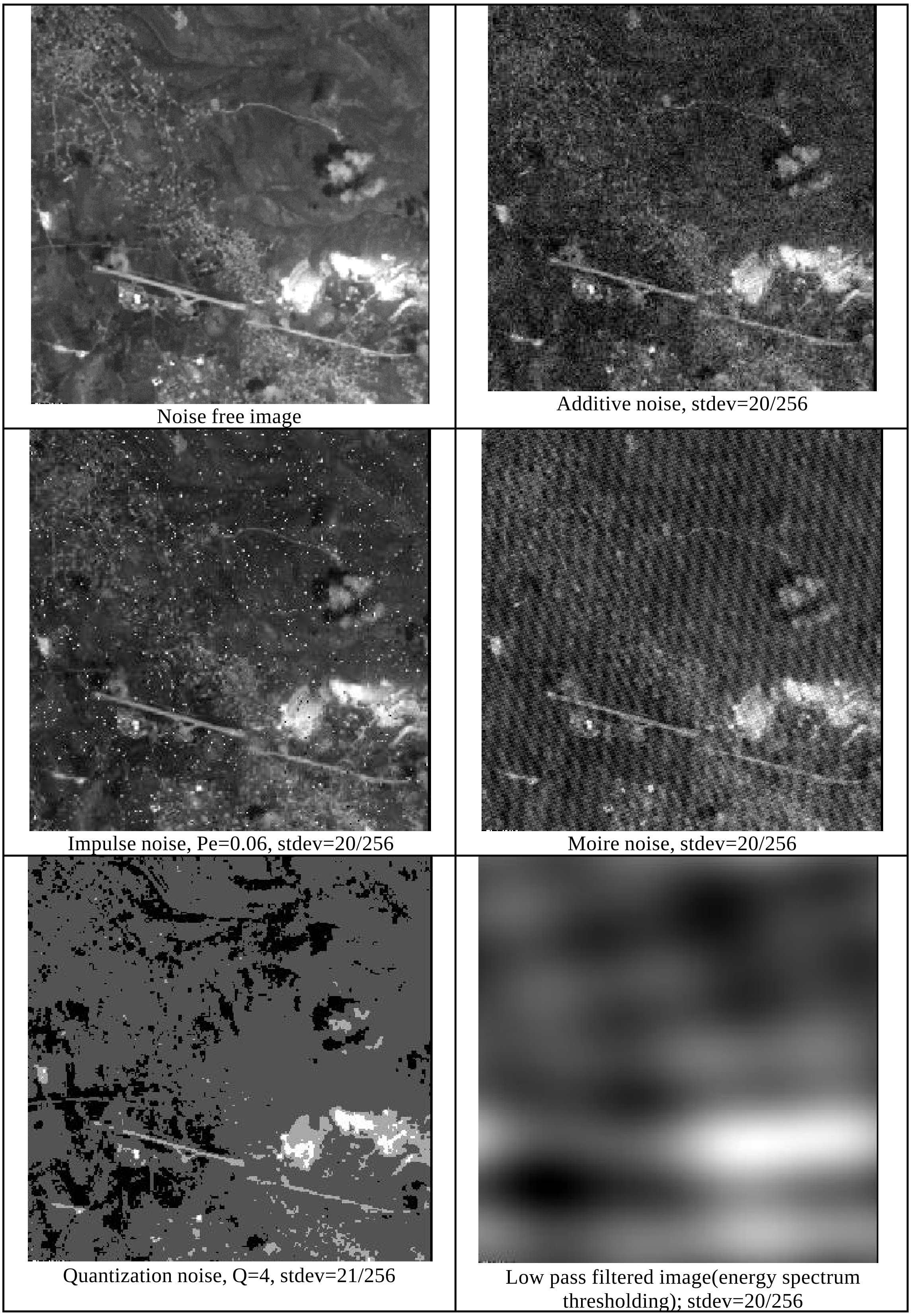
Correlation function of white noise $\boldsymbol{CF}(\boldsymbol{n})$ is a delta function that appears, in empirical measurements, as a narrow peak in the correlation function origin of co-ordinates. Image signal correlation function is usually a function that decays to zero much more slowly. Therefore, signal correlation function value in the origin of co-ordinates can be accurately enough predicted from noisy signal correlation function values in adjacent points outside the extent of noise correlation function. Difference between the value of noisy signal correlation function in co-ordinate origin and the predicted one provides estimate for variance of additive white noise. Fig.2.3 illustrates the method.

2.4.4 Diagnostics of impulse noise.

Main statistical characteristic of impulse noise is the probability of appearance of noise impulses. Impulse noise manifests itself as easily detectable contrast anomalies directly in the noisy signal. Therefore, probability of noise impulses can be estimated by detecting noise impulses directly in the noisy signal. In practice, this process is a part of noise cleaning algorithms that work in two steps: detection of noise impulses and replacing signal samples detected as substituted by noise by values approximated from neighbouring not distorted samples. Fig. 2.4 illustrates impulse noise in an image and its influence on the prediction error for image samples from their neighbours. This error can be used for the detection of noise impulses.

2.4.5 Quantization noise

Quantization noise is characterized by the size of signal quantization interval. Signal statistical characteristic in which one can easily detect the presence of quantization noise and evaluate quantization intervals is image histogram for quantization noise manifests itself via appearance in image histogram of isolated and easily detectable peaks (Fig. 2.5)



Noise free image

Additive noise, stdev=20/256

Impulse noise, Pe=0.06, stdev=20/256

Moire noise, stdev=20/256

Quantization noise, Q=4, stdev=21/256

Low pass filtered image(energy spectrum thresholding); stdev=20/256

Fig. 2.1 Different image distortions with the same standard deviation of “additive noise”

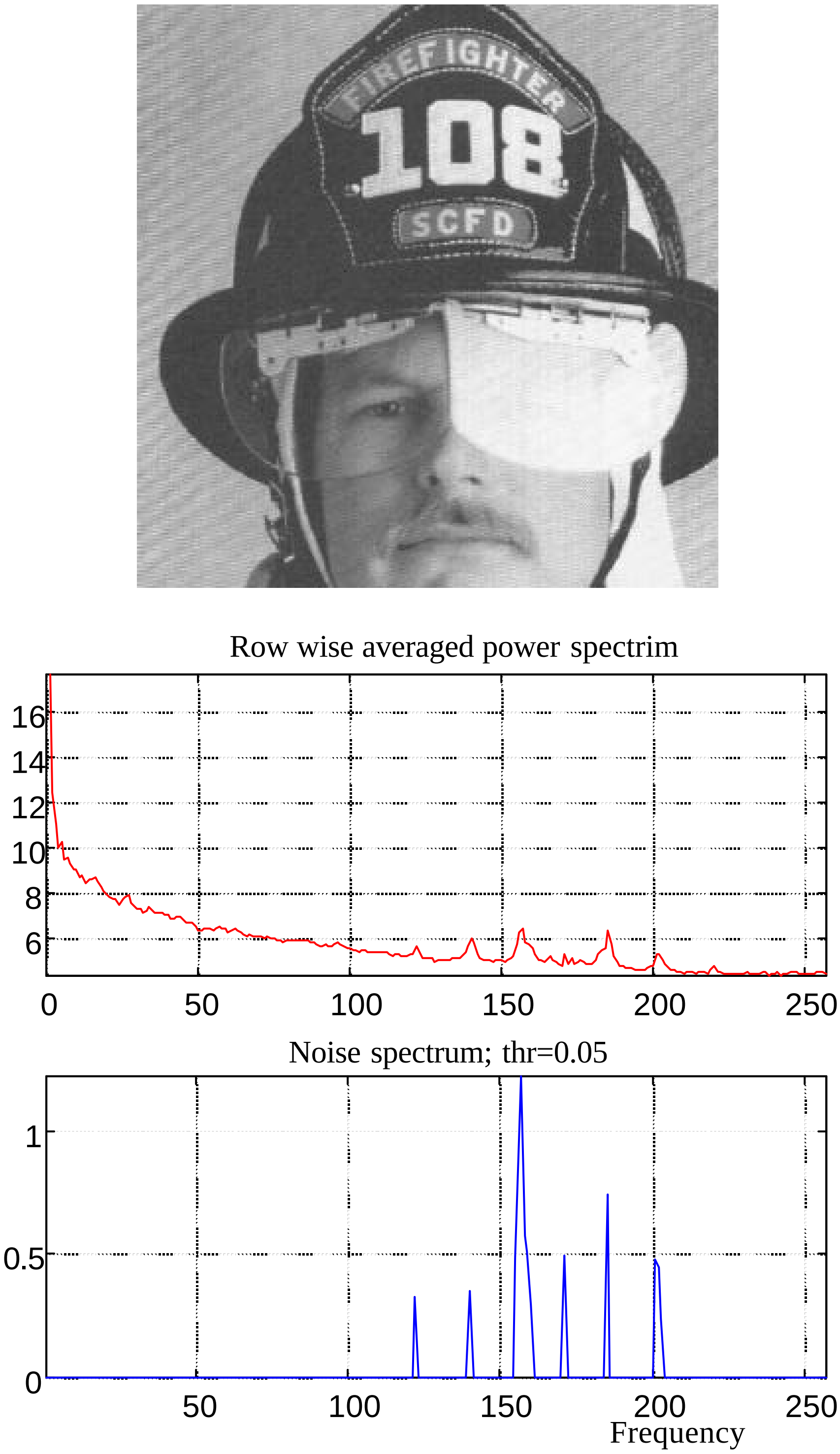


Fig. 2.2 Diagnostics of “moire” noise. Noise spectral peaks are detected in row wise averaged power spectrum of noisy image by comparison of prediction error of spectrum samples predicted (from left to right)) with a threshold which is an algorithm free parameter. The threshold and the prediction method of spectrum samples depend on a priori knowledge regarding images and noise.

Initial and additively noised image (STDnoise=20)

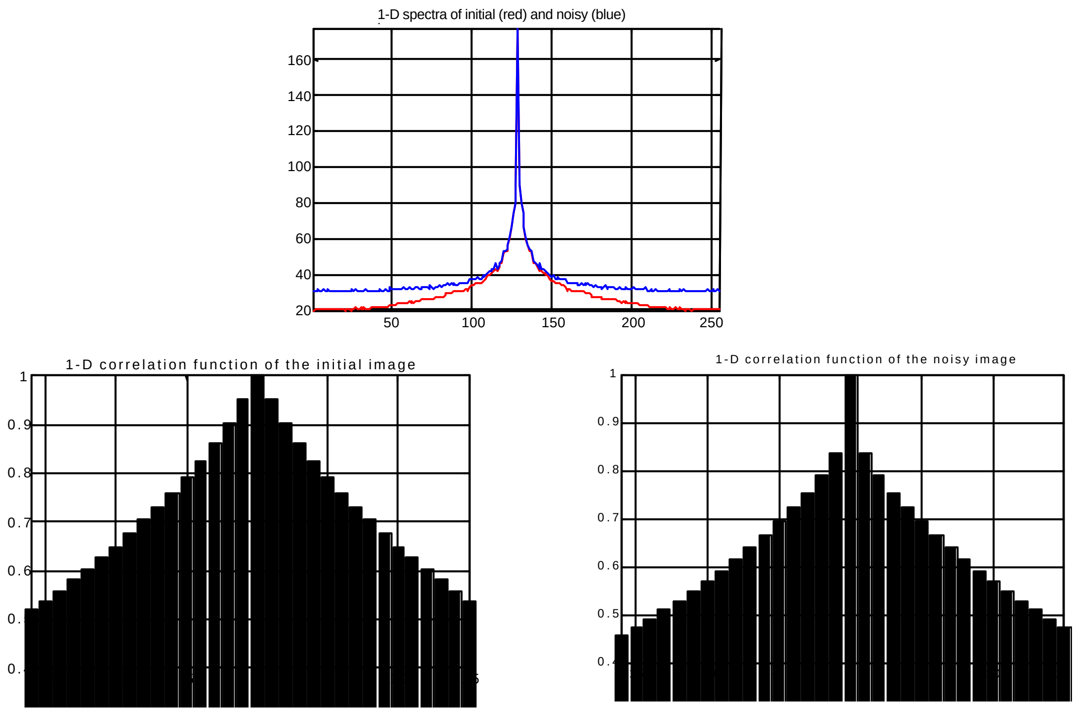


Fig. 2.3 Diagnostics of additive white noise in images. Plots show 1-D spectra (top) and 1-D correlation functions of noise free (bottom left) and noisy (bottom right) images. One can easily see peak in the correlation function of noisy image due to noise present.

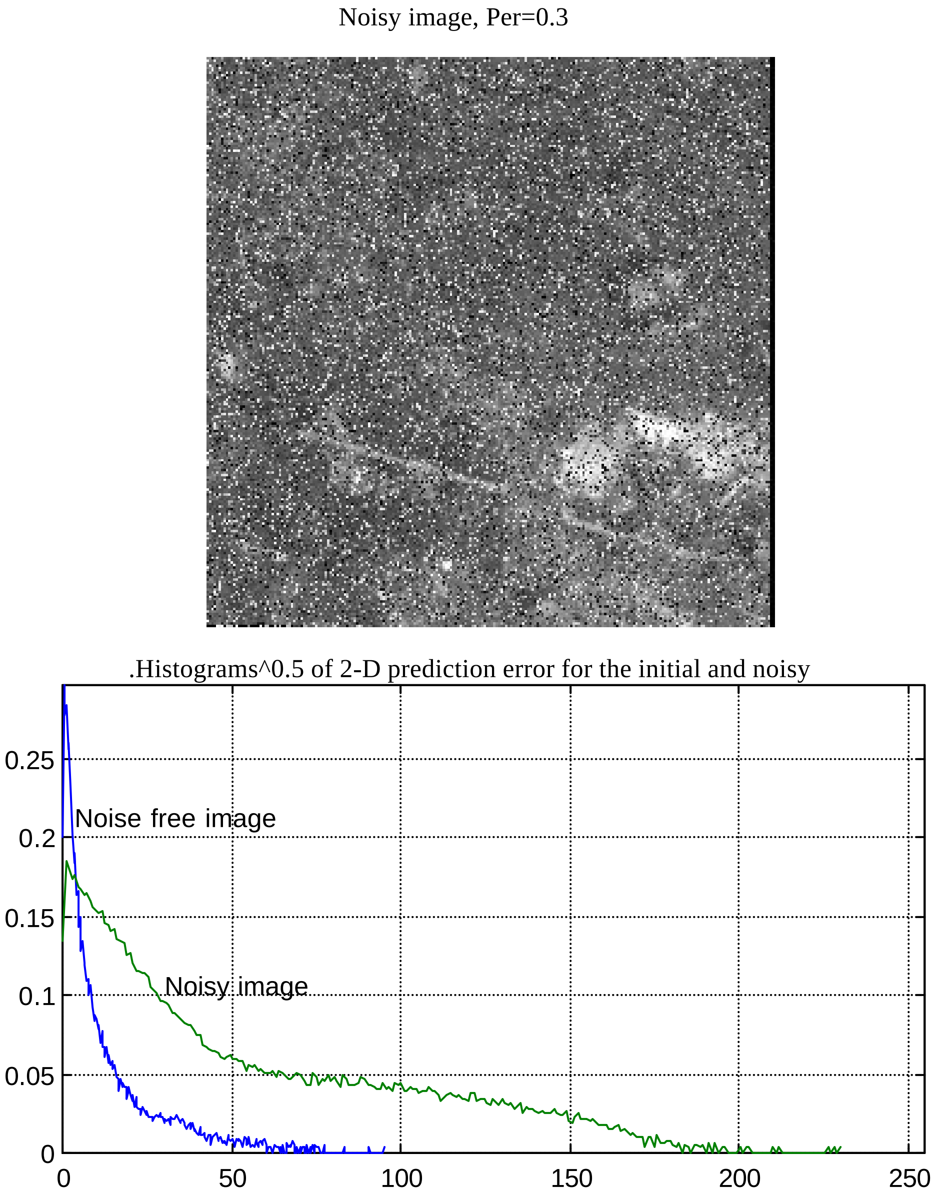


Fig. 2.4 Diagnostics of impulse noise. Impulse noise manifests itself as contrast isolated peaks in the image and appearance of heavy tails in histogram of prediction error of pixel grey levels from those of adjacent pixels

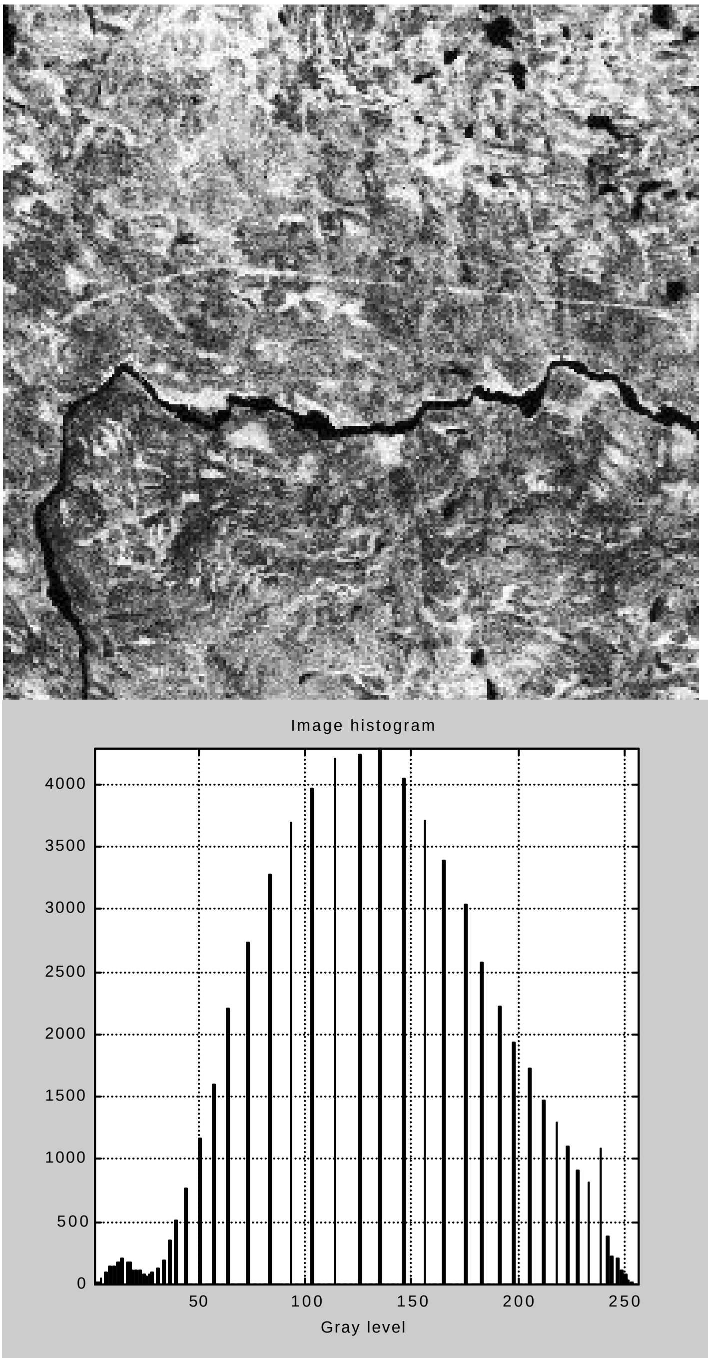


Fig. 2.5 Quantization noise diagnostics. Quantization noise manifests itself via appearence of isolated peaks in image histogram.