LECTURE 3.1

Image restoration, enhancement and segmentation: Linear filtering.

3.1.1 MSE Optimal Linear Filters

For $\{a_k^{tr}\}$ as samples of a "true" signal $\mathbf{A}^{tr}$ that has to be approximated (restored) from an observed input signal $\mathbf{A}^{inp} = \{a_k^{inp}\}$, $k = 0,1,...,N-1$, the Mean Squared Error (MSE) optimal restored signal $\mathbf{A}^{rst} = \{a_k^{rst}\}$ is the one whose pixel wise deviation from signal $\{a_k^{tr}\}$ squared and averaged over the available set of samples and over random factors involved in the imaging process is minimal among all possible mappings of observed signal $\{a_k^{inp}\}$ into the estimation:

$$\{a_k^{rst}\} = \underset{\{a_k^{inp}\}\to\{a_k^{rst}\}}{arg\,min}\, \mathbf{AV}\left(\sum_{k=1}^{N-1}\left|a_k^{tr} - a_k^{rst}\right|^2\right), \qquad (3.1.1)$$

where $\mathbf{AV}(\cdot)$ is the averaging operator.

3.1.2 Transform domain MSE optimal linear scalar filters

Assume that signal restoration will be carried out by linear filtering of input signal. General linear filtering of a discrete signal can be vector-matrix formalizm expressed as multimilcation of signal vector $\mathbf{A}^{inp}$ by a matrix $\mathbf{H}$ that represents the linear filter:

$$\mathbf{A}^{rst} = \mathbf{H}\mathbf{A}^{inp}. \qquad (3.1.2)$$

For signals of N samples, filter matrix $N \times N$ matrix whose specification (filter design) requires has the complexity $O(N^2)$. A solution that can reduce this complexity is filtering in transform domain when filter matrix $\mathbf{H}$ is approximated as a product of three matrices, a direct transform matrix $\mathbf{T}$, a diagonal matrix $\mathbf{H}_d$ that is specified its N diagonal elements and an inverse transform matrix $\mathbf{T}^{-1}$:

$$A^{rst} = T^{-1}H_d T\left(A^{inp}\right). \qquad (3.1.3)$$

We will call such an implementation of linear filtering transform domain scalar filtering. For transform domain scalar filtering, transform domain representations of output signal $\{\alpha_r^{rst}\} = \mathbf{TA}^{rst}$ and of input signal $\{\alpha_r^{inp}\} = \mathbf{TA}^{inp}$, $(r = 0,1,...,N-1)$ are connected by the relationship:

$$\alpha_r^{rst} = \eta_r \alpha_r^{inp} \qquad (3.1.4)$$

where $\{\eta_r\}$ are diagonal elements of the filter matrix.

For orthogonal transforms and other invertible transforms, MSE restoration error can be equally measured in transform domain which implies that one can obtain MSE optimal transform domain scalar filter coefficients $\{\eta_r^{opt}\}$ as a solution of the equation:

$$\{\alpha_r^{rst}\} = \underset{\{\eta_r\}}{arg\,min}\, \mathrm{AV}\left[\sum_{k=1}^{N-1} \left|\alpha_r^{tr} - \eta_r \alpha_r^{inp}\right|^2\right], \qquad (3.1.5)$$

where The solution of this equation can be easily found and is known as a signal "orthogonal projection":

$$\eta_r^{opt} = \frac{\mathrm{AV}\left(\alpha_r^{tr}\left(\alpha_r^{inp}\right)^*\right)}{\mathrm{AV}\left[\left|\alpha_r^{inp}\right|^2\right]}. \qquad (3.1.6)$$

3.1.3 Scalar filters for restoration of signal observed in additive signal independent noise

Assume observed input signal $\mathbf{A}^{inp}$ to be restored can be modelled as an additive mixture of an ideal signal $\mathbf{A}^{tr}$ at the output of a linear filter $\mathbf{L}$ and a signal independent noise $\mathbf{N}$:

$$\mathbf{A}^{inp} = \mathbf{LA}^{tr} + \mathbf{N} \qquad (3.1.7)$$

Assume also that a transform exists in which domain signal and noise relationship of Eq. (3.1.7) can be written as

$$\alpha_r^{inp} = \lambda_r \alpha_r^{tr} + \nu_r, \qquad (3.1.8)$$

where $\{\nu_r\}$ are transform domain representation (spectrum) of realisations of noise $\mathbf{N}$. Then optimal transform domain filter coefficients of Eq. (3.1.6) can be found as:

$$\eta_r^{opt} = \frac{1}{\lambda_r}\frac{\mathrm{AV}\left[|\lambda_r|^2\left|\alpha_r^{tr}\right|^2\right]}{AV\left[\left|\alpha_r^{inp}\right|^2\right]} = \frac{1}{\lambda_r}\frac{\mathrm{AV}\left[|\lambda_r|^2\left|\alpha_r^{id}\right|^2\right]}{\mathrm{AV}\left[|\lambda_r|^2\left|\alpha_r^{tr}\right|^2\right] + \mathrm{AV}\left[|\nu_r|^2\right]} \qquad (3.1.9)$$

The filter implementation requires knowledge of linear filter transform domain representation coefficients $\{\lambda_r\}$ and power spectra $\mathrm{AV}\left[|\nu_r|^2\right]$ and $\mathrm{AV}|\alpha_r^{tr}|^2$ of noise and "ideal" signal.

Linear filter transform domain representation coefficients $\{\lambda_r\}$ and noise power spectrum are frequently known from the specification of imaging system. In case they are not known, blind restoration methods are needed that determine them from observed images. Methods of additive noise diagnostics see in Lect. 2. As for the spectrum of " true" signal, in classical MSE approach that dates back to N. Wiener it is supposed to be known for the specification of signal class. However, in practical image processing this assumtion leeds to the results that hardly can be accepted since filtering based on spectra averaged over an image ensemble tends to produce "average" image thus almost completely ignors individuality of images restoration of which is in fact the restoration goal. For instance, images restored on the base of averaged image spectra usually loose edge sharpnes and tiny image details. One can remove this drawback of Wiener filtering defined by Eq. (3.1.9) by requesting minimization of mean squared restoration error for each individual image under processing. Removing image ensemble averaging in the critrion of Eq. 3.1.5 we arrive at the following optimal filter:

$$\eta_r^{opt} = \frac{1}{\lambda_r}\frac{\left[|\lambda_r|^2|\alpha_r^{tr}|^2\right]}{AV_n\left[|\alpha_r^{inp}|^2\right]} = \frac{1}{\lambda_r}\frac{\left[|\lambda_r|^2|\alpha_r^{id}|^2\right]}{\left[|\lambda_r|^2|\alpha_r^{tr}|^2\right] + \mathrm{AV}_n\left[|\nu_r|^2\right]} \qquad (3.1.10)$$

The implementation of this filter requires estimation of "true" spectra of the individual image under processing for input image. This makes filter adaptive.

The estimate of "true" image spectrum for additive signal independend model of Eq. 3.1.8 can be obtained from the relationship:

$$AV_n\left[|\alpha_r^{inp}|^2\right] = \left[|\lambda_r|^2|\alpha_r^{tr}|^2\right] + \mathrm{AV}_n\left[|\nu_r|^2\right] \qquad (3.1.11)$$

which leeds to the filter:

$$\eta_r^{opt} = \frac{1}{\lambda_r}\frac{\left[|\lambda_r|^2|\alpha_r^{inp}|^2\right] - \mathrm{AV}_n\left[|\nu_r|^2\right]}{\mathrm{AV}_n\left[|\alpha_r^{inpr}|^2\right]}. \qquad (3.1.12)$$

Filter of Eq. (3.1.12), for known noise spectrum $\mathbf{AV}_n\left(\left|\nu_r\right|^2\right)$, still assumes estimation of spectrum of input image by means of statistical averaging over an ensemble of imaging system noise $\mathbf{N}$. Although this can, in principle, be attempted, one can, as a “zero order” approximation to it , use for the filter design spectrum of input image. In this way we arrive at the filter:

$$\eta_r^{opt} = \frac{1}{\lambda_r} max\left(\frac{\left|\alpha_r^{inp}\right|^2 - \mathbf{AV}_n\left(\left|\nu_r\right|^2\right)}{\left|\alpha_r^{inp}\right|^2}, 0\right). \qquad (3.1.13)$$

We will call this filter ***Empirical Wiener filter***.

A useful practical modification of this filter is “***Rejective***” (transform shrinkage) filter in which Empirical Wiener filter coeficients are quantized to two levels:

$$\eta_r = \begin{cases} \frac{1}{\lambda_r}, & \left(\left|\alpha_r^{inp}\right|^2\right) \geq Thr_r \\ 0, & \left(\left|\alpha_r^{inp}\right|^2\right) < Thr_r \end{cases}. \qquad (3.1.14)$$

It follows from comparison of Rejective filter with Empirical Wiener filter that the rejection threshold is of the order of magnitude of noise spectral coefficients:

$$Thr_r = O\left(\mathbf{AV}_n\left(\left|\nu_r\right|^2\right)\right). \qquad (3.1.15)$$

Recently, similar filters were described for filtering additive white noise that work in the domain of wavelet transform ([4]):

***Wavelet shrinkage: “soft thresholding*”**

$$\eta_r^{opt} = max\left(\frac{\left|\alpha_r^{inp}\right| - Thr}{\left|\alpha_r^{inp}\right|}, 0\right); \qquad (3.1.16)$$

Wavelet shrinkage: “hard thresholding”.

$$\eta_r = \begin{cases} 1, & \left|\alpha_r^{inp}\right| \geq Thr_r \\ 0, & \left|\alpha_r^{inp}\right| < Thr_r \end{cases}. \qquad (3.1.17)$$

For blind image restoration and image enhancement, a "***Fractional power***" filter

$$\eta_r = \begin{cases} \left|\alpha_r^{inp}\right|^{p-1}, & \left(\left|\alpha_r^{inp}\right|^2\right) \geq Thr_r \\ 0, & \left(\left|\alpha_r^{inp}\right|^2\right) < Thr_r \end{cases} \qquad (3.1.18)$$

can be recommended which is a modification of the rejective filter in which input signal transform coefficients that survived thresholding are risen, by magnitude, to a power p. If $p < 1$, signal energy is redistributed in favour of low energy spectral coefficients thus resulting in their amplification.

In the conclusion note that since transform domain weight coefficients of the described filters are found on the base of modification of spectra of input images, the filters are inherently adaptive.

3.1.4 Global and local transform domain scalar filtering

Above described filters can be applied in images processing both globally and locally. In global filtering, images are subjected to filtering as a whole, while in local filtering images are processed in fragment-wise fashion.

Global filters are global adaptive: in the filter design, global image spectral charateristics prevail over local ones. Experimental experience shows that global filters can be successfully applied for restoration of images contaminated with highly correlated narrow band noise with transform domain spectrum concentrated in a few spectral coefficients. Detecting and eliminating these coefficients with Empirical Wiener or rejecting filter almost completely removes noise while keeping unchanged the waste majority of image spectral coefficients. This is illustrated in Figs. 3.1.1 and 3.1.2 on examples of filtering periodical ("moire") noise (Fig. 3.1.1) and stripe noise (Fig. 3.1.2). Periodical noise manifests itself in transform (for instance, DFT) domain via easily detectable sharp peaks. This makes it simple to diagnose noise spectrum (see Lect. 2), and then to build an empirical Wiener or rejecting filter that successfully removes noise without noticeable degradation of the image. For stripe noise, appropriate transform is image projection along the direction of stripes (Radon transform). In the projection, stripes manifest themselves as chaotic peaks on the background of relatively slow changing image mean values in the projection direction (Fig. 3.1.2, top right). Smoothing peaks caused by stripes in the projection allows to restore image projection (in Fig. 3.1.2, bottom right, it is approximated as a constant). Difference between smoothed and non smoothed projections provides strip noise values for every stripe that can be then subtracted from the input image to produce an output image filtered image (Fig. 3.1.2, bottom left).

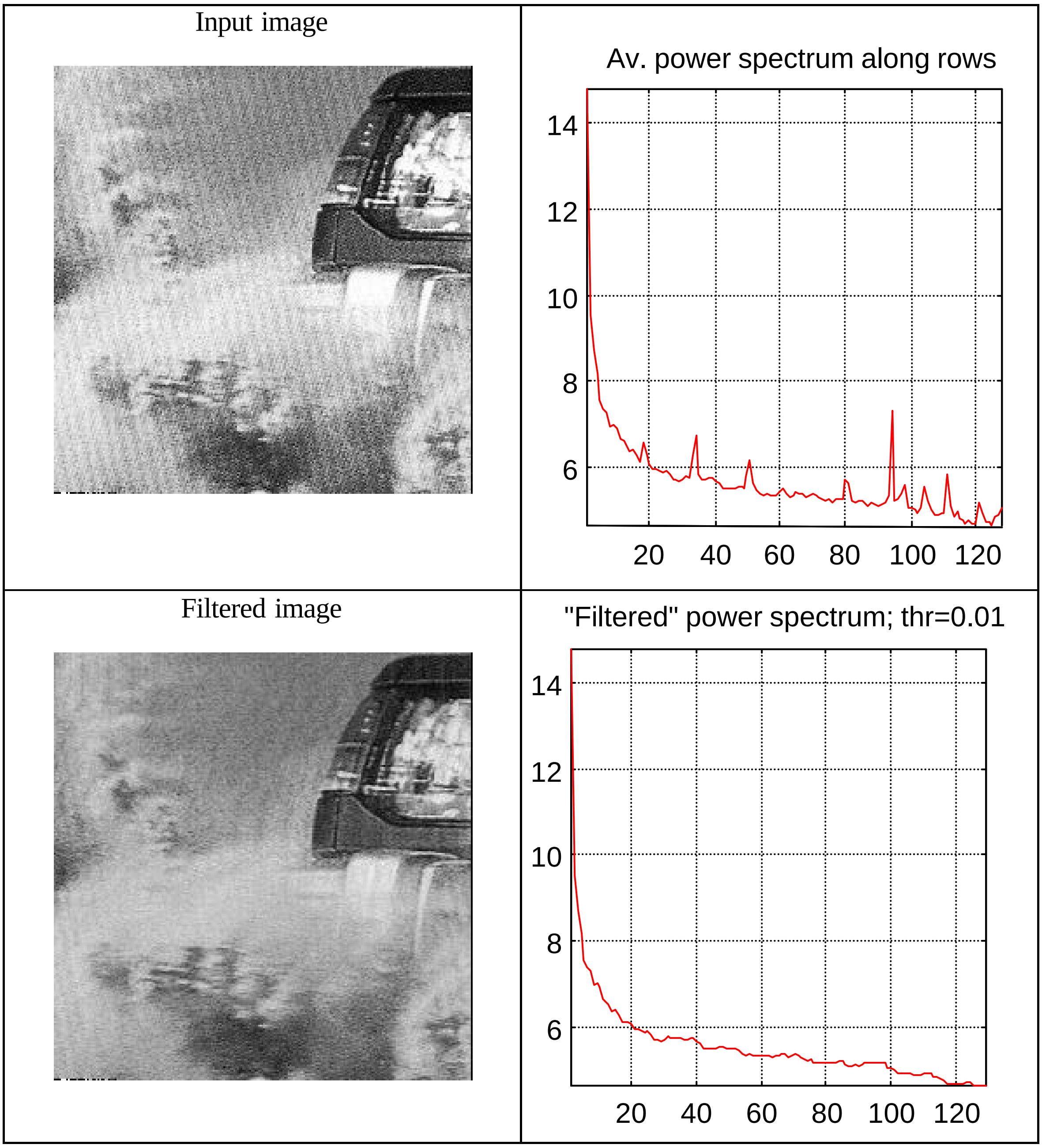


Fig. 3.1.1. Global adaptive Empirical Wiener filtering of periodic noise in an image.

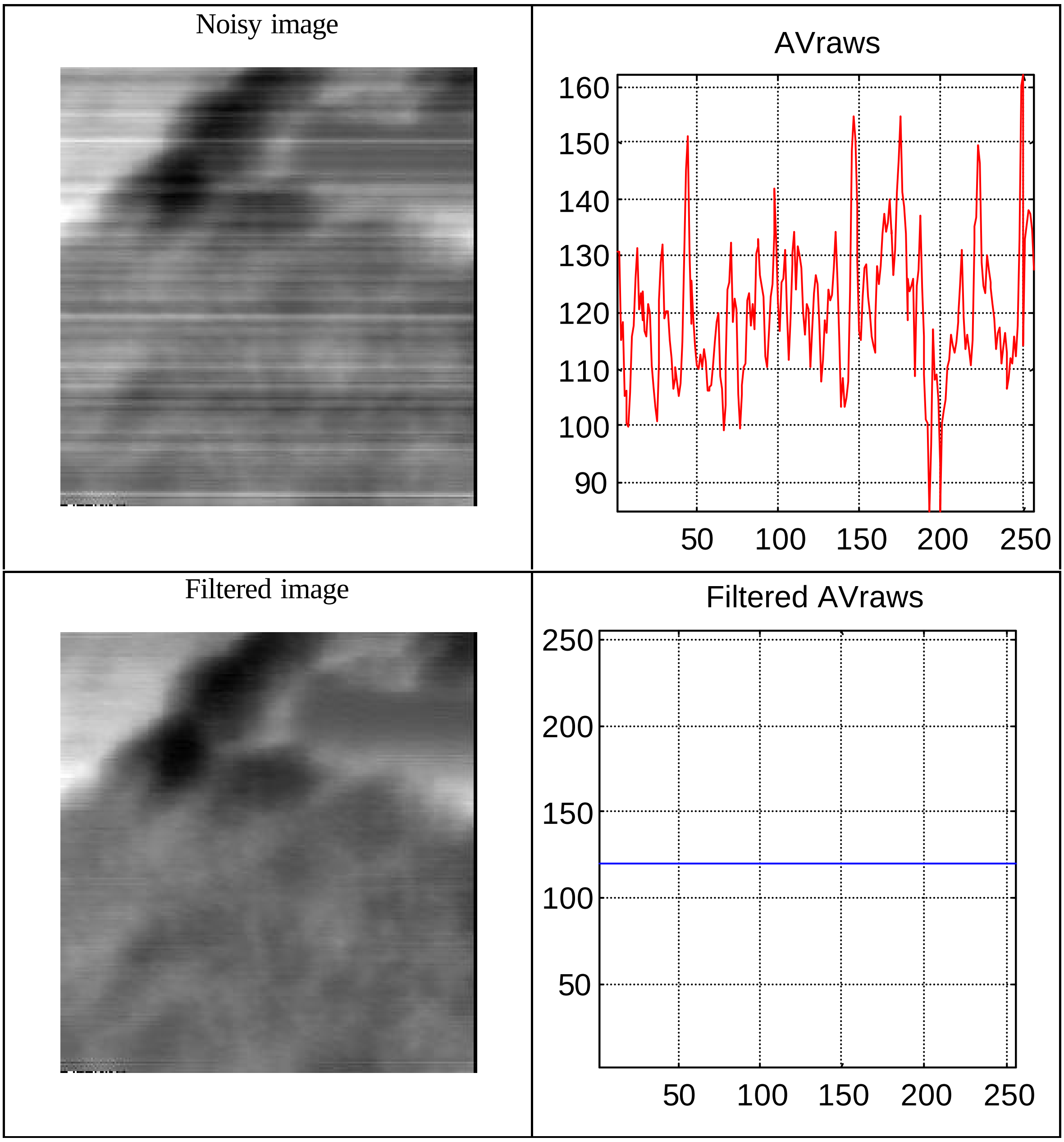


Fig. 3.1.2 Filtering stripe noise.

Global filtering however is very inefficient for denoising images contaminated with broad band noise with a spectrum more or less uniformly distributed among spectral coefficients in the transform domain. In this case the only way not to loose image edges and other tiny details is to process images locally. Fig. 3.1.3 compares global and local treatment of images. One can see that local peculiarities of images such as edges and tiny details are much better pronounced and detectable in image fragments while their global contribution in global image spectrum is mixed up and can easily be lost in global spectrum thresholding.

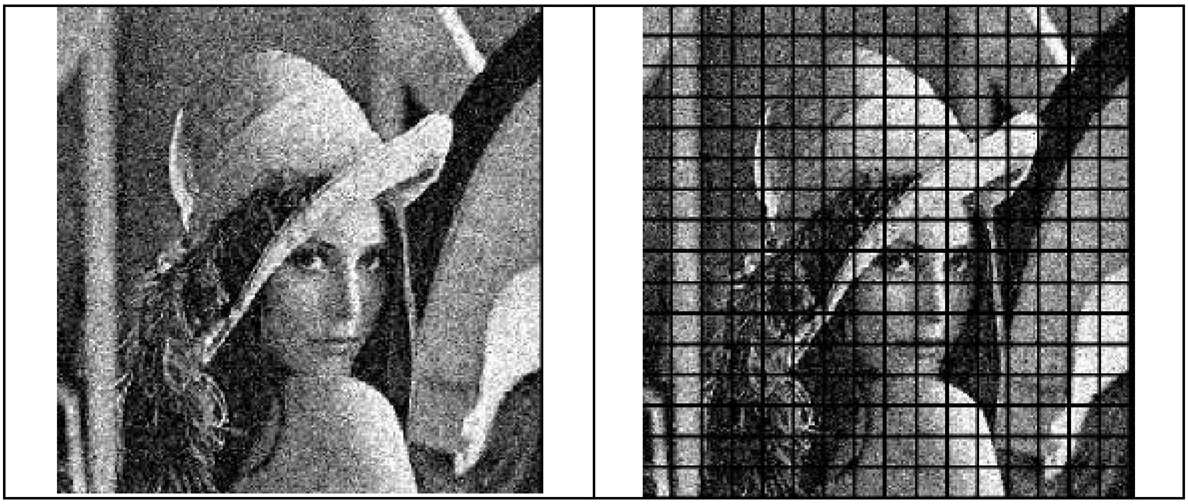


Fig. 3.1.3 Global (left) and local (right) treatment of images.

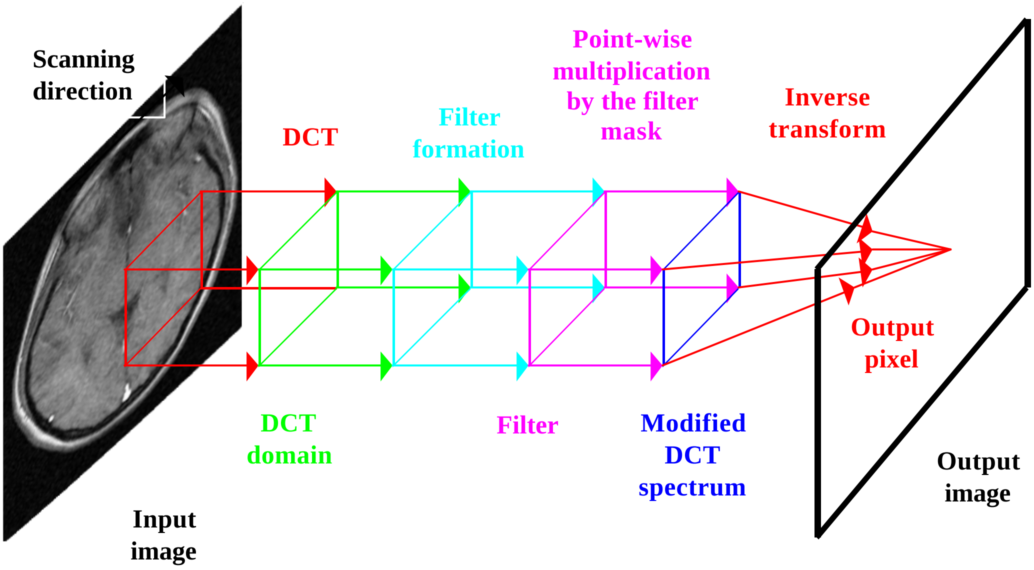
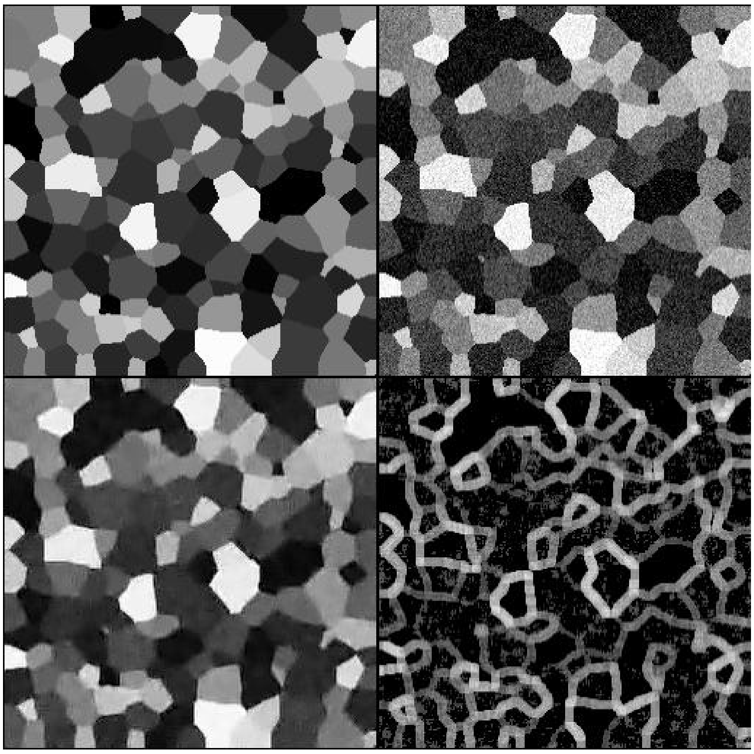


Fig. 3.1.4 Flow diagram of sliding window local adaptive filtering in DCT domain

Wavelet shrinkage filtering allows, to a certain degree, to compensate this drawback of global filtering since wavelet transform combines globality and locality. However, ultimate solution will be local image processing.

Local processing can be implemented block-wise such as, for instance, it is in transform image coding, and in a sliding window. Block wise processing is potentially vulnerable to "blocking effects" very well known in image coding. Sliding window is completely free of "blocking effects" but may be very computationally expensive since it assumes computing and modifying image spectra in each position of the window. Fortunately, such transforms as DFT and its derivatives such as DCT can be computed in moving window recursively by simple modification of local spectra in the given position of the window from the spectrum found in the window previous position. Note that in the sliding window processing, inverse transform for returning from transform to signal domain is necessary to compute only for the central pixel of the window. Experiments show that one of the best transform candidates for sliding window transform domain scalar filtering is DCT. Fig. 3.1.4 illustrates flow diagram of such a DCT domain filtering

Some illustrative examples of image restoration (denoising and blind deblurring) are shown in Figs. 3.1.6 – 3.1.8



Initial (upper left), noisy (upper right), filtered (bottom left) images and filter “transparency” map

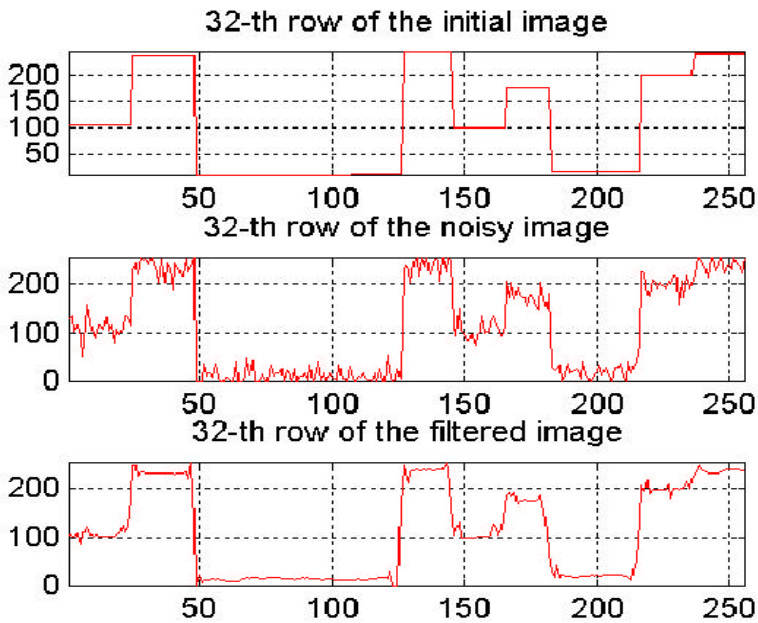
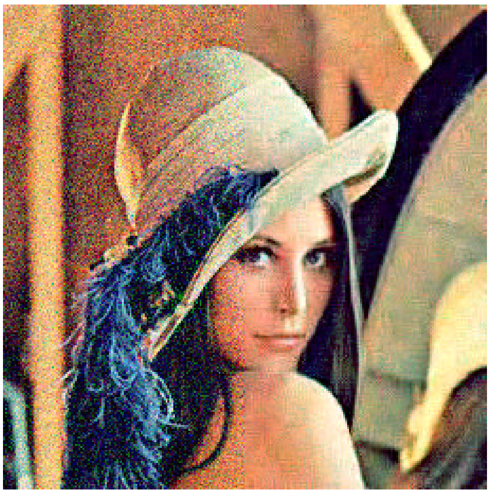
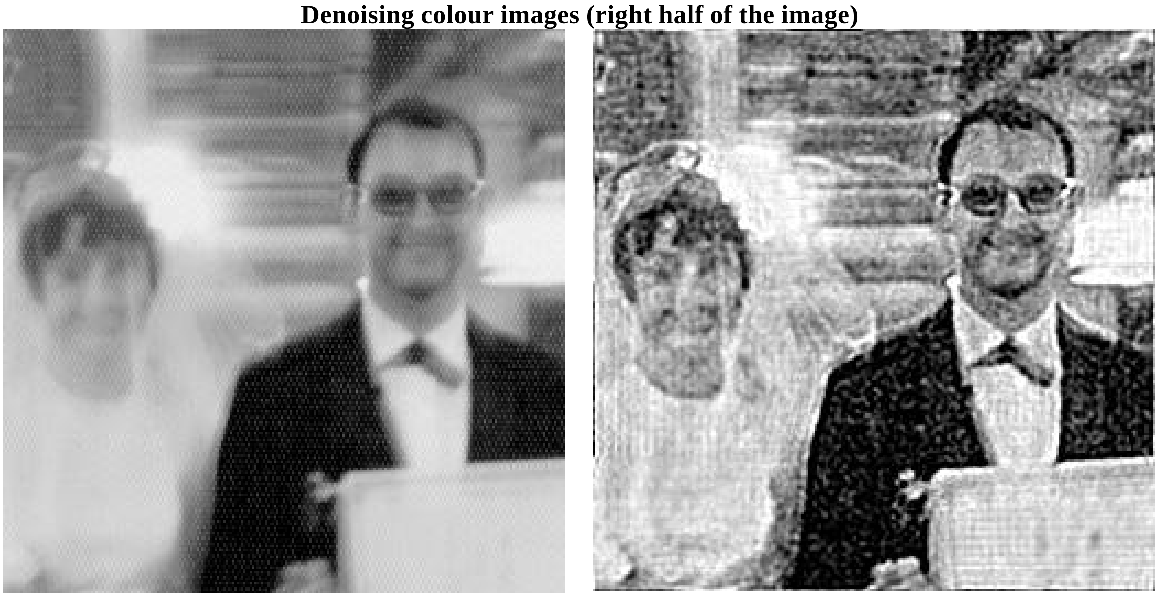


Fig.3.1.5 Sliding window DCT domain denoising of a test piece-wise constant image. Filter “transparency” map displays fraction of nonzero rejective filter coefficients. Note that filter is practically completely “transparent” in the vicinity of edges and almost completely opaque in plane areas of the image.



Denoising colour images (right half of the image)



Blind image restoration: initial and restored photographs

Blind restoration of a colour image

Fig. 3.1.6 Illustrative examples of image denoising and restoration by means of sliding window DCT domain filtering

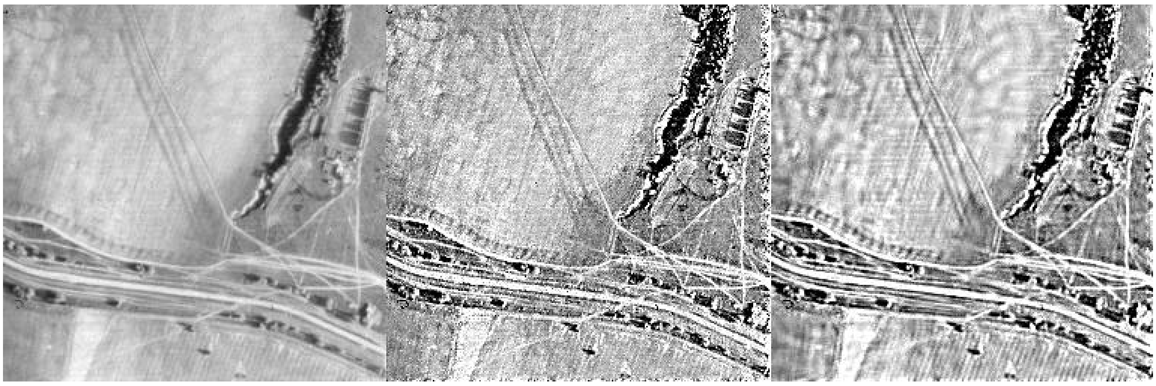


Fig. 3.1.7 Image enhancement by sliding window “fractional power” filter of Eq. 3.1.18. From left to right: input image, enhanced image (window $\mathbf{15 \times 15}$, $\boldsymbol{p = 0.5}$) without thresholding and enhanced image with thresholding,

3.1.5 Local adaptive filtering signal dependent noise in US images

Described sliding window transform domain filters can also be successfully used for eliminating signal dependent noise such as, for instance, speckle noise in ultrasound images. The main peculiarity of speckle noise is that its variance is proportional to signal value. Therefore, making noise threshold in rejective filter of Eq. (3.1.14) proportional to signal local mean, or, equivalent, to the dc component $\boldsymbol{\alpha}_0^{inp}$ of local spectrum

$$\boldsymbol{h}_r = \begin{cases} \mathbf{1}, & \left(\left|\boldsymbol{\alpha}_r^{inp}\right|\right) \geq \boldsymbol{Thr}\left|\boldsymbol{\alpha}_0^{inp}\right| \\ \mathbf{0}, & \boldsymbol{otherwise} \end{cases} \qquad (3.1.19)$$

one can build a rejective filter for elimination speckle noise. An illustrative example of such filtering is shown in Fig. 3.1.8 ([10]).

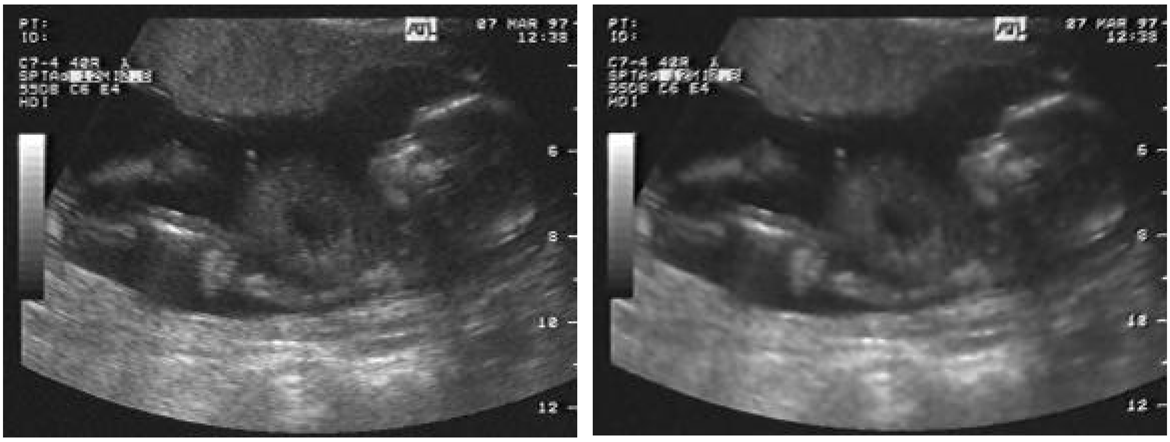


Fig. 3.1. 8 An example of denoising ultrasound images (left – initial image, right – filtered image).

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