Biomedical signal and image processing **(Course 055-355-5501)**

Lect. 10. Signal reconstruction and restoration: Linear filters

Imaging devices, distortions and inverse problem.

Wiener (least squares) restoration: $AVERR = AV_{im.sys} AV_{im.par}\left(\sum_{k=1}^{N-1}\left|a_k^{id} - a_k^{rst}\right|^2\right)$

Linear restoration filters: $A^{rst} = \mathbf{LF}\left(A^{inp}\right)$.

Filtering in transform domain. Scalar filters: $A^{rst} = T^{-1} H_d T\left(A^{inp}\right)$; $\alpha_r^{rst} = \eta_r \alpha_r^{rst}$

$$\eta_r^{opt} = \frac{AV_{im.sis} AV_{im.par}\left(\alpha_r^{id}\left(\alpha_r^{inp}\right)^*\right)}{AV_{im.sis} AV_{im.par}\left(\left|\alpha_r^{inp}\right|^2\right)}$$

Scalar filters for suppressing additive noise: $A^{inp} = A^{id} + N$

$$\eta_r^{opt} = \frac{AV_{im.par}\left(\left|\alpha_r^{id}\right|^2\right)}{AV_{im.par}\left(\left|\alpha_r^{id}\right|^2\right) + AV_{im.sys}\left|\nu_r\right|^2}$$

Empirical Wiener filters and power spectra estimation

$$\eta_r^{opt} = \left[\frac{AV_{im.par}\left(\left|\alpha_r^{inp}\right|^2\right) - AV_{im.sys}\left|\nu_r\right|^2}{AV_{im.par}\left(\left|\alpha_r^{inp}\right|^2\right)}\right] \approx \max\left[0, \left(\frac{\left|\alpha_r^{inp}\right|^2 - \sigma_{oise}^2}{\left|\alpha_r^{inp}\right|^2}\right)\right].$$

Rejecting filters (transform shrinkage): $\eta_r = \begin{cases} 1, AV_{im.par}\left(\left|\alpha_r^{inp}\right|^2\right) \geq thr \\ 0, AV_{im.par}\left(\left|\alpha_r^{inp}\right|^2\right) < thr \end{cases}$

Examples: filtering periodic noise; filtering strip-noise
Drawbacks of Wiener-type filtering.
Image deblurring:

$$A^{inp} = LA^{id} + N, \text{ where } L \text{ is a linear operator } \quad \alpha_r^{inp} = \lambda_r \alpha_r^{id} + \nu_r$$

$$\eta_r^{opt} = \frac{1}{\lambda_r} \frac{AV_{im.par}\left(\left|\lambda_r\right|^2\left|\alpha_r^{id}\right|^2\right)}{AV_{im.par}\left(\left|\lambda_r\right|^2\left|\alpha_r^{id}\right|^2\right) + AV_{im.sys}\left|\nu_r\right|^2} = \frac{1}{\lambda_r}\frac{SNR_r}{1 + SNR_r}$$

Inverse filter: $\eta_r^{inv} = 1/\lambda_r$

<u>Local adaptive linear filters</u>:

Local criteria: $AVLOSS(k,l) = AV_{stat}\left\{\sum_{m,n} LOC(m,n/k,l)\, LOSS\left(\hat{a}_{m,n}, a_{m,n}\right)\right\}$

Filter implementation in sliding window in transform domain.
Advantages and disadvantages of different bases for implementing local adaptive filters
Recursive implementation of linear local adaptive filters in the bases of DFT and DCT.
Local adaptive filtering speckle noise in US imaging

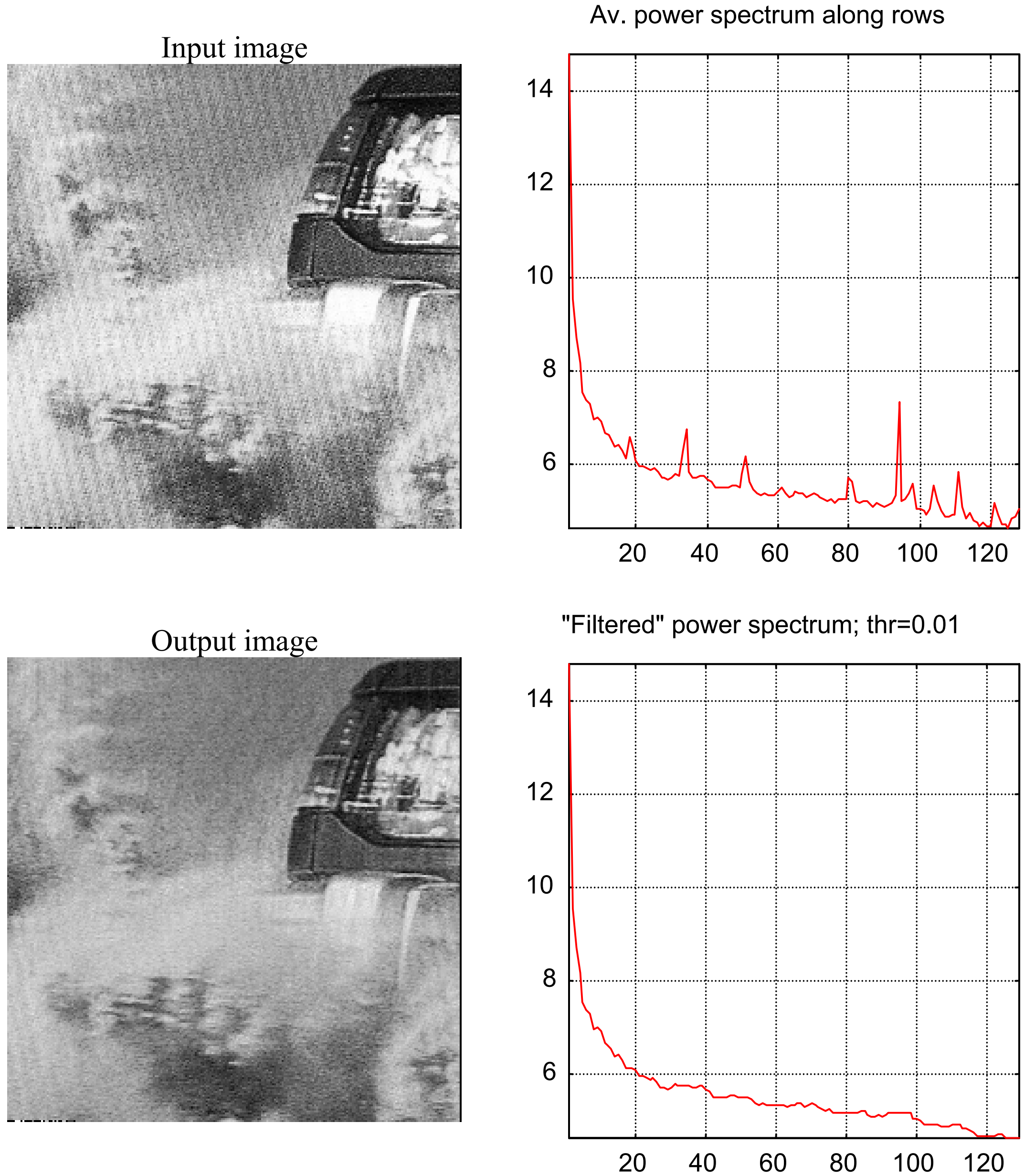
<u>Multi component signal restoration</u>

Data fusion and multi component signal denoising and deblurring
"Super resolution" from multiple images. Super resolution in video sequences.

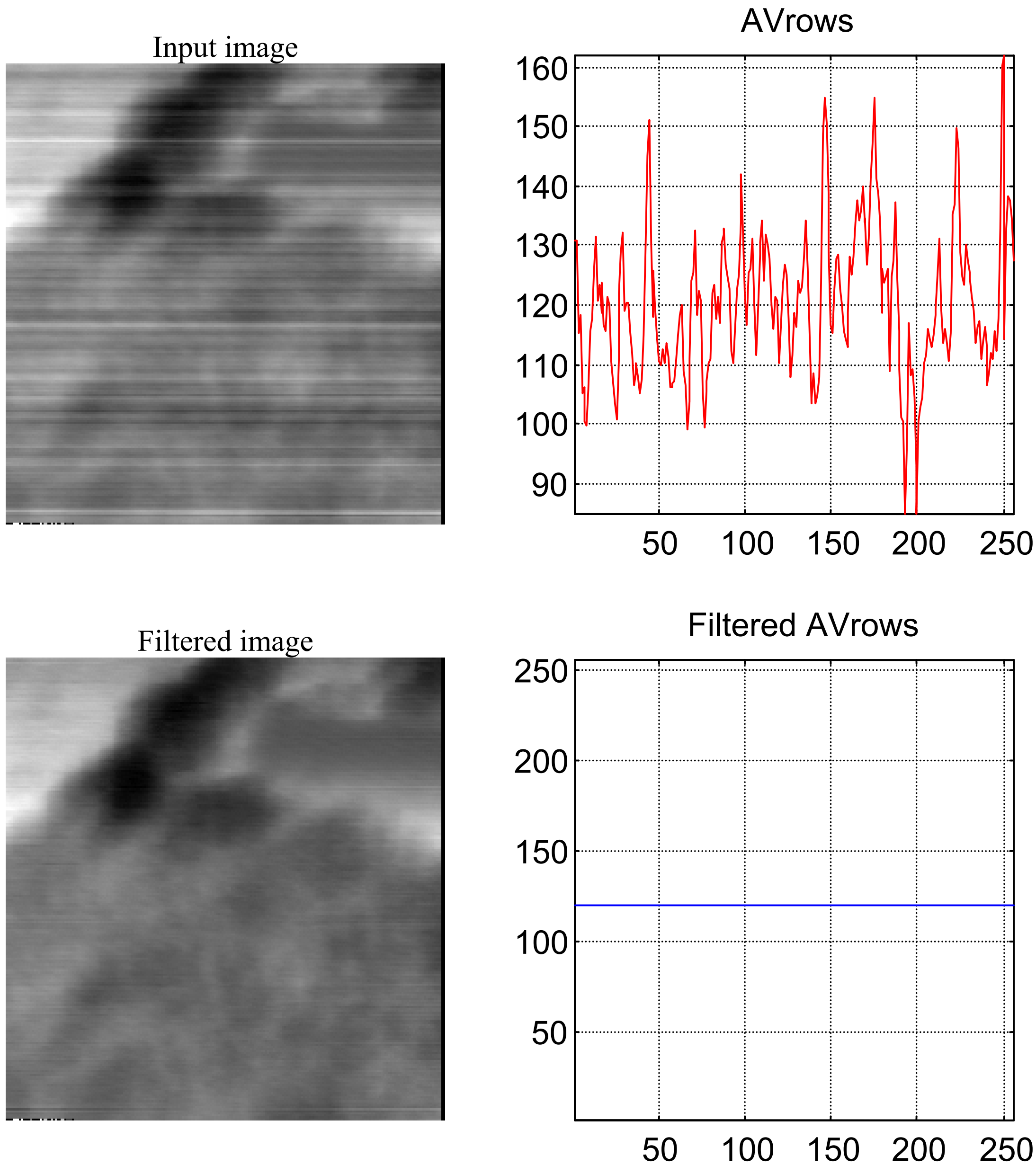
<u>Problems</u>:

1. Formulate the signal restoration problem and least squares approach to solving it.
2. Derive Wiener and empirical Wiener scalar filters for signal denoising.
3. Explain drawbacks of Wiener filtering and give examples its application for signal/image denoising.
4. Explain how Wiener, inverse and pseudo-inverse filters for signal deblurring work.
5. Describe approaches to synthesis and computer implementation of local adaptive filters

<u>Home work</u>: Simulate empirical Wiener filtering for sinusoidal signals of arbitrary frequency and additive noise.

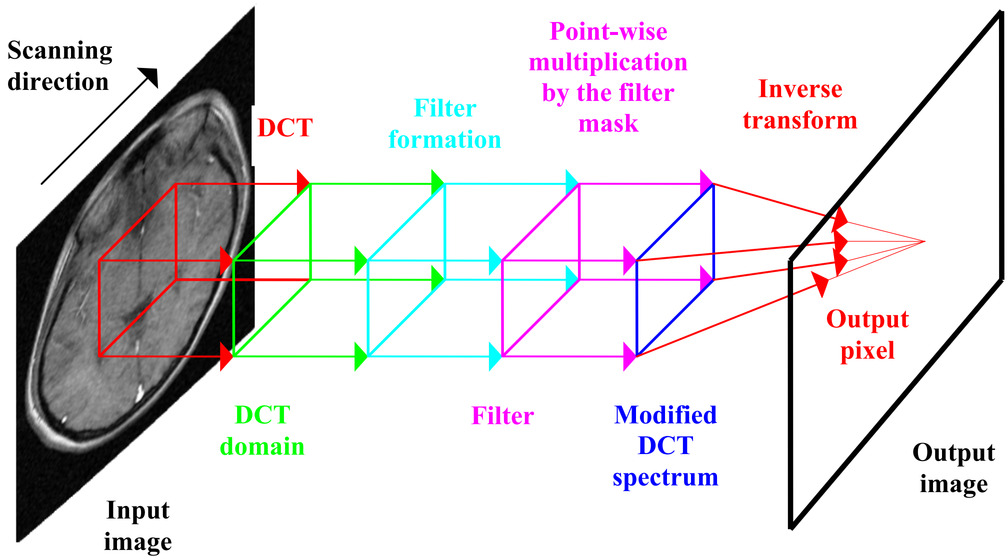


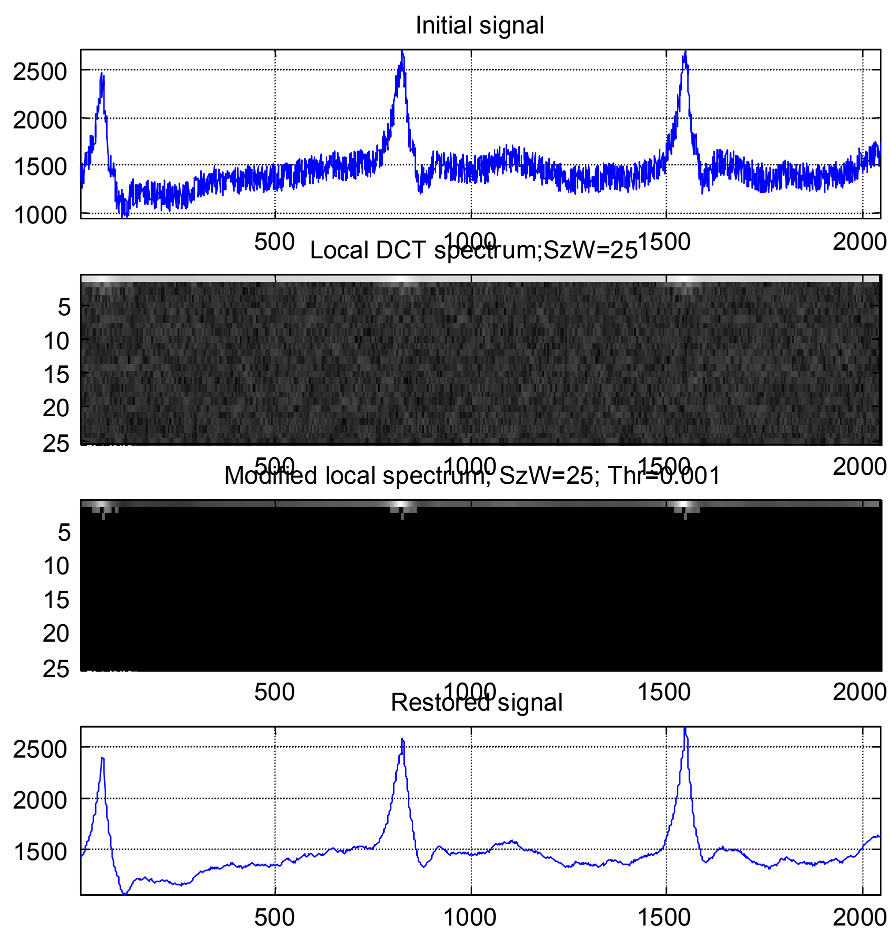
Empirical Wiener filtering periodic noise



Empirical Wiener filtering stripe noise

LOCAL ADAPTIVE FILTERING IN TRANSFORM DOMAIN





Local adaptive denoising electrocardiogram

Filtering speckle noise in US images:original (top) and filtered I(bottom) images