Biomedical signal and image processing **(Course 055-355-5501)**

Lect. 4. Element-wise quantization

Principles of optimal quantization

$$\varepsilon(r)=\alpha-\hat{\alpha}^{(r)} \quad Q_I=\sum_{r=1}^{M-2}\int_{\alpha^{(r-1)}}^{\alpha^{(r)}} p(\alpha)D(\varepsilon(r))d\alpha;\ Q_L=\int_{-\infty}^{\alpha^{(0)}} p(\alpha)D(\varepsilon(r))d\alpha;\quad Q_R=\int_{\alpha^{(M)}}^{\infty} p(\alpha)D(\varepsilon(r))d\alpha;$$

MMSE Max-Lloyd- quantizer: $\{\alpha_r,\hat{\alpha}_r\}=\arg\min\left\{\sum_{r=1}^{M-2}\int_{\alpha^{(r-1)}}^{\alpha^{(r)}} p(\alpha)|\varepsilon(r)|^2 d\alpha\right\}$; $p(\alpha)$ - distribution density.

Compressor-expander (compander) quantization.

Criterion $Q_I=\sum_{r=1}^{M-2}\int_{\alpha^{(r-1)}}^{\alpha^{(r)}} p(\alpha)D(\varepsilon(r))d\alpha$ is replaced by $Q_I=\int_{\alpha^{(0)}}^{\alpha^{(M)}} p(\alpha)D(\Delta_u/w'(\alpha))d\alpha;$

Optimal solution is obtained as a solution of Euler-LaGrange equation:

$$\frac{\partial}{\partial w'}\{p(\alpha)D(\Delta_u/w'(\alpha))\}=const$$

Examples:

1. Threshold criterion: $D\left(\alpha^{(r)}-\alpha^{(r+1)}\right)=\begin{cases}0, & \left|\alpha^{(r)}-\alpha^{(r+1)}\right|<\Delta_{thr}\\ 1, & \left|\alpha^{(r)}-\alpha^{(r+1)}\right|>\Delta_{thr}\end{cases}\Rightarrow$ uniform quantization

2. Threshold criterion: $D\left(\alpha^{(r)}-\alpha^{(r+1)}\right)=\begin{cases}0, & \left|\alpha^{(r)}-\alpha^{(r+1)}\right|<\Delta_{thr}=\delta_0\alpha\\ 1, & \left|\alpha^{(r)}-\alpha^{(r+1)}\right|>\Delta_{thr}=\delta_0\alpha\end{cases}\Rightarrow$ Uniform quantization in logarithmic scale

$$\frac{w(\alpha)-w(\alpha_{\min})}{w(\alpha_{\max})-w(\alpha_{\min})}=\frac{\ln(\alpha/\alpha_{\min})}{\ln(\alpha_{\max}/\alpha_{\min})};\ M=(\ln(\alpha_{\max}/\alpha_{\min}))/\delta_0$$

False contours and image quantization. Why 256 levels?

3. $D\left(\alpha^{(r)}-\alpha^{(r+1)}\right)=\left(\alpha^{(r)}-\alpha^{(r+1)}\right)^{2n}\Rightarrow\frac{w(\alpha)-w(\alpha_{\min})}{w(\alpha_{\max})-w(\alpha_{\min})}=\frac{\int_{\alpha_{\min}}^{\alpha}(p(\alpha))^{1/(2n+1)}d\alpha}{\int_{\alpha_{\min}}^{\alpha_{\max}}(p(\alpha))^{1/(2n+1)}d\alpha}$

$$D\left(\alpha^{(r)}-\alpha^{(r+1)}\right)\propto\left(\frac{\alpha^{(r)}-\alpha^{(r+1)}}{\alpha^{(r)}+\alpha^{(r+1)}}\right)^{2n}\Rightarrow\frac{w(\alpha)-w(\alpha_{\min})}{w(\alpha_{\max})-w(\alpha_{\min})}=\frac{\int_{\alpha_{\min}}^{\alpha}\left(\frac{p(\alpha)}{\alpha^n}\right)^{1/(2n+1)}d\alpha}{\int_{\alpha_{\min}}^{\alpha_{\max}}\left(\frac{p(\alpha)}{\alpha^n}\right)^{1/(2n+1)}d\alpha}$$

for $p(\alpha)=1/(\alpha_{\max}-\alpha_{\min})$, $\frac{w(\alpha)-w(\alpha_{\min})}{w(\alpha_{\max})-w(\alpha_{\min})}=\frac{\alpha^{1/(2n+1)}-\alpha_{\min}^{1/(2n+1)}}{\alpha_{\max}^{1/(2n+1)}+\alpha_{\min}^{1/(2n+1)}}$

Practical aspects of element-wise quantization.

P-th law quantization $\frac{w(\alpha)-w(\alpha_{\min})}{w(\alpha_{\max})-w(\alpha_{\min})}=\frac{\alpha^P-\alpha_{\min}^P}{\alpha_{\max}^P-\alpha_{\min}^P}$.

Quantization in the presence of noise. Quantization in reconstructive tomography.

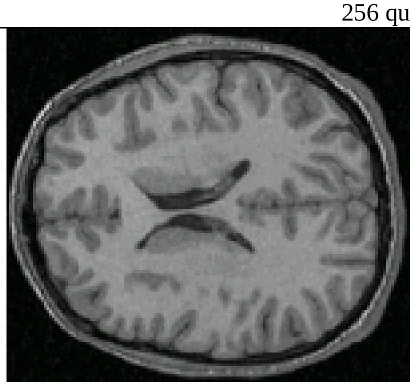
Problems:

1. What is quantization noise and how it is characterized.
2. What role play loss function and signal probability density in optimization of nonuniform quantization
3. Describe and compare Max-Lloyd quantization and compandor/expander quantization
4. Why 256 quantization levels in logarithmic scale were selected for image quantization? What is ***P***-th law quantization?
5. How additive noise may hide quantization artifacts?

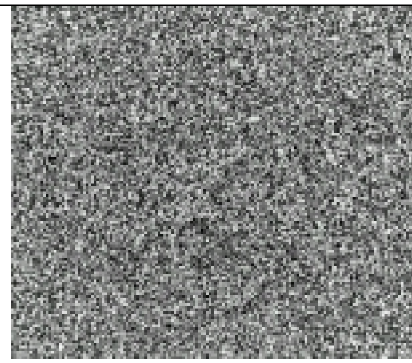
Home work: Simulate (in Matlab) ***P***-th law quantization and demonstrate improvement with respect to uniform quantization

“False contours” and other quantization artifacts

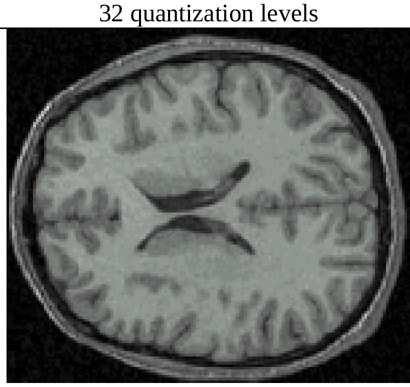
256 quantization levels



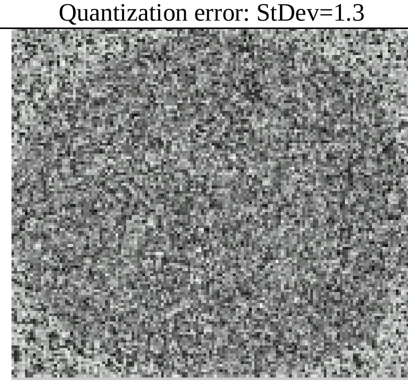
32 quantization levels



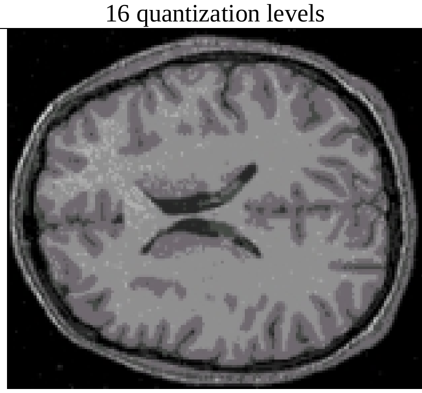
Quantization error: StDev=1.3



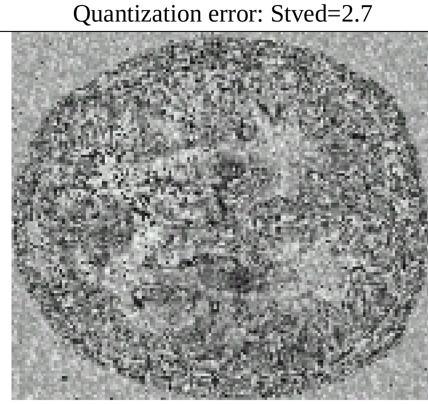
16 quantization levels



Quantization error: Stved=2.7



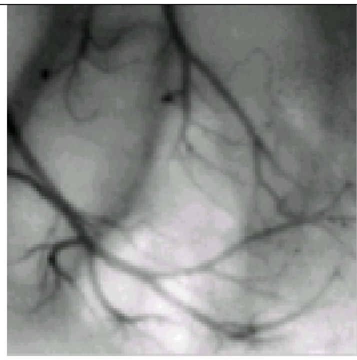
8 quantization levels



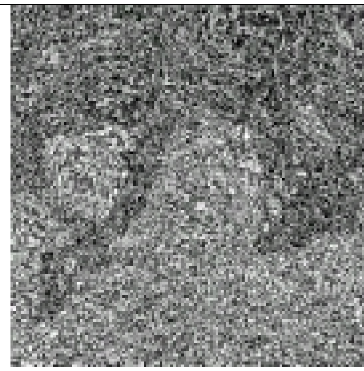
Quantization error: (St. Dev 4.9)

“False contours” and other quantization artifacts

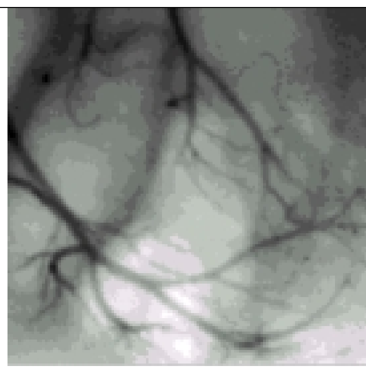
256 quantization levels



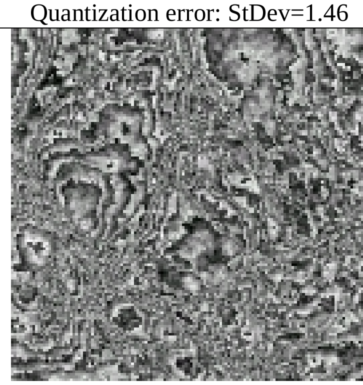
32 quantization levels



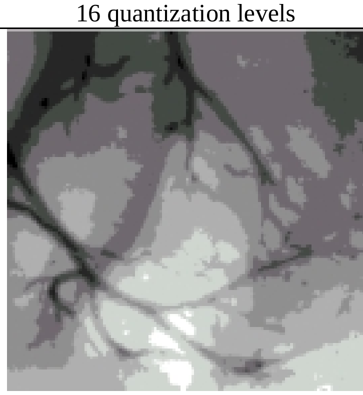
Quantization error: StDev=1.46



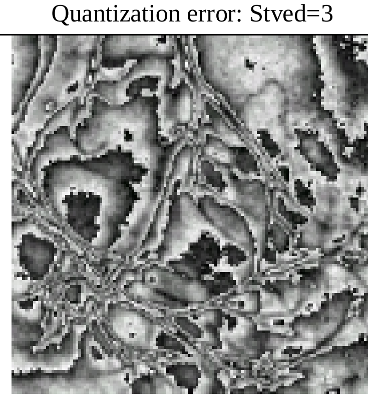
16 quantization levels



Quantization error: Stved=3

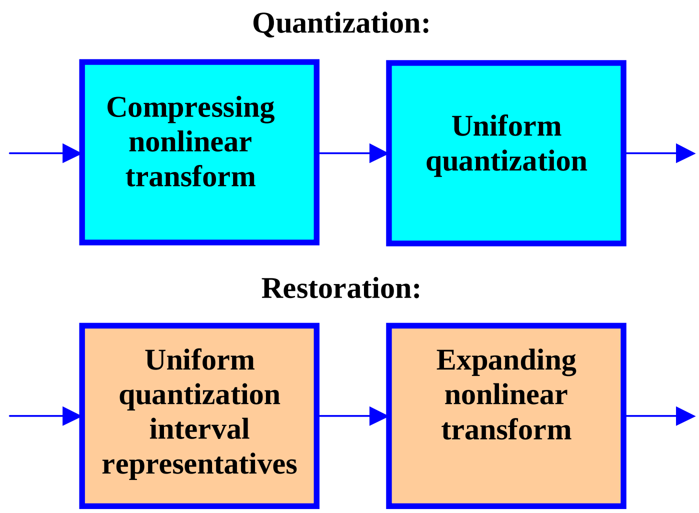


8 quantization levels

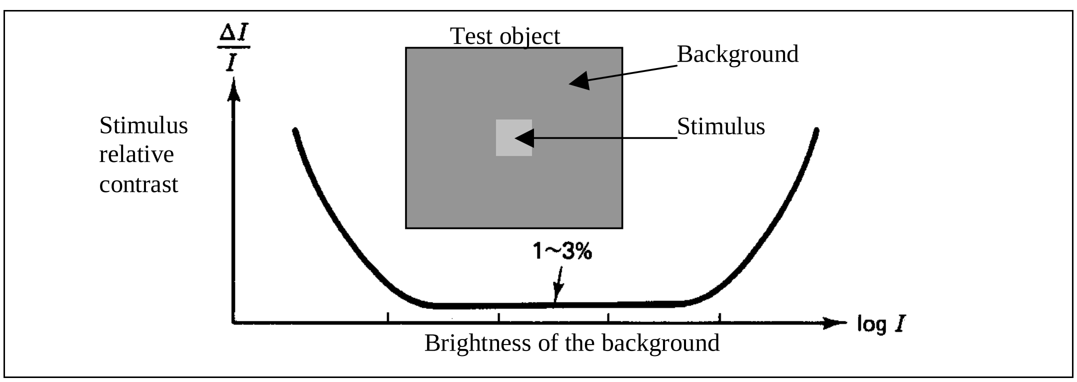


Quantization error: (St. Dev 6.4)

Element-wise quantization: compression-expanding method

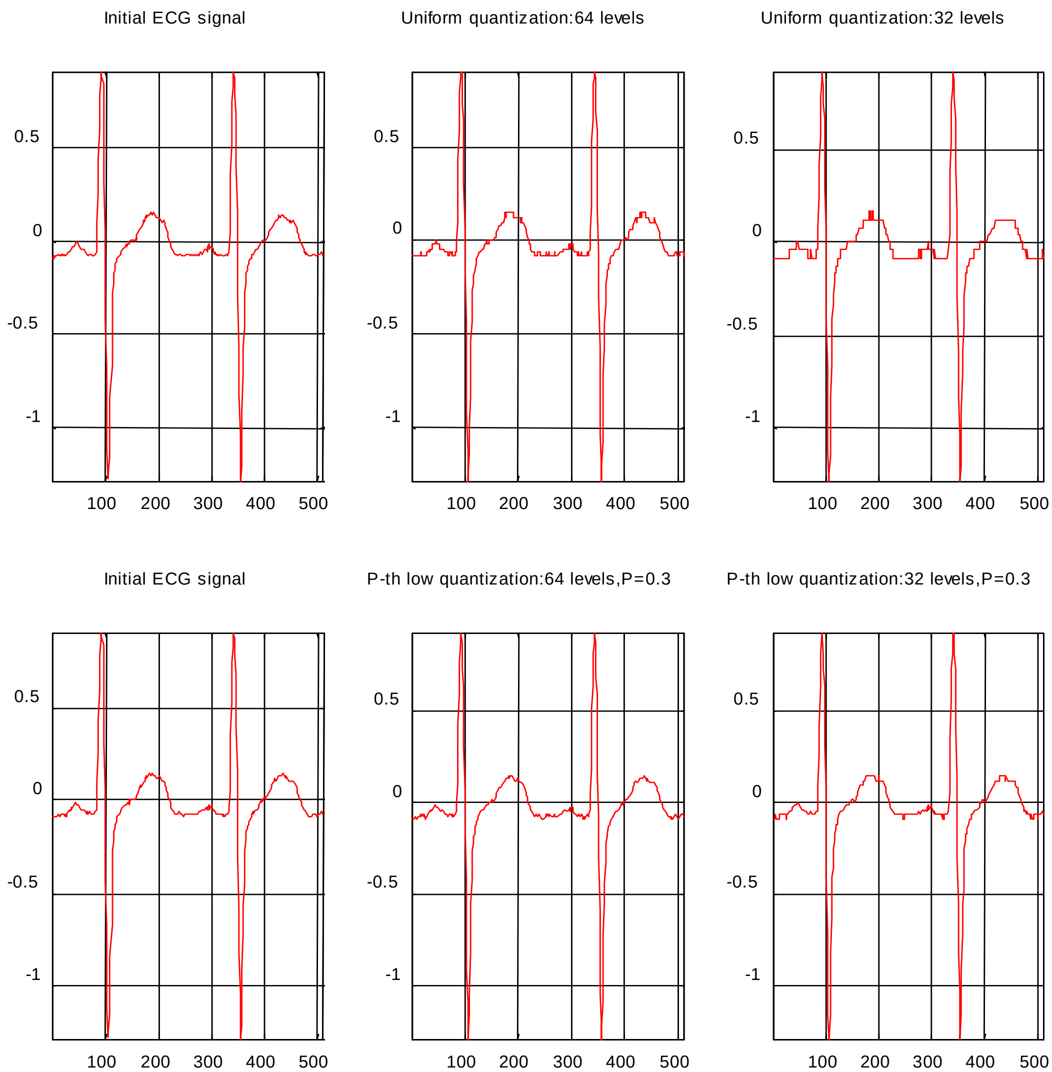


Weber-Fechner's law

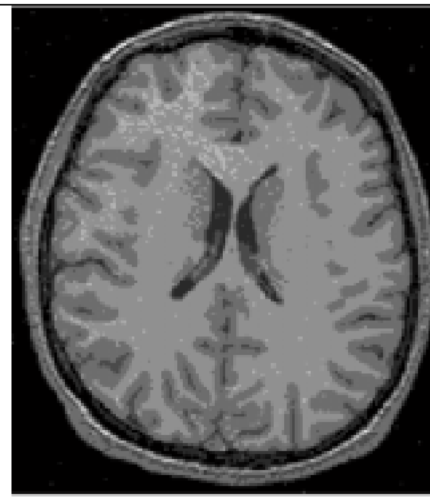


Contrast sensitivity of vision

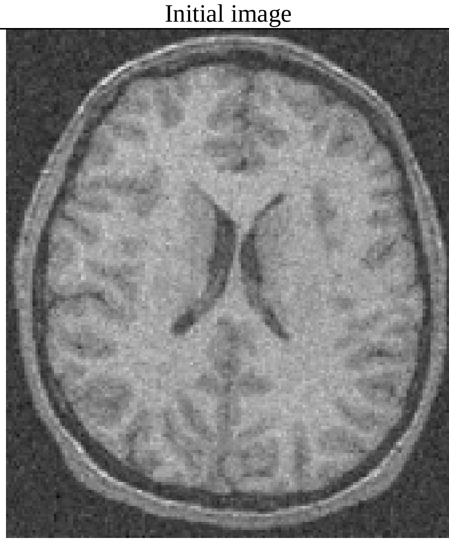
Quantization with *P*-th law nonlinearity



Initial image

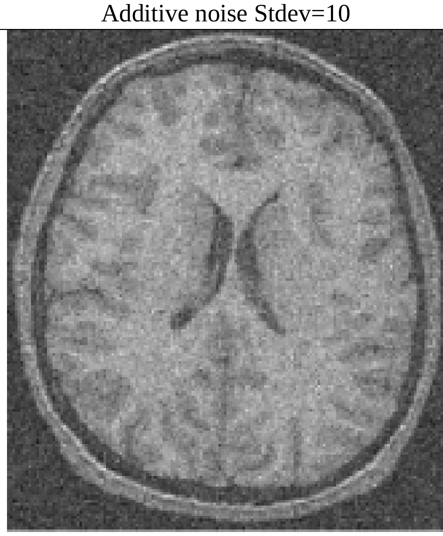


8 quantization levels

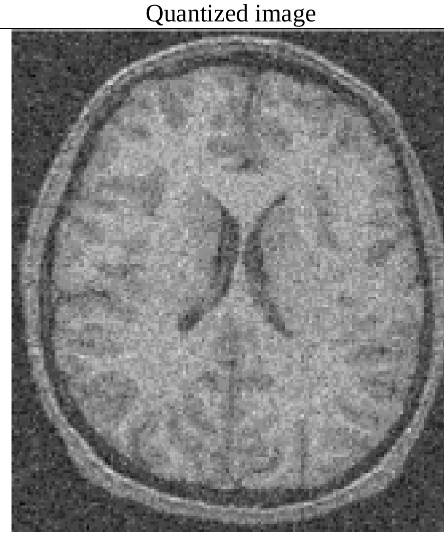


Additive noise Stdev=10

Quantized image



Additive noise Stdev=15



Quantized image

Quantization with nioise