Biomedical signal and image processing **(Course 055-355-5501)**

Lect. 6 Signal transformations

Signal transform as mapping in signal space. Basic types of signal transforms: element-wise and linear transforms. Linear transforms: superposition principle

Representation of linear transforms: $b(x) = \int_X a(\xi) h(x,\xi) d\xi$;

Impulse response (point spread function) of the linear transform.

Shift invariant transforms and signal convolution: $h(x,\xi) = h(x-\xi)$

$$b(x) = \int_{-\infty}^{\infty} a(\xi) h(x-\xi) d\xi \quad \delta(x,\xi) = \lim_{\Delta x \to 0} rect[(x-\xi)/\Delta x]$$

Fourier Transform.

$$\alpha(f) = \int_{-\infty}^{\infty} a(x)\exp(i2\pi f x)dx; \; a(x) = \lim_{F\to\infty} \int_{-F}^{F} \alpha(f)\exp(-i2\pi f x)df; \; \delta(x) = \lim_{F\to\infty} 2F \operatorname{sinc}(2\pi F(x-\xi))$$

Relationship between the signal convolution and Fourier transforms:

$$\beta(f) = \eta(f)\alpha(f); \qquad \eta(f) = \int_{-\infty}^{\infty} h(x)\exp(i2\pi f)dx; \qquad H(f,p) = \eta(f)\delta(f-p)$$

Point-spread function and frequency response of a general linear transform (linear filter):

$$\beta(f) = \int_F \alpha(p) H(f,p) dp \quad H(f,p) = \int_{-\infty}^{\infty} h(x,\xi)\exp[i2\pi(fx - p\xi)]dx d\xi$$

Basic properties of the Fourier Transform. Applications in acoustics, optics and MRI imaging.

Radon transform

$$\alpha(\xi_1,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a(x_1,x_2)\delta(x_1\cos\theta + x_2\sin\theta - \xi_1)dx_1 dx_2$$

$$= \int_{-\infty}^{\infty} a(\xi_1\cos\theta - \xi_2\sin\theta, \xi_1\sin\theta + \xi_2\cos\theta)d\xi_2$$

Projection theorem: $\int_{-\infty}^{\infty} \alpha(\xi_1,\theta)\exp(i2\pi f\xi_1)d\xi_1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a(x_1,x_2)\exp(i2\pi(fx_1\cos\theta + fx_2\sin\theta))dx_1 dx_2$

Principles of computer tomography.

Discrete representation of signal transformations

Digital representation of transforms parallels that of signals

Digital implementation of point-wise transforms: look-up-table; polynomial.

Digital representation of linear transforms. The conformity principle between analogue and digital transformations.

Digital filters: $\mathbf{b} = \{b_k\} = \mathbf{H}\cdot \boldsymbol{a} = \left\{\sum_n h_{k,n} a_k\right\}$. Shift invariant digital filters: $\mathbf{b} = \{b_k\} = \left\{\sum_n h_n a_{k-n}\right\}$

Continuous and discrete impulse response and frequency response of a digital filter:

$$h_{cnt}(x,\xi) = \sum\sum h_n \varphi_d[x-(k-n)\Delta x]\varphi_r(\xi - n\Delta x); \quad \mathbf{h}_{dscr} = \{h_n\}; H_{dscr}(f) = \left[\sum_{n=0}^{N_h-1} h_n \exp(i2\pi f n\Delta x)\right]$$

$$H_{cnt}(f,p) = \left[\sum_{n=0}^{N_h-1} h_n \exp(i2\pi p n\Delta x)\right]\left[\sum_{k=0}^{N_b-1}\exp(i2\pi(f-p)k\Delta x)\right]\Phi_r(-p)\Phi_d(f)$$

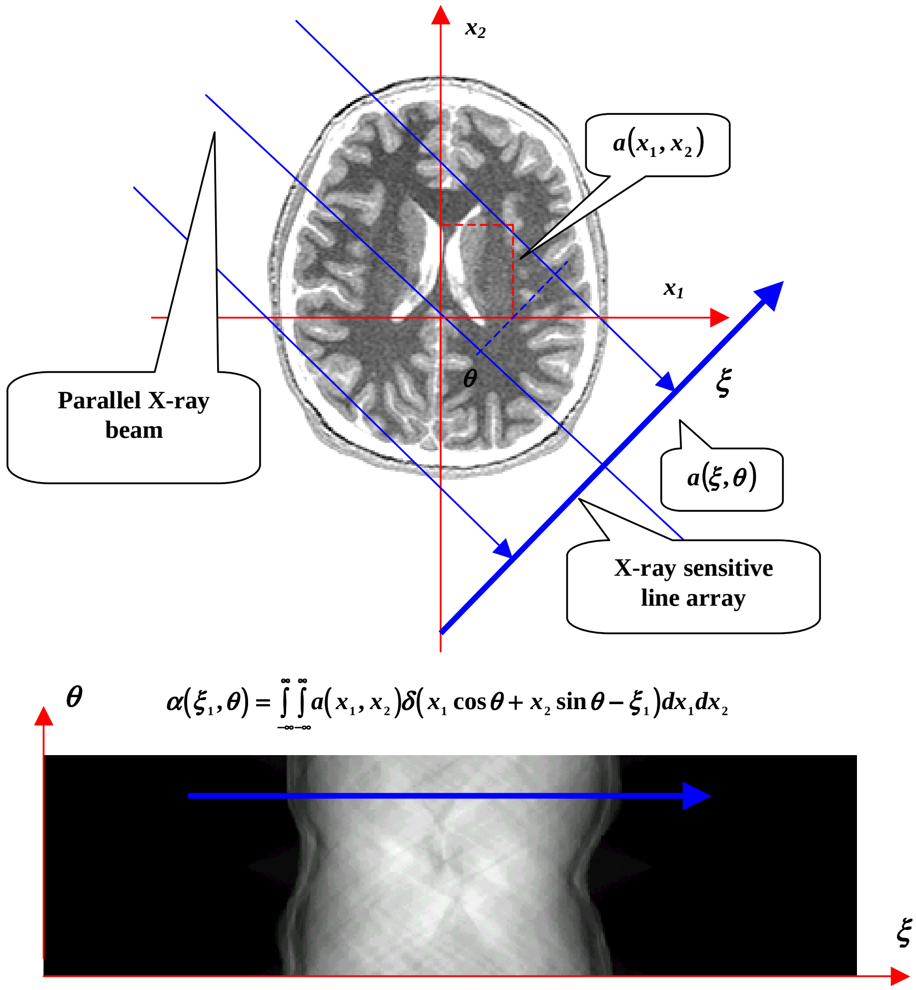
Problems:

1. What types of signal transforms do you know? Linear transforms and their characterization.
2. Basic properties of convolution and Fourier integral transforms.
3. Radon Transform and its relation to Fourier transform.
4. From the convolution integral, derive basic formula of a digital filter.

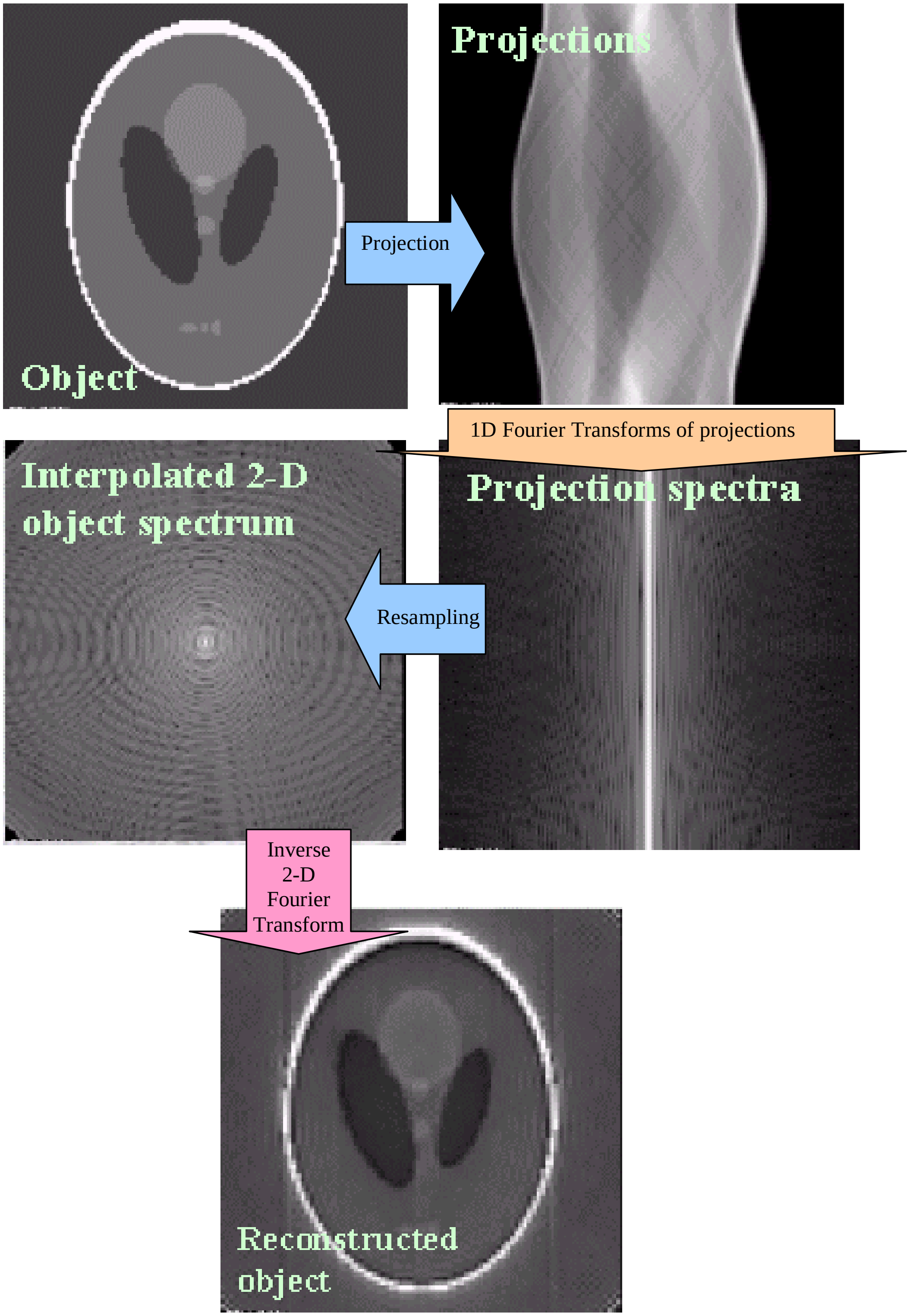
Home work: Derive discrete frequency response of a digital filter that computes signal local mean:

$$b_k = \frac{1}{2N+1}\sum_{n=-N}^{N} a_{k-n}$$

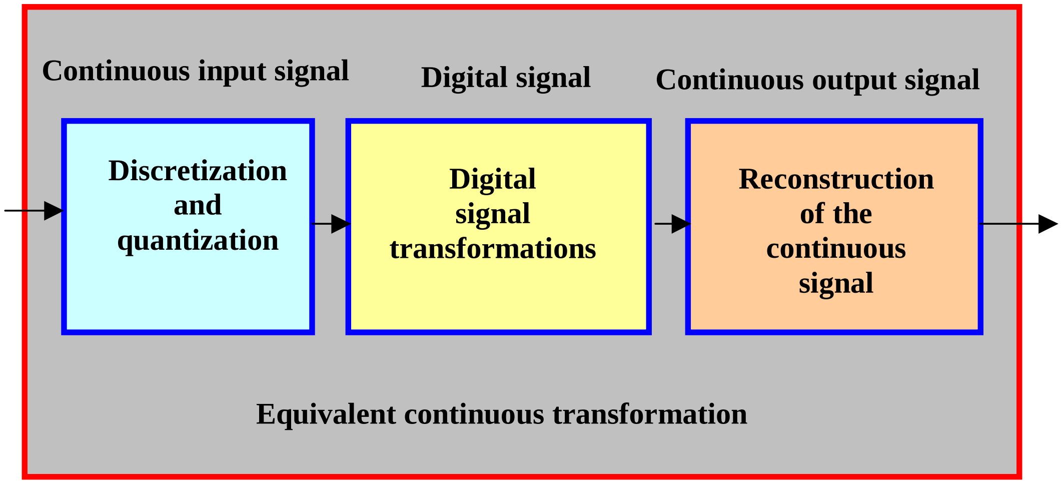
Illustrate it for several different values of ***N*.**



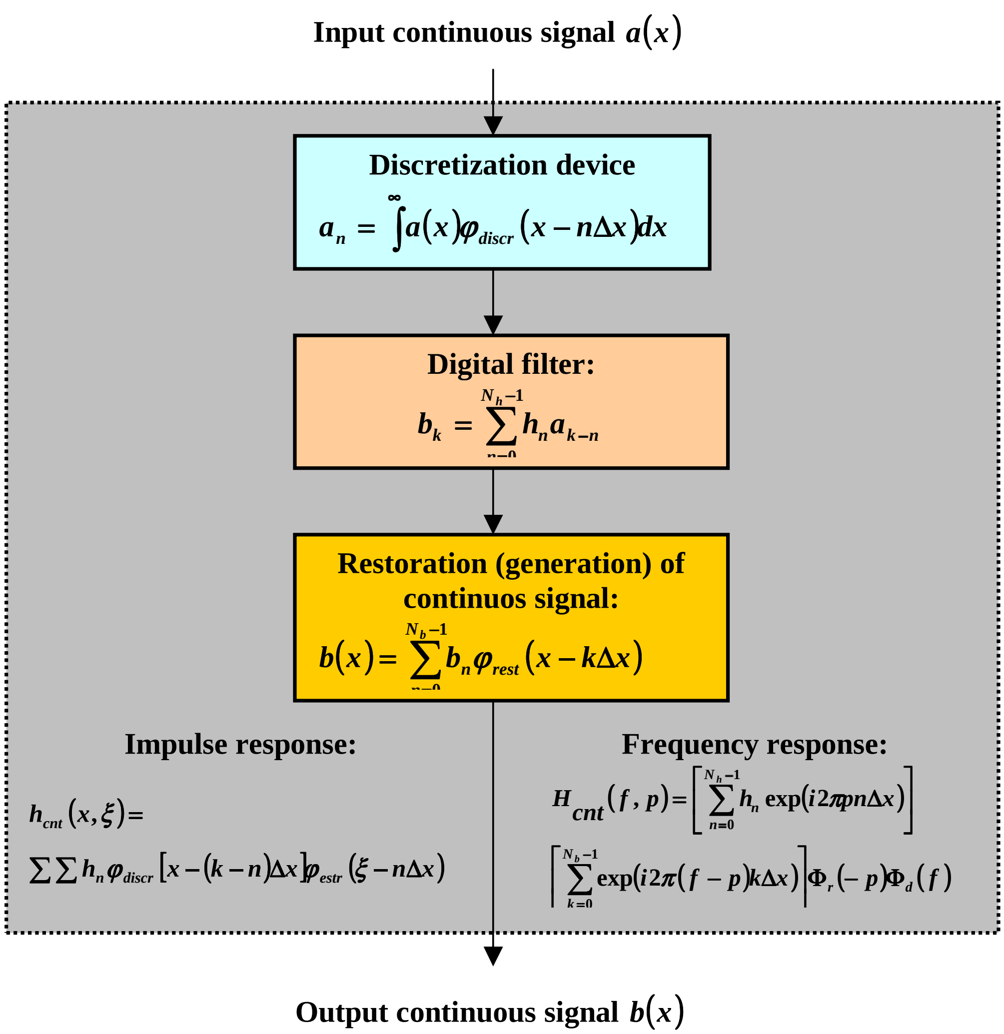
Schematic diagram of parallel beam projection tomography



Inverse Radon Transform via Fourier Transform: Fourier method of tomographic reconstruction



The conformity principle between analogue and digital transformations



Digital filtering and its equivalent continuous impluse and frequency response