Lect. 9. Signal parameter estimation and recognition.

Statistical formulation.

Additive white gaussian noise (AWGN) model:

$$r(x) = s(x,\rho) + n_x;\ \rho - \textit{unknown parameter}$$

Statistically optimal estimates: maximum a posteriori probability and maximum likelihood estimates.

$$p(\rho / r(x)) = \frac{p(r(x)/\rho)\,p(\rho)}{p(r(x))} \propto p_n(n(x) = r(x) - s(x,\rho))\,p(\rho) =$$

$$= \exp\Big(-\frac{1}{2\sigma_n^2}\sum_{k=0}\left|r_k - s_k(\rho)\right|^2 + \ln(p(\rho))\Big) \propto \exp\Big(-\frac{1}{2\sigma_n^2}\sum_{k=0}^{N-1}\left|s_k(\rho)\right|^2 + \frac{1}{\sigma_n^2}\sum_{k=0}^{N-1} r_k s_k(\rho) + \ln(p(\rho))\Big)$$

MAP-estimate: $\rho_{MAP}^{opt} = \arg\max_{\rho}\Big(\exp\Big(-\frac{1}{2\sigma_n^2}\sum_{k=0}^{N-1}\left|s_k(\rho)\right|^2 + \frac{1}{\sigma_n^2}\sum_{k=0}^{N-1} r_k s_k(\rho) + \ln(p(\rho))\Big)\Big)$

ML-estimate: $\rho_{ML}^{opt} = \arg\max_{\rho}\Big(\exp\Big(-\frac{1}{2\sigma_n^2}\sum_{k=0}^{N-1}\left|s_k(\rho)\right|^2 + \frac{1}{\sigma_n^2}\sum_{k=0}^{N-1} r_k s_k(\rho)\Big)\Big)$

Estimation of signal position (additive white Gaussian noise model).

$$(\hat{x}_0, \hat{y}_0) = \arg\max_{(x_0,y_0)}\left\{\frac{1}{\sigma_n^2}\sum_{k=0}^{N-1} r_k s_k(x_0,y_0) + \ln(p(x_0,y_0))\right\}$$

$$(\hat{x}_0, \hat{y}_0) = \arg\max_{(x_0,y_0)}\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} r(x,y)\,s(x-x_0, y-y_0) + N_0 \ln(p(x_0,y_0))\right\}$$

Correlation in frequency domain (matched filtering): $H_{opt}(f_x, f_y) = \alpha^{*}(f_x, f_y)$.

Non-white noise: arbitrary noise spectral density $N_n(f_x, f_y)$.

Whitening" filter: $H_w(f) = 1/(N_n(f))^{1/2}$

"Optimal filter"

$$H_{opt}(f_x, f_y) = \alpha^{*}(f_x, f_y) / N_n(f_x, f_y)$$

Optimality of "optimal filter" in terms of ratio of signal peak to standard deviation of noise at the filter output :

$$H_{opt}(\vec{f}) = \arg\max(PSNR) = \arg\max_{H}\left(\frac{\int \alpha(\vec{f})H(\vec{f})df}{\left(\int N_n(f)\left|H(f)\right|^2 df\right)^{1/2}}\right)$$

Accuracy and reliability of parameter estimation.

Normal and anormalous localization errors.

Variance of the localization error and probability of anomalous errors. Trade-off between localization accuracy and reliability. Reliability of signal (image) recognition, and distance in the signal space. Recognition of signals (objects) known to the accuracy of some unknown parameters (scale, rotation, etc.). Trade-off between the recognition invariance to unknown parameters and reliability of recognition.

Applications: measurement of physiological data such as heart beat variations, lung volume, dimensions of organs and blood vessels, velocity of blood stream, tumor detection, image and signal alignment

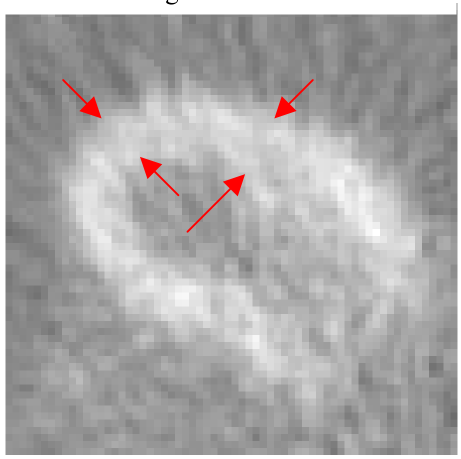
Problems:

1. Formulate statistical approach to signal parameter estimation. MAP and ML estimates.
2. Prove optimality of correlation techniques.
3. Explain matching filtering as a method for determining signal shift and target location.
4. Describe two types of errors in parameter estimation.
5. Give examples of application of correlation techniques for processing biomedical signals.

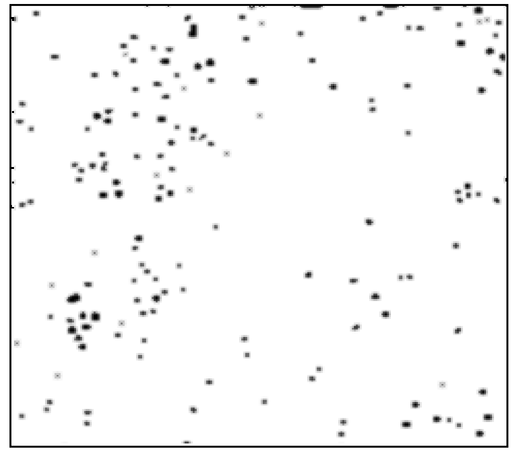
Home work: Using Matlab functions, demonstrate template matching on 1D signals or/and images

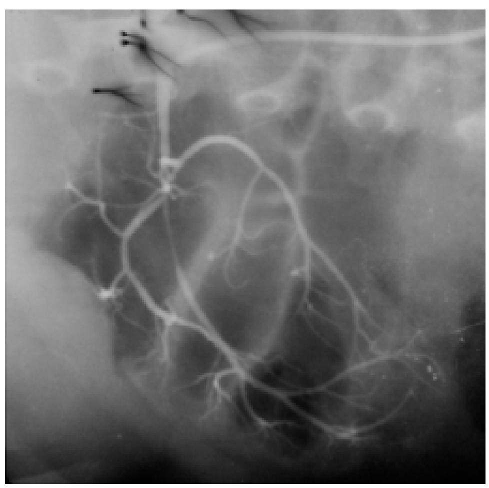
Biomedical signal parameter estimation tasks: examples

Measuring heart wall thickness

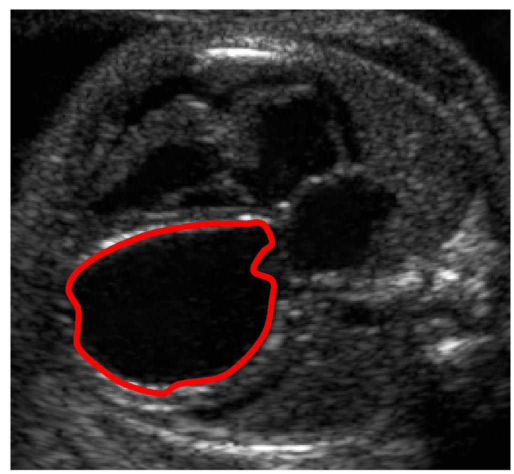


Counting particles and measuring sizes



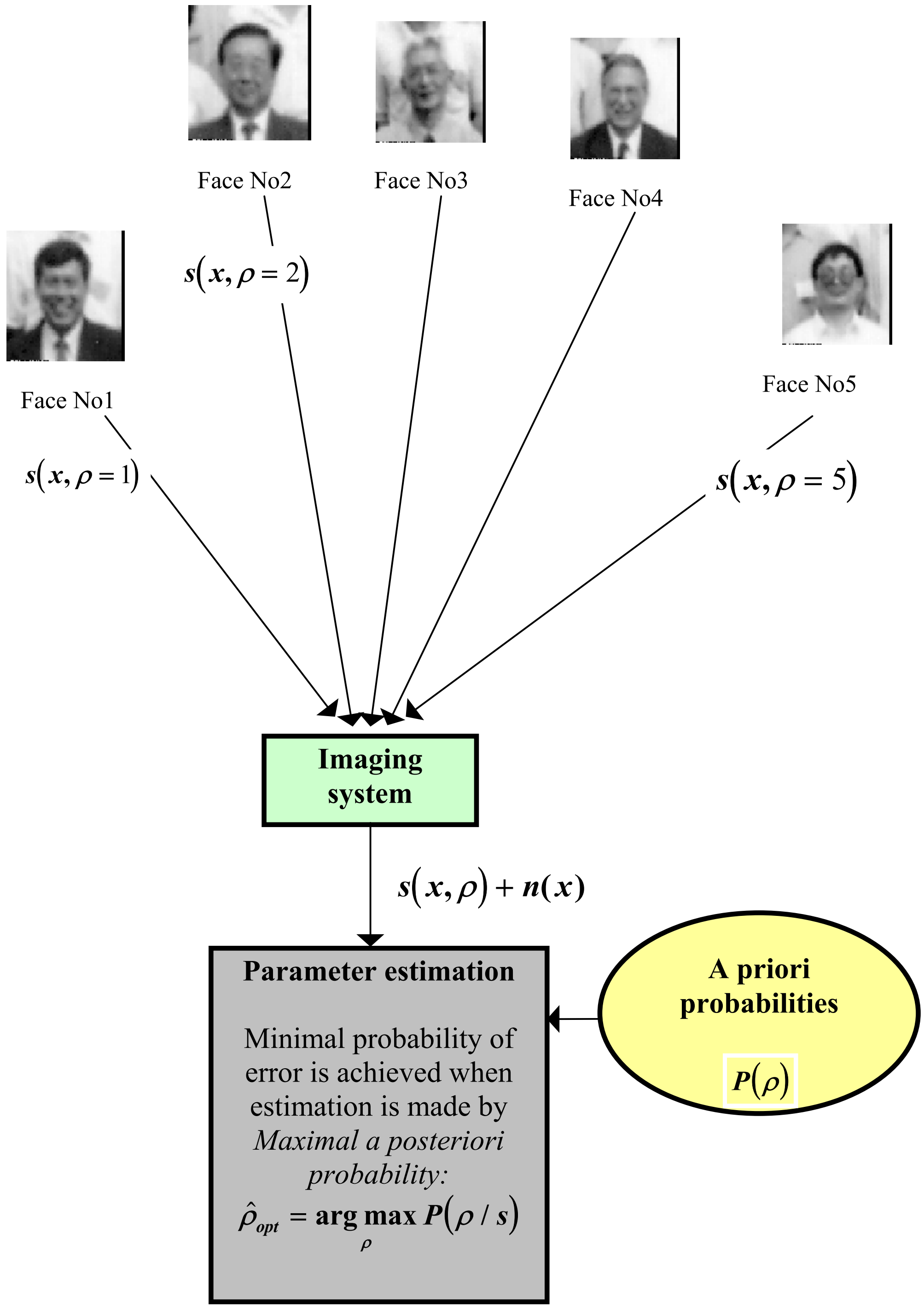


Measuring blood flow

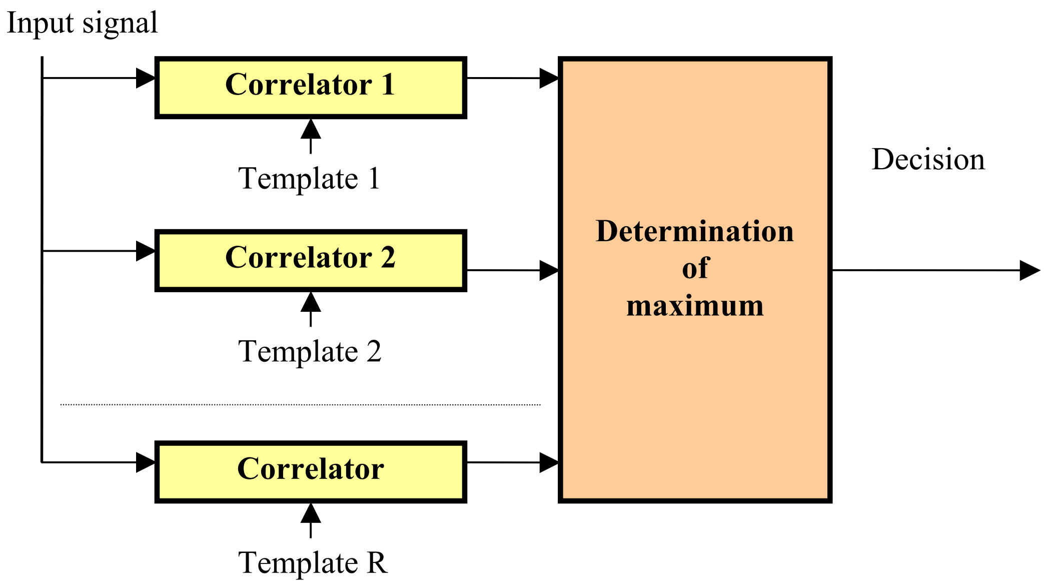


Measuring organs' dimensions

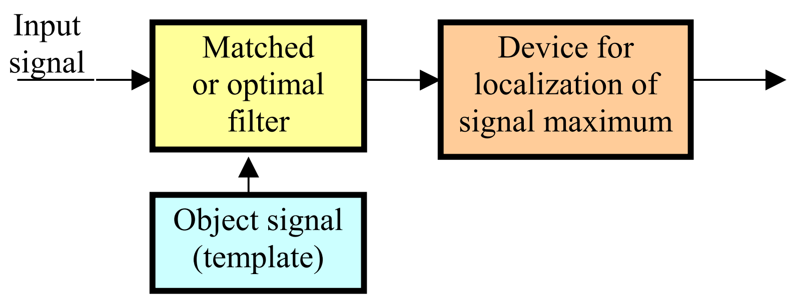
Detecting microcalcifications in mammograms



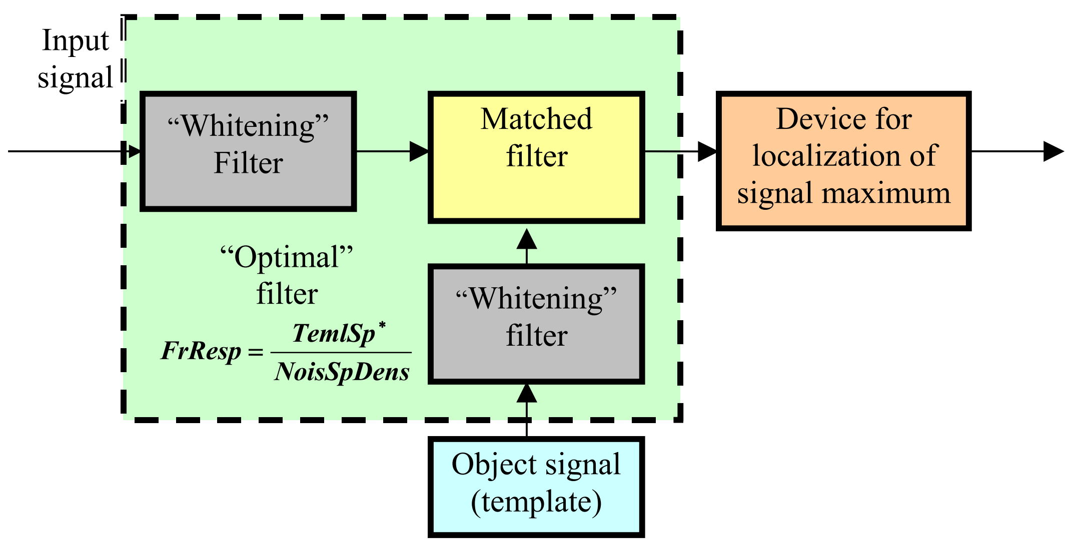
Parameter estimation statistical formulation



White Gaussian noise model: optimal recognition device

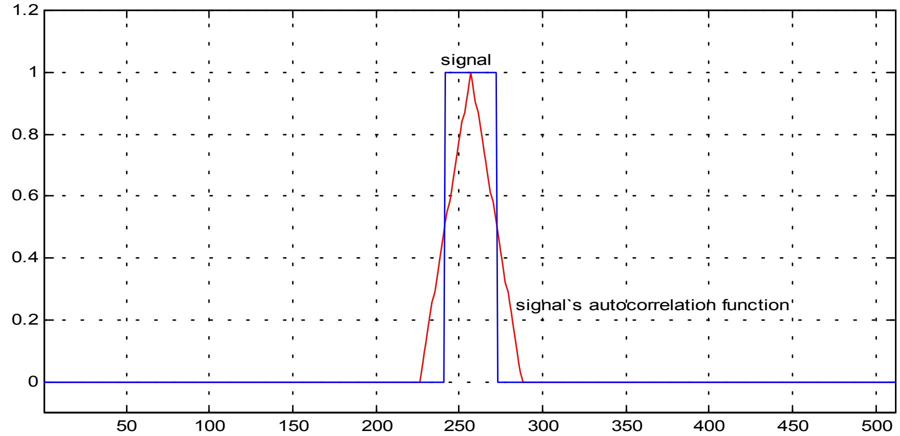


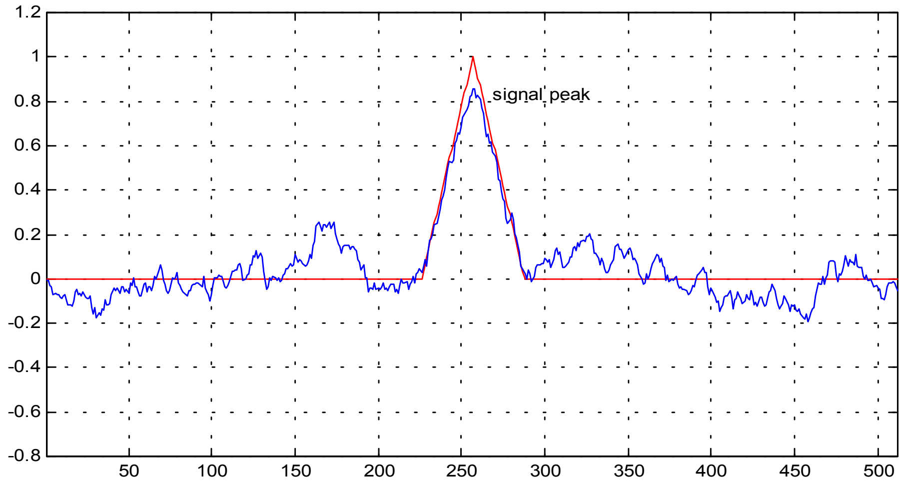
White Gaussian noise model: optimal localization device (template matching)



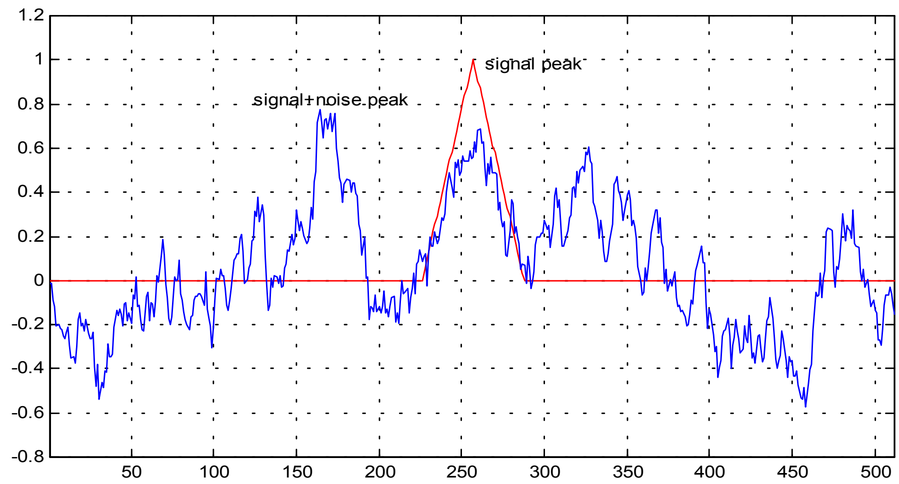
“Optimal filter”: template matching for non-white noise model

TWO TYPES OF THE LOCALIZATION ERRORS





Correlator’s output: the case of “normal” error



Correlator’s output: the case of anomalous error

VARIANCE OF THE LOCALIZATION ERRORS

$$\sigma_x^2 = \frac{1}{4\pi^2} \frac{\bar{f}_y^2}{\bar{f}_x^2 \bar{f}_y^2 - \left(\bar{f}_{xy}^2\right)^2} \frac{N_0}{E_a};$$

$$\sigma_y^2 = \frac{1}{4\pi^2} \frac{\bar{f}_x^2}{\bar{f}_x^2 \bar{f}_y^2 - \left(\bar{f}_{xy}^2\right)^2} \frac{N_0}{E_a};$$

$$\sigma_{xy}^2 = \frac{1}{4\pi^2} \frac{\bar{f}_{xy}^2}{\bar{f}_x^2 \bar{f}_y^2 - \left(\bar{f}_{xy}^2\right)^2} \frac{N_0}{E_a}.$$

where

$$E_a = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left|\alpha\left(f_x, f_y\right)\right|^2 df_x df_y \text{ - signal energy}$$

$$\bar{f}_x^2 = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_x^2 \left|\alpha\left(f_x, f_y\right)\right|^2 df_x df_y}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left|\alpha\left(f_x, f_y\right)\right|^2 df_x df_y}$$

$$\bar{f}_y^2 = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_y^2 \left|\alpha\left(f_x, f_y\right)\right|^2 df_x df_y}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left|\alpha\left(f_x, f_y\right)\right|^2 df_x df_y}$$

$$\bar{f}_{xy}^2 = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_x f_y \left|\alpha\left(f_x, f_y\right)\right|^2 df_x df_y}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left|\alpha\left(f_x, f_y\right)\right|^2 df_x df_y}$$

For one-dimensional signals:

$$\sigma_x^2 = \frac{1}{\bar{f}_x^2} \frac{N_0}{4\pi^2 E_a} = \frac{N_0}{E_{aa}},$$

where $E_{aa} = 4\pi^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_x^2 \left|\alpha\left(f_x, f_y\right)\right|^2 df_x df_y$ - energy of signal's derivative.

LOCALIZATION RELIABILITY

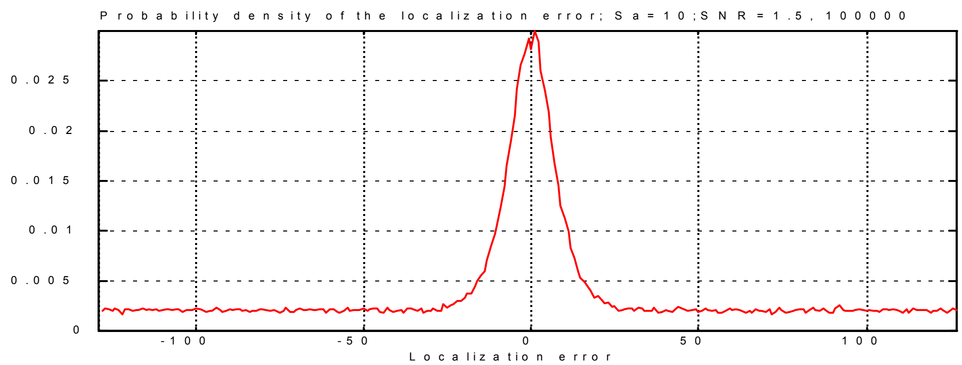
Probability of anomalous (false detection) errors:

$$P_{ae} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{n^2}{2}\right) \left\{ 1 - \left[\Phi\left(\sqrt{\frac{E_a}{N_0}} + n \right) \right]^{Q-1} \right\} dn$$

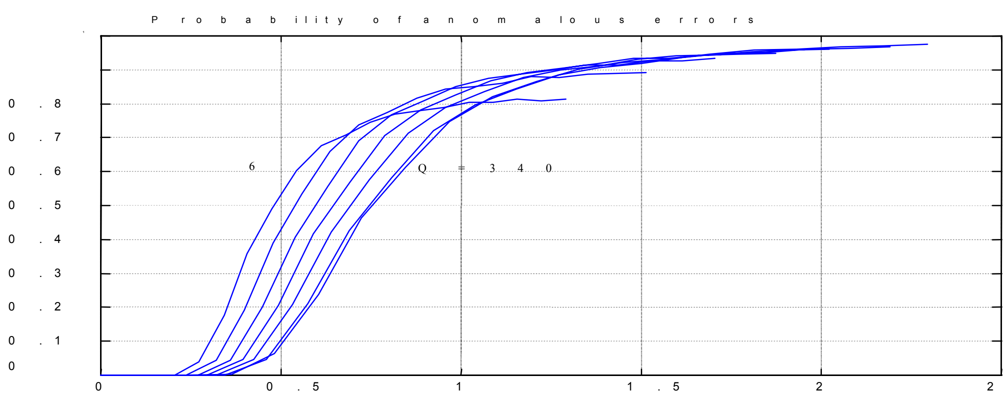
where $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{n^2}{2}\right) dn$, $Q \approx \dfrac{AreaOfSearch}{SignalCorrelationInterval}$

The localization reliability threshold:

$$\lim_{Q\to\infty} P_{ae} = \begin{cases} 1, \ if \ E_a / 2N_0 \le \ln Q \\ 0, \ if \ E_a / 2N_0 > \ln Q \end{cases}$$

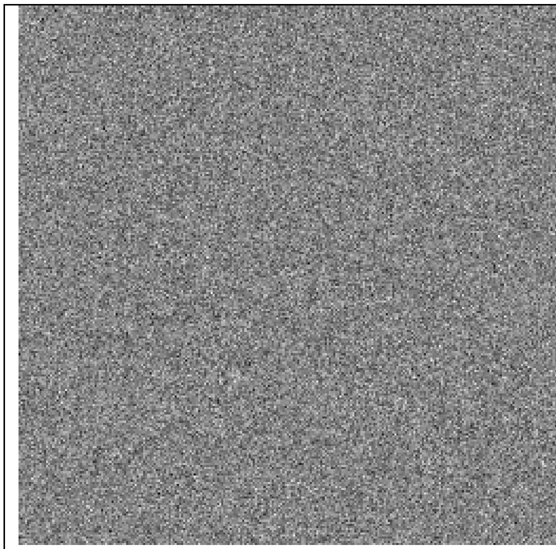


Experimental distribution density of the localization error for Gaussian-shaped impulse ($\sigma_a = 10$) and $\sqrt{E_a / N_0} = 1.5$ (100000 realizations)

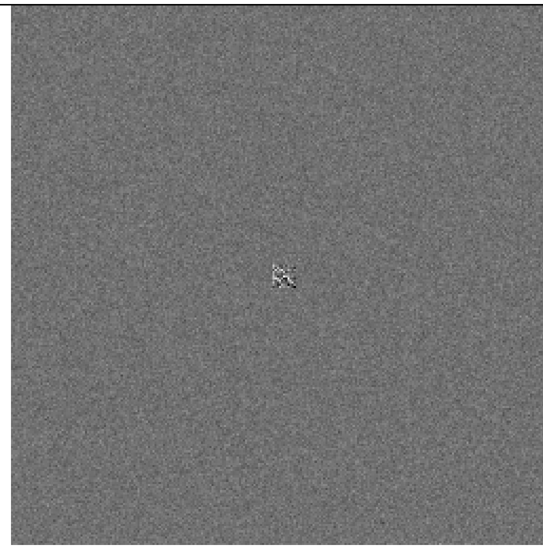


Probability of anomalous errors as a function of normalized noise-to-signal ratio $\sqrt{N_0 \ln Q / E_a}$ for localization of rectangle impulses of 2;5;11;21;41;81;161 samples within an interval of 1024 samples (10000 realizations). The theoretical threshold value of the normalized noise-to-signal ratio is $\sqrt{2}/2 \approx 0.707$

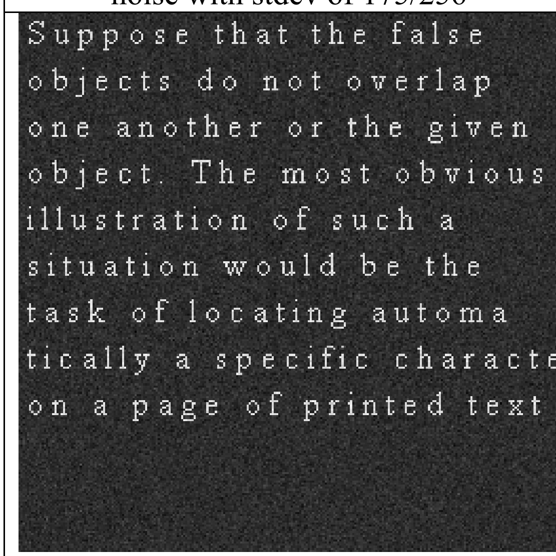
RELIABILITY OF LOCALIZATION OF A TARGET IN THE ABSENSE AND PRESENCE OF NONOVERLAPPING FOREIGN OBJECTS



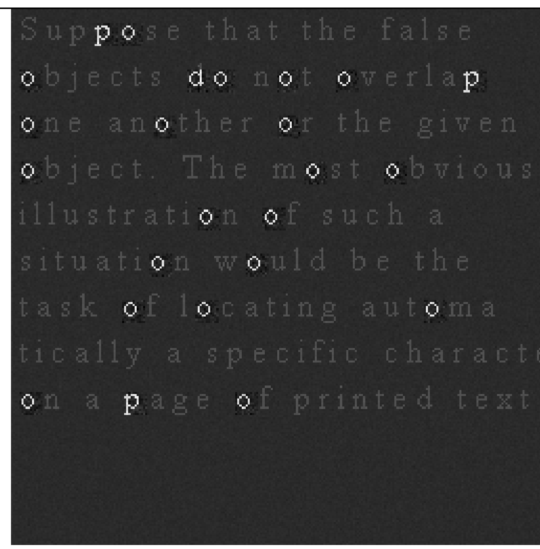
Character “O” hidden in the observation noise with stdev of 175/256



Result of localization with matched filter



A fragment of a printed text with the observation noise with stdev of 15/256;



Result of localization of character “O” with matched filter