Lecture 12. Correlational accumulation (averaging) as a method for signal restoration

Applications where multiple copies of signals are available: Periodical and quasi periodical physiological signals (such as ECG, EEG), electron micrographs, video frame sequencies of still objects.

The task of signal restoration from multiple copies.

1. Mathematical model:

$$\{b_q(k) = a(k - k_q) + n_q(k)\}, \qquad q = 1,...,Q,$$

where $\{b_q(k)\}$ are observed signal copies; $a(k)$ - signal to be restored; $\{n_q(k)\}$ - realizations of sensor noise, Q -number of copies

2. Correlational Accumulation: principle and optimality

$$\hat{a}(k) = \underset{\{k_q\}\{a(k)\}}{\arg\min}\left\{\sum_{q=1}^{Q}\frac{1}{\sigma_q^2}\sum_{k=0}^{N-1}\left|b_q(k) - a(k - k_q)\right|^2\right\} =$$

$$\underset{\{k_q\}\{a(k)\}}{\arg\min}\left\{\sum_{q=1}^{Q}\frac{1}{\sigma_q^2}\sum_{k=0}^{N-1}\left|b_q(k)\right|^2 - 2\sum_{q=1}^{Q}\frac{1}{\sigma_q^2}\sum_{k=0}^{N-1}b_q(k)a(k - k_q) + \sum_{q=1}^{Q}\frac{1}{\sigma_q^2}\sum_{k=0}^{N-1}\left|a(k - k_q)\right|^2\right\} \qquad \Rightarrow$$

- Signal shifts $\{\hat{k}_q\}$ are first estimated that maximize the correlation term
- Signal estimation is found by averaging aligned observed signals:

$$\hat{a}_k = \left(\sum_{q=1}^{Q}\frac{1}{\sigma_q^2}b_q(k - \hat{k}_q)\right) \Big/ \left(\sum_{q=1}^{Q}\frac{1}{\sigma_q^2}\right)$$

3. Expected performance of the Correlational Accumulation

Given channel noise levels $\{\sigma_q\}$, signal estimation quality is affected by:

- misalignment errors of noisy signal copies $\{b_q(k - \hat{k}_q)\}$ (normal and anomalous ones);
- the number of signal copies available

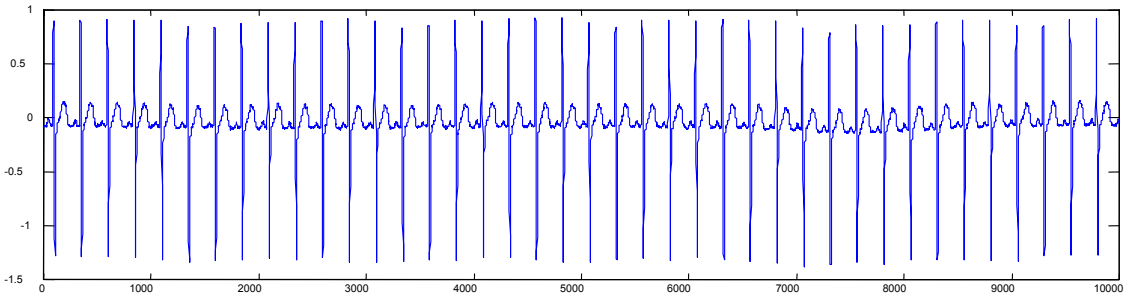
Normal misalignment errors limit additive noise variance reduction factor to $Q \cdot (1 - P_{an.err})$ and cause restored signal blur with PSF determined by variance of normal errors.

Anomalous misalignment errors cause a spurious signal resulted from accumulation of noise realizations that exhibited high correlation with the signal template.

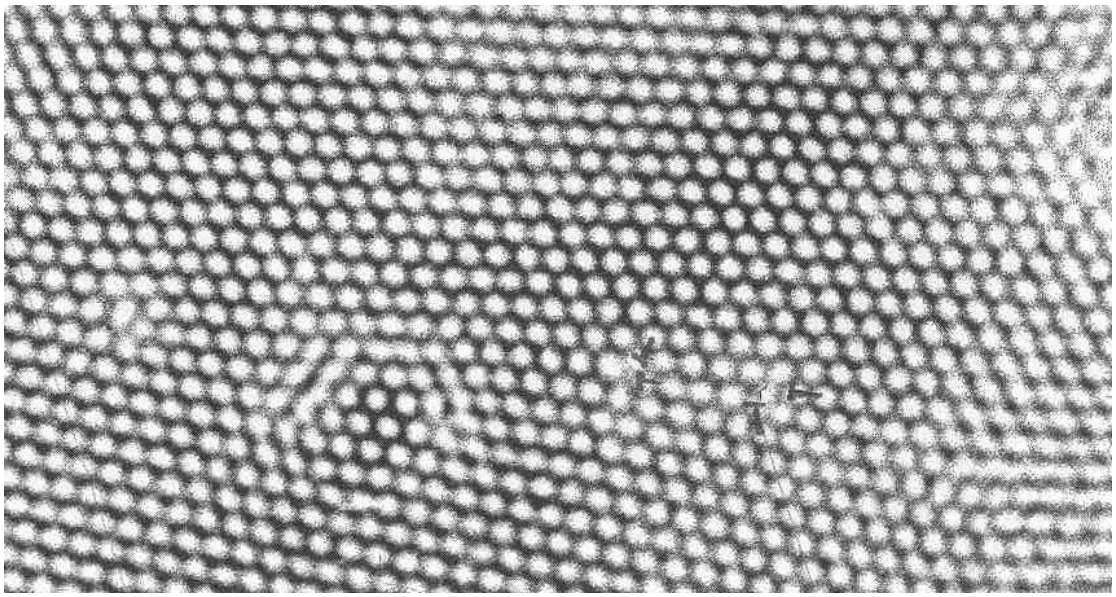
4. Iterative restoration algorithm: one of the noisy realizations is used as an intial template which is then replaced, at each new iteration, by the estimated signal obtained on the previous iteration.
5. Two-channel algorithm: correlational signal accumulation is supplemented with accumulation of the signal power Fourier spectra. The former is then used to compute phase component of the restored signal Fourier spectrum; the latter provides estimation of the magnitude of the spectrum. This allows to reduce signal blur due to misalignment errors.

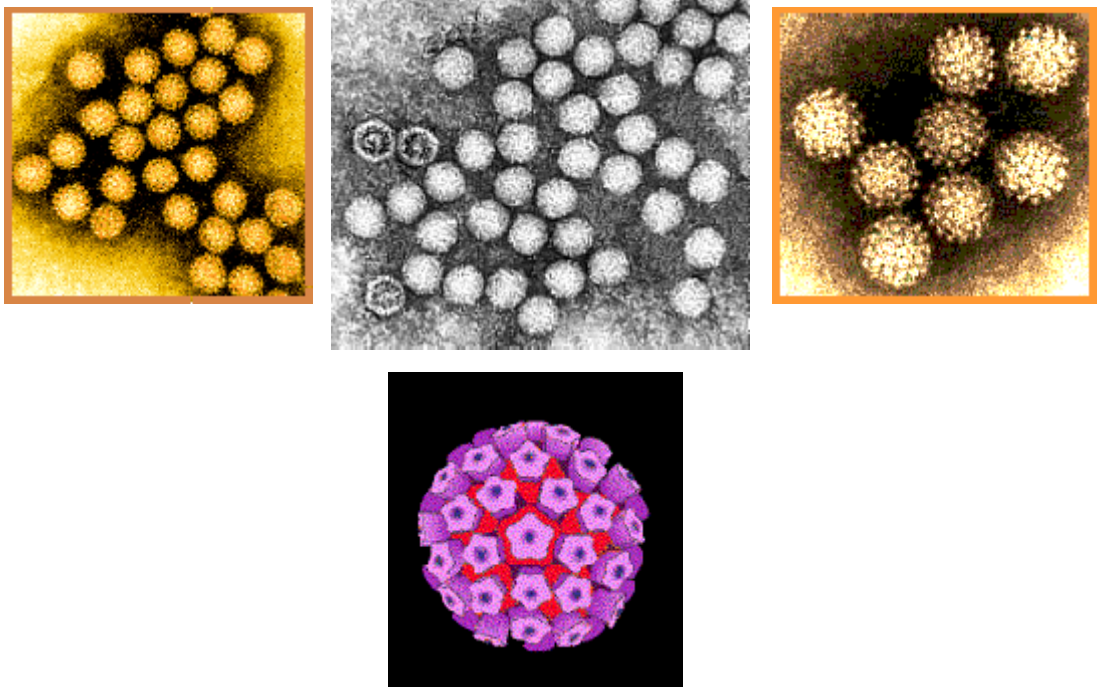
References:

1. A. Cohen, Biomedical Signal Processing, v. 1., Time and Frequency Domain Analysis, CRC Press, Roca Raton, F1, 1986
2. N. El Sherif, G. Turitto, eds., High Resolution Electrocardiography, Futura Mount Kisco, N.Y., 1992
3. L. Yaroslavsky, M. Eden, Correlational Accumulation as a Method for Signal Restoration, Signal Processing, 39 (1994) pp. 89-106

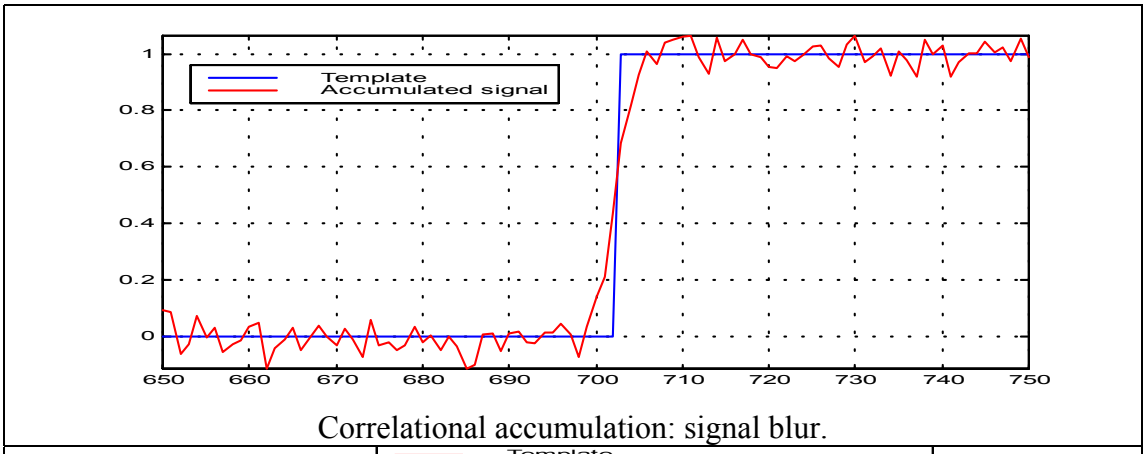


Electrocardiogram signal

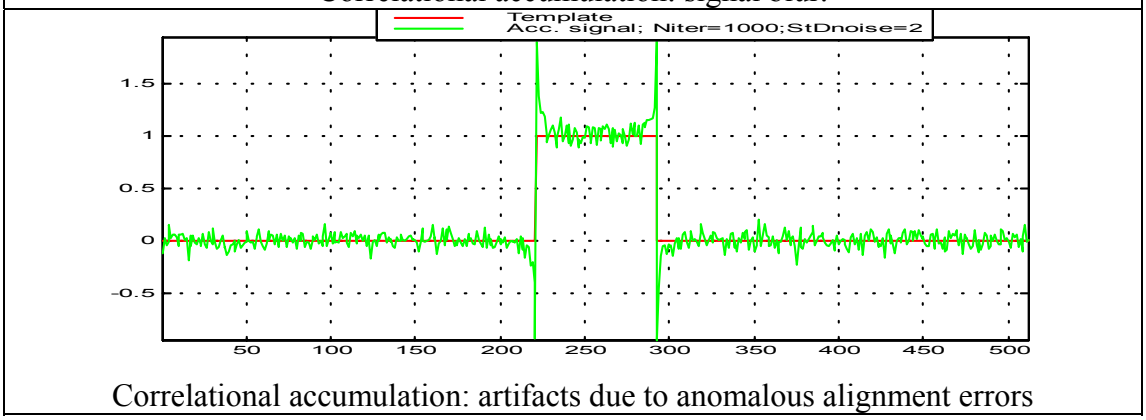




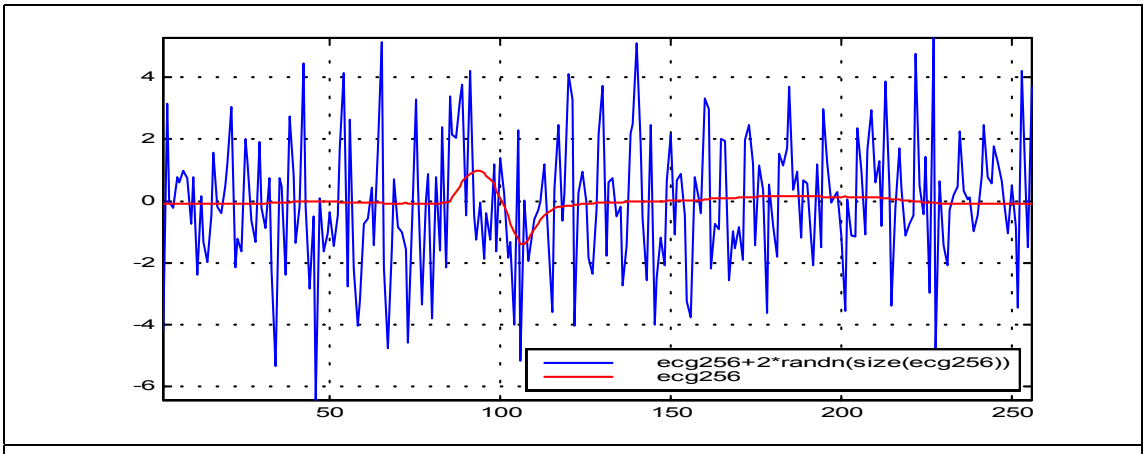
Examples of electron micrograph virus particles and 3D reconstruction of virus

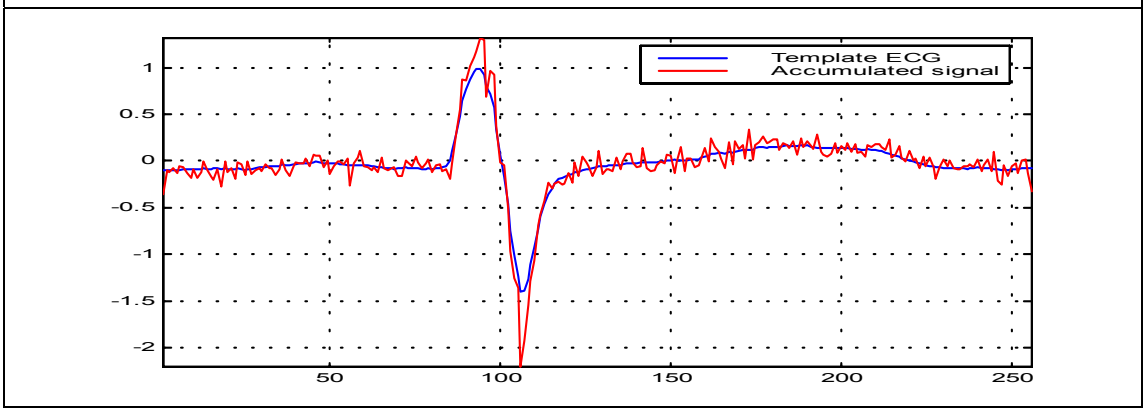


Correlational accumulation: signal blur.

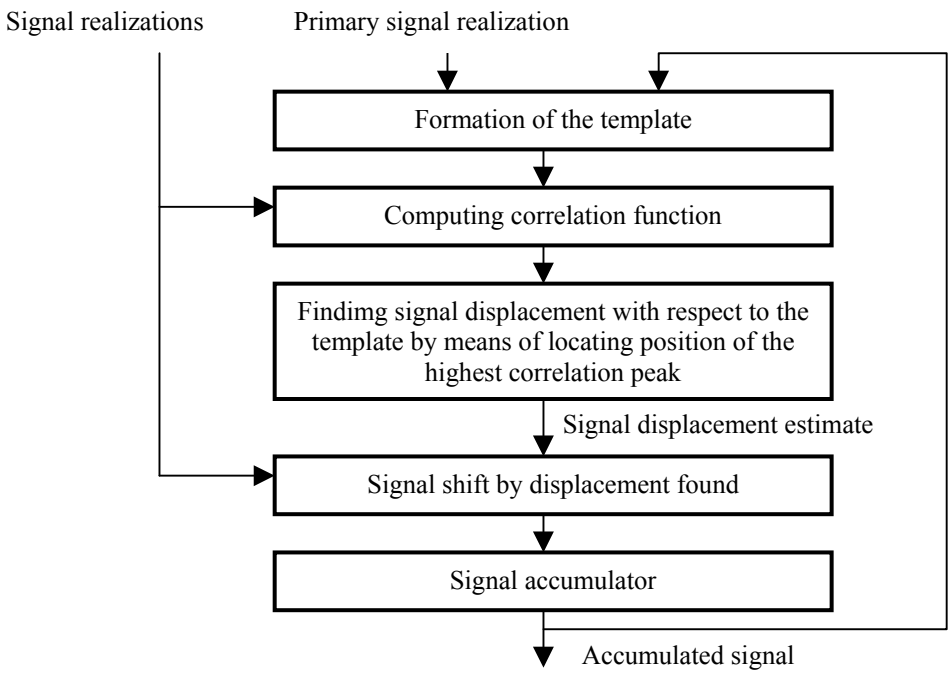


Correlational accumulation: artifacts due to anomalous alignment errors

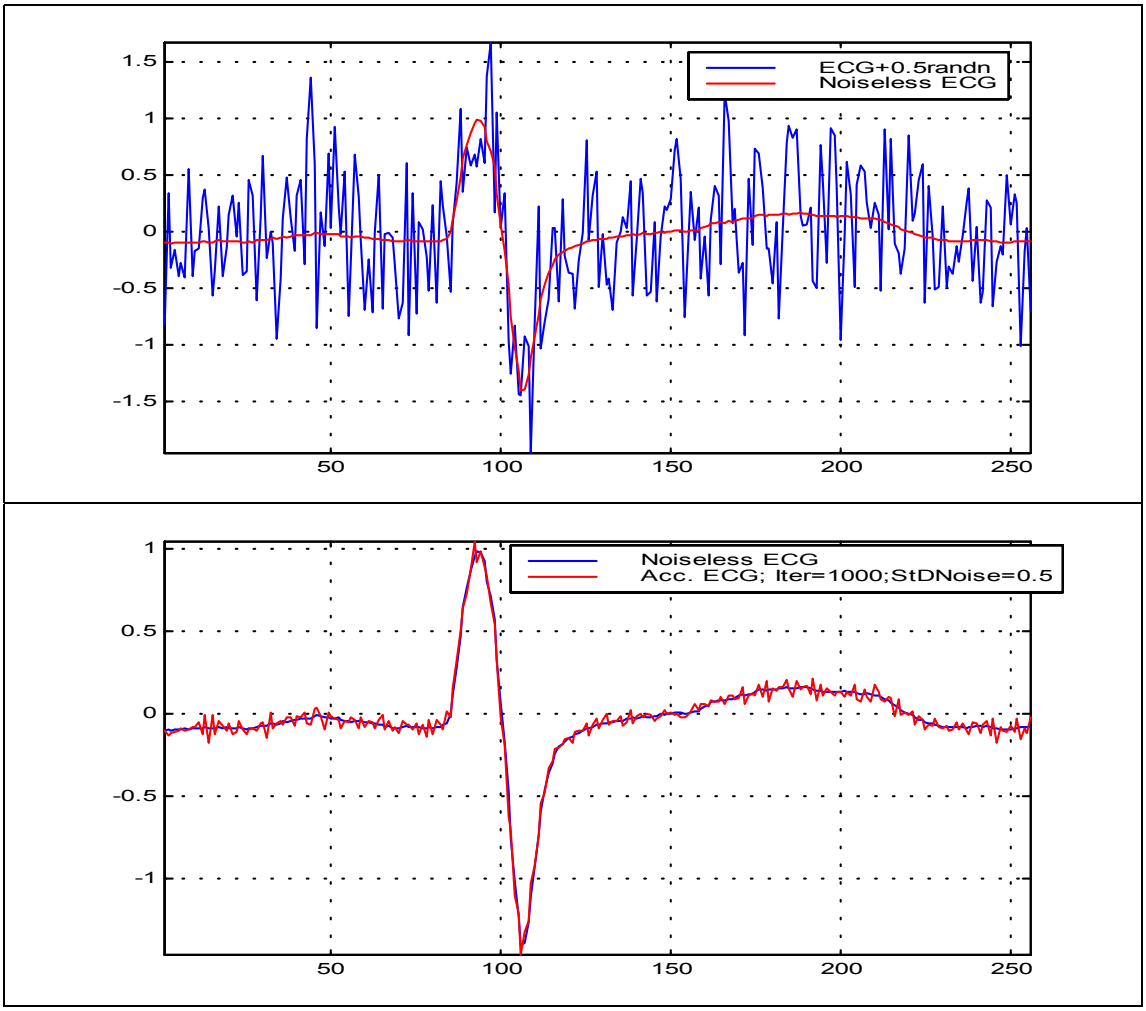




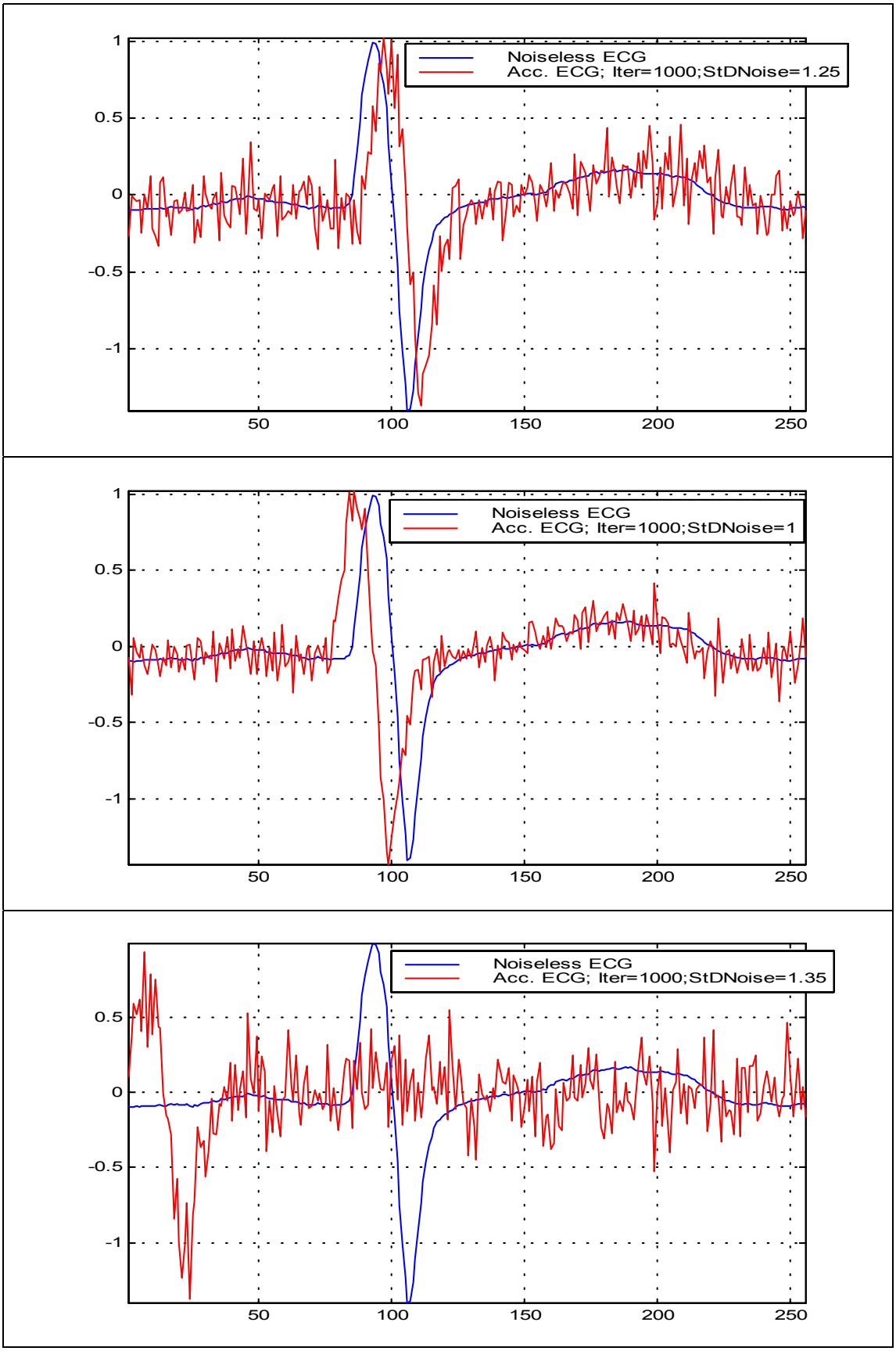
Correlational accumulation of a piece of ECG: 1000 iterations; StDevNoise=2



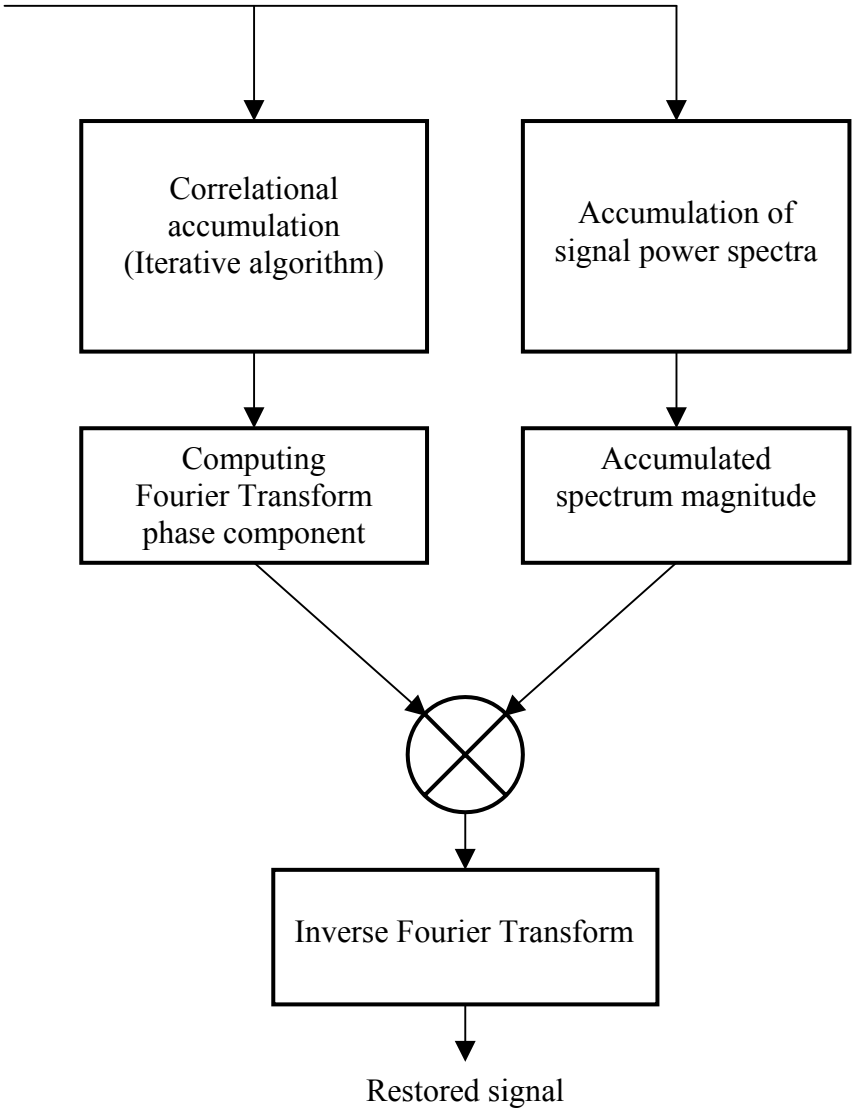
Iterative correlational accumulation flow diagram



Iterative correlational accumulation



Iterative correlational accumulation: threshold effect



Two channel restoration algorithm