

## Lecture 12. Correlational accumulation (averaging) as a method for signal restoration

Applications where multiple copies of signals are available: Periodical and quasi periodical physiological signals (such as ECG, EEG), electron micrographs, video frame sequences of still objects.

The task of signal restoration from multiple copies.

1. Mathematical model:

$$\{b_q(k) = a(k - k_q) + n_q(k)\}, \quad q = 1, \dots, Q,$$

where  $\{b_q(k)\}$  are observed signal copies;  $a(k)$  - signal to be restored;  $\{n_q(k)\}$  - realizations of sensor noise,  $Q$  - number of copies

2. Correlational Accumulation: principle and optimality

$$\hat{a}(k) = \underset{\{k_q\}_{\{a(k)\}}}{\operatorname{arg\,min}} \left\{ \sum_{q=1}^Q \frac{1}{\sigma_q^2} \sum_{k=0}^{N-1} |b_q(k) - a(k - k_q)|^2 \right\} =$$

$$\underset{\{k_q\}_{\{a(k)\}}}{\operatorname{arg\,min}} \left\{ \sum_{q=1}^Q \frac{1}{\sigma_q^2} \sum_{k=0}^{N-1} |b_q(k)|^2 - 2 \sum_{q=1}^Q \frac{1}{\sigma_q^2} \sum_{k=0}^{N-1} b_q(k) a(k - k_q) + \sum_{q=1}^Q \frac{1}{\sigma_q^2} \sum_{k=0}^{N-1} |a(k - k_q)|^2 \right\} \Rightarrow$$

- Signal shifts  $\{\hat{k}_q\}$  are first estimated that maximize the correlation term
- Signal estimation is found by averaging aligned observed signals:

$$\hat{a}_k = \left( \sum_{q=1}^Q \frac{1}{\sigma_q^2} b_q(k - \hat{k}_q) \right) / \left( \sum_{q=1}^Q \frac{1}{\sigma_q^2} \right)$$

3. Expected performance of the Correlational Accumulation

Given channel noise levels  $\{\sigma_q\}$ , signal estimation quality is affected by:

- misalignment errors of noisy signal copies  $\{b_q(k - \hat{k}_q)\}$  (normal and anomalous ones);
- the number of signal copies available

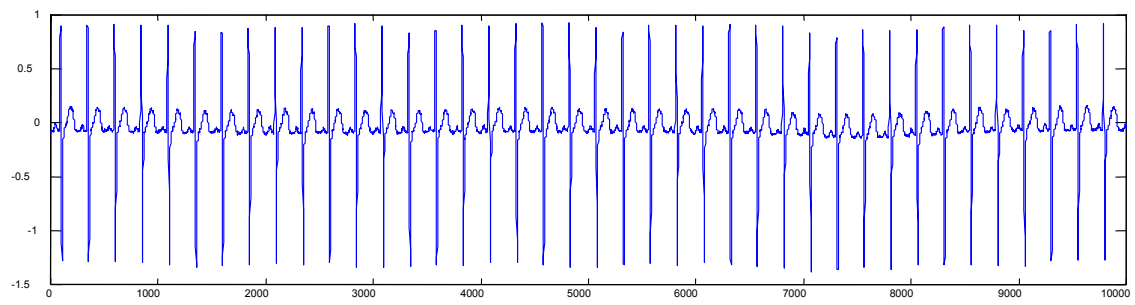
Normal misalignment errors limit additive noise variance reduction factor to  $Q \cdot (1 - P_{an.err})$  and cause restored signal blur with PSF determined by variance of normal errors.

Anomalous misalignment errors cause a spurious signal resulted from accumulation of noise realizations that exhibited high correlation with the signal template.

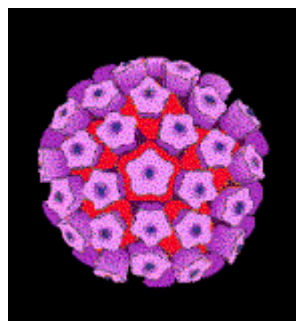
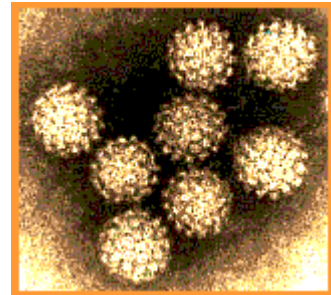
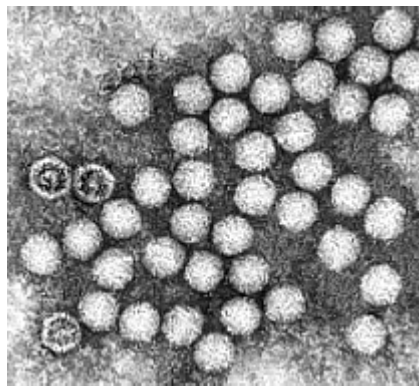
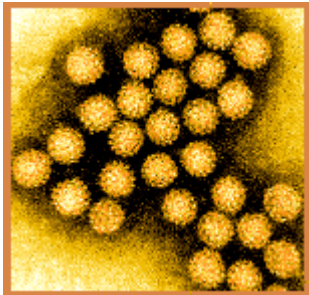
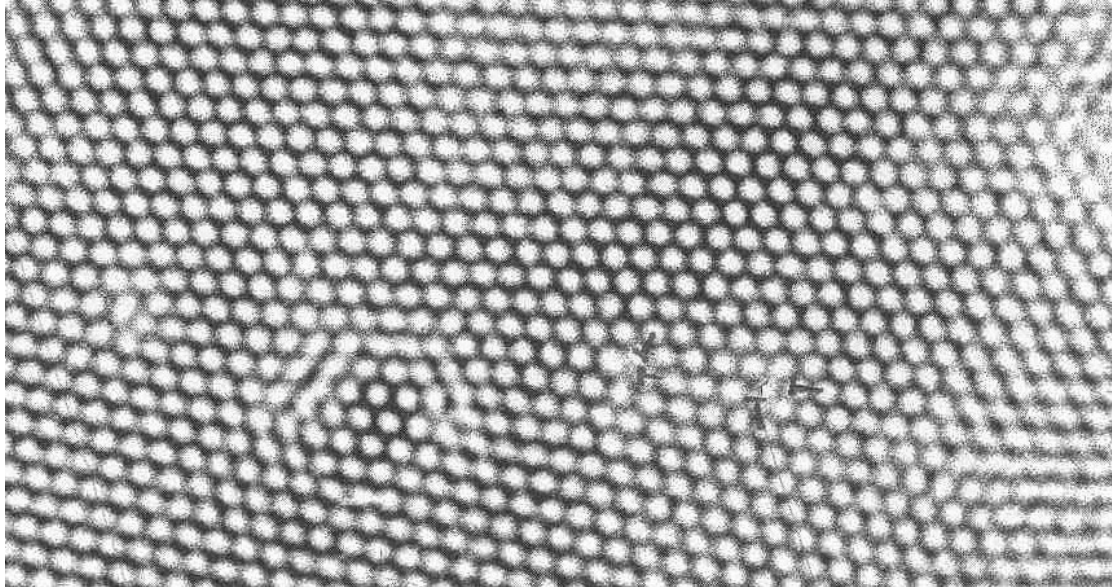
4. Iterative restoration algorithm: one of the noisy realizations is used as an initial template which is then replaced, at each new iteration, by the estimated signal obtained on the previous iteration.
5. Two-channel algorithm: correlational signal accumulation is supplemented with accumulation of the signal power Fourier spectra. The former is then used to compute phase component of the restored signal Fourier spectrum; the latter provides estimation of the magnitude of the spectrum. This allows to reduce signal blur due to misalignment errors.

References:

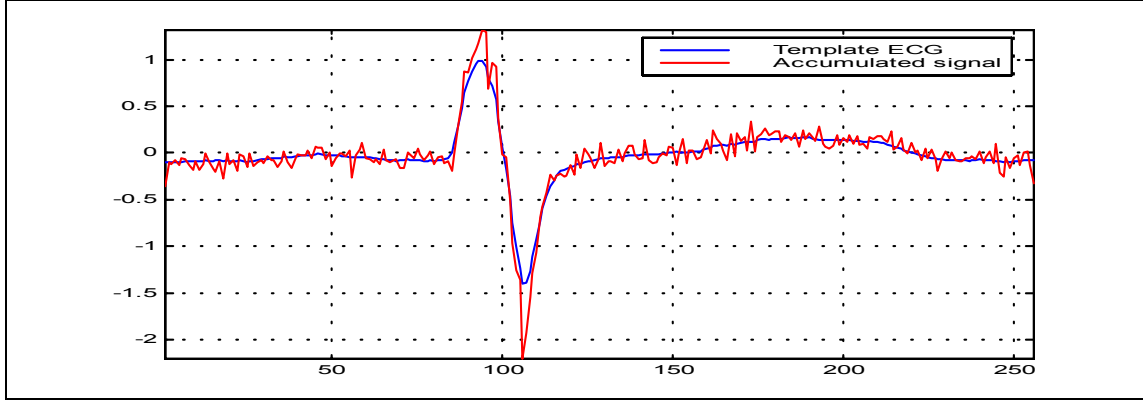
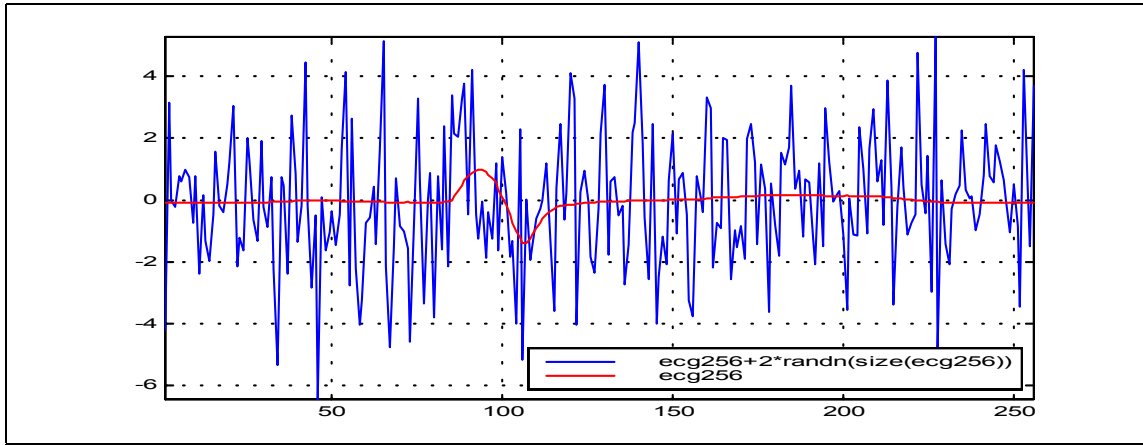
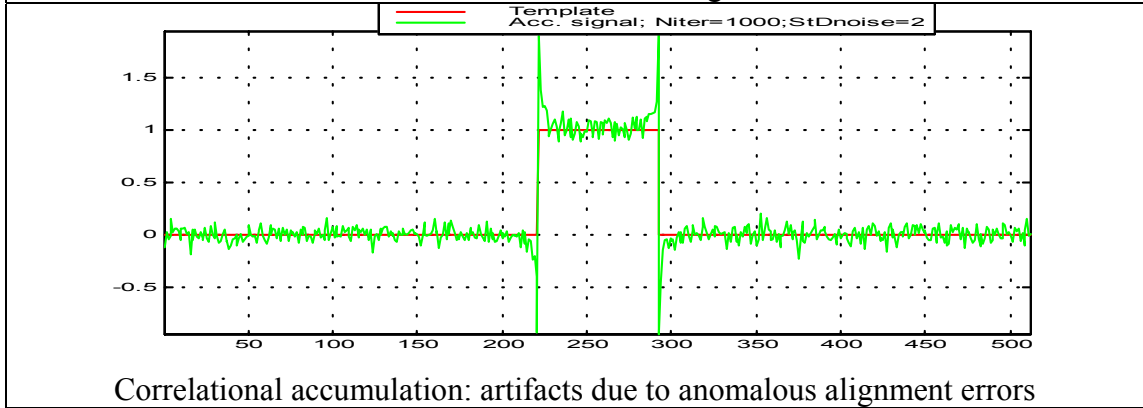
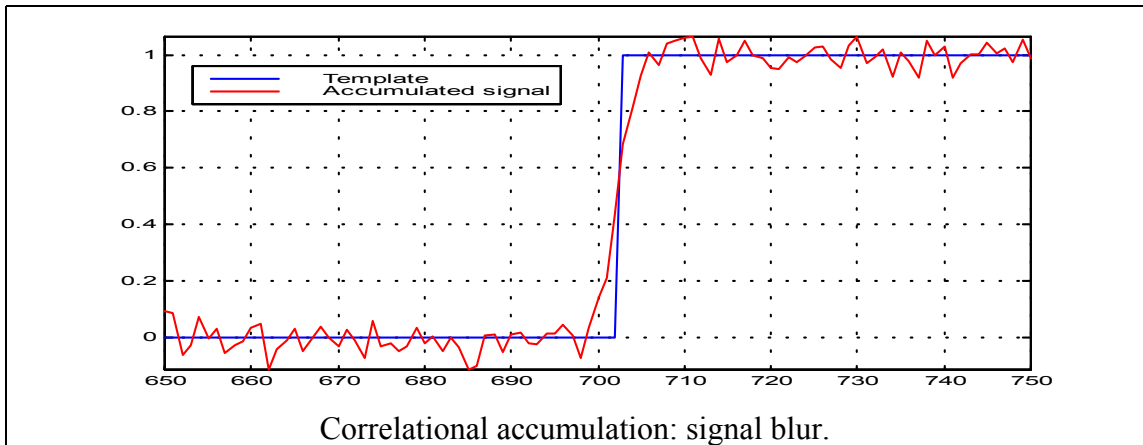
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2. N. El Sherif, G. Turitto, eds., High Resolution Electrocardiography, Futura Mount Kisco, N.Y., 1992
3. L. Yaroslavsky, M. Eden, Correlational Accumulation as a Method for Signal Restoration, Signal Processing, 39 (1994) pp. 89-106

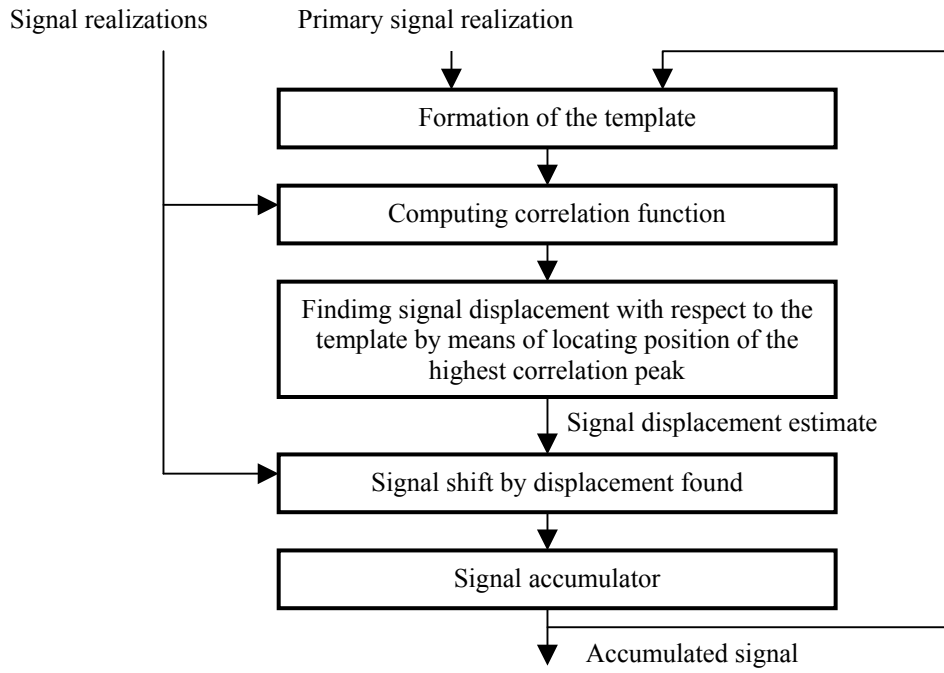


Electrocardiogram signal

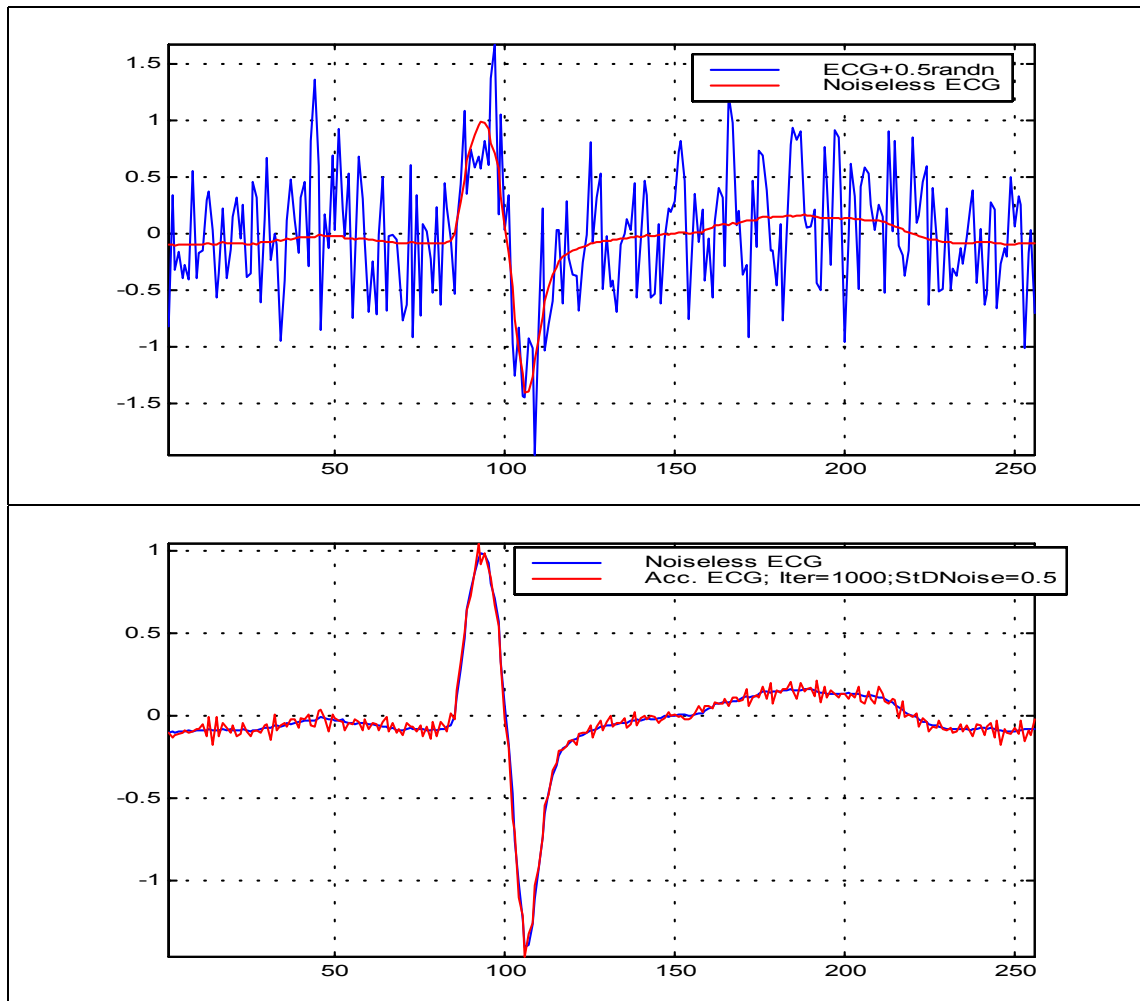


Examples of electron micrograph virus particles and 3D reconstruction of virus

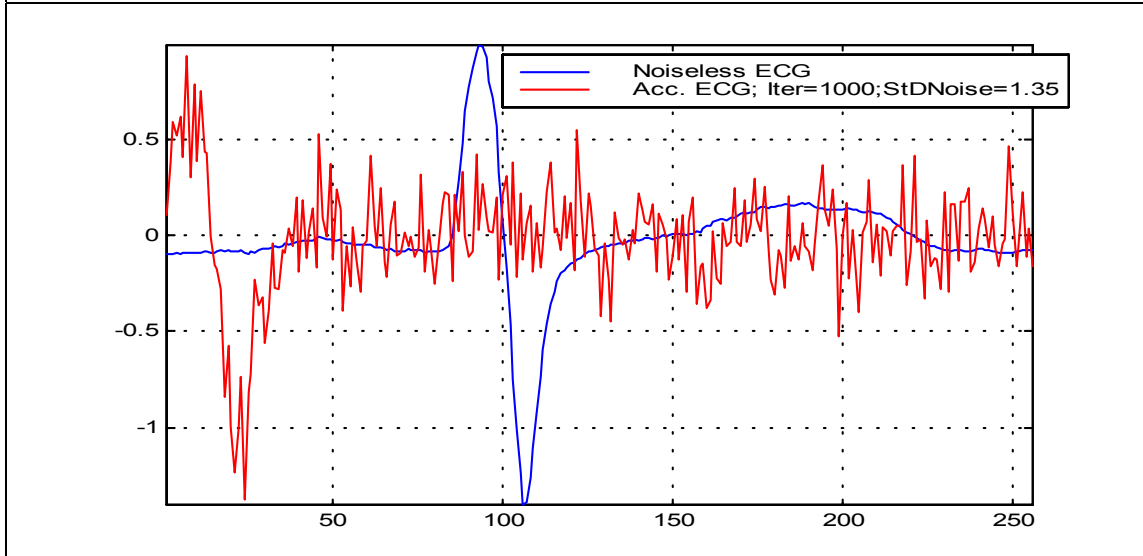
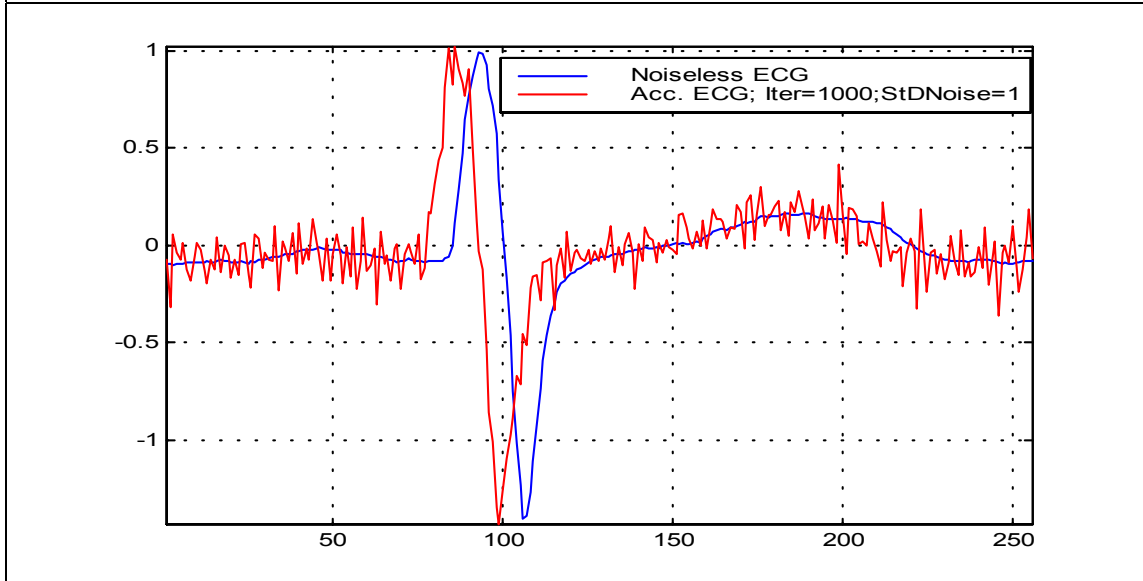
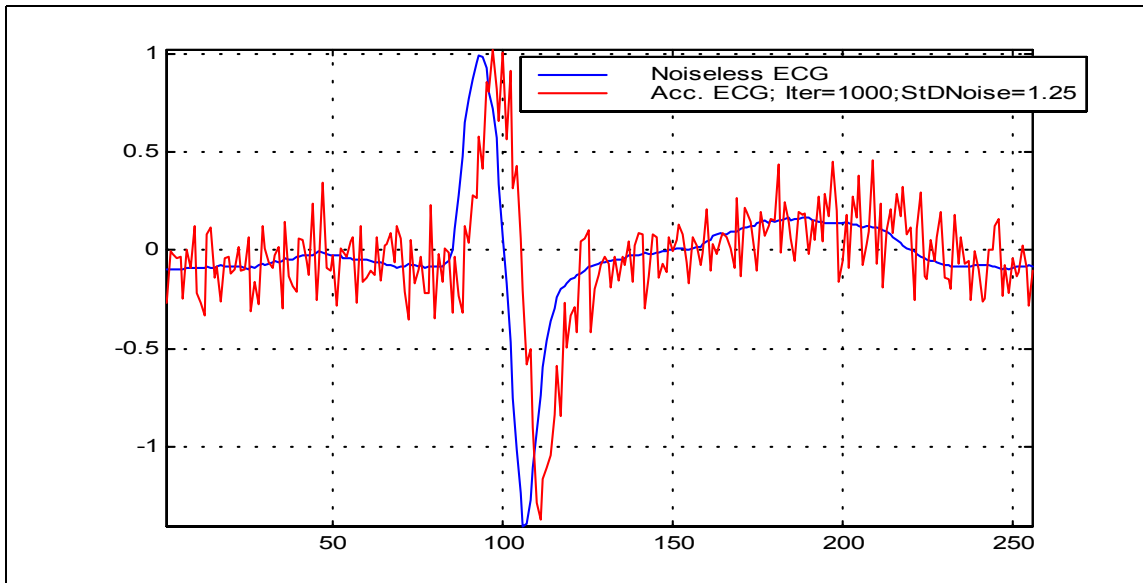




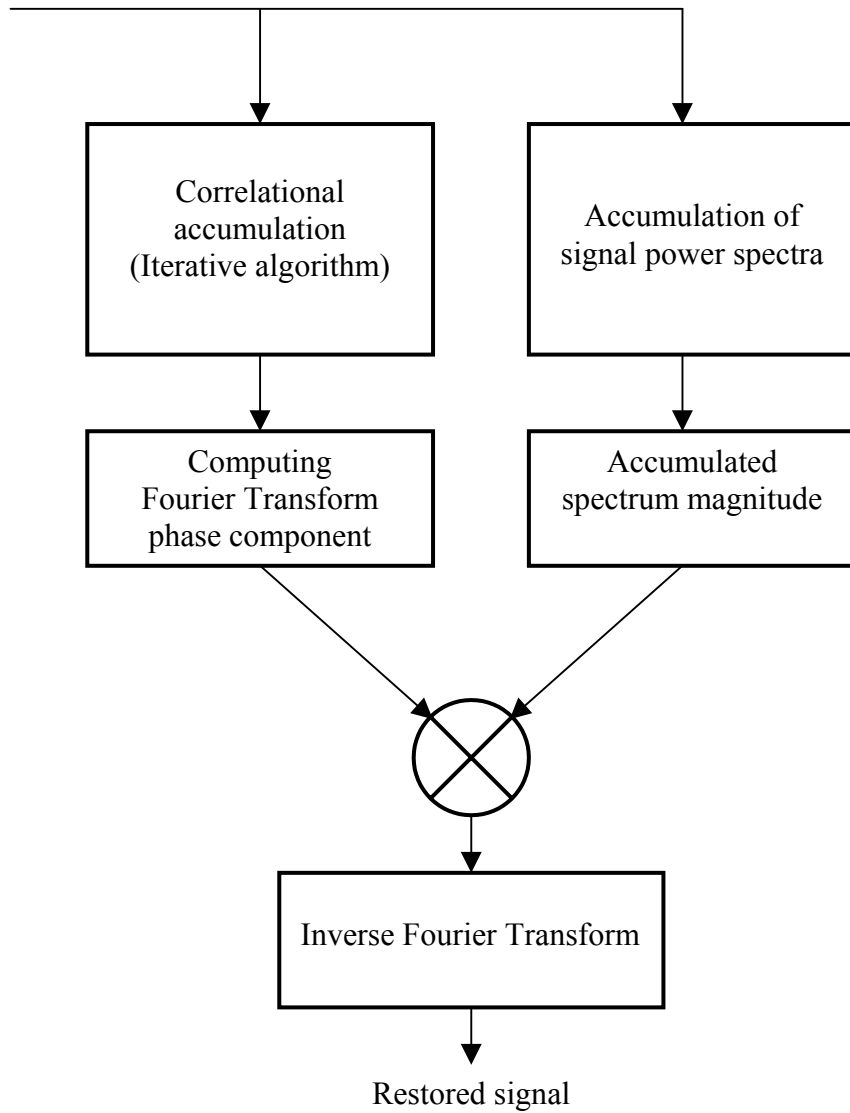
**Iterative correlational accumulation flow diagram**



**Iterative correlational accumulation**



**Iterative correlational accumulation: threshold effect**



**Two channel restoration algorithm**