Lecture 12. Correlational accumulation (averaging) as a method for signal restoration

Applications where multiple copies of signals are available: Periodical and quasi periodical physiological signals (such as ECG, EEG), electron micrographs, video frame sequencies of still objects.

The task of signal restoration from multiple copies.

1. Mathematical model:
   \[ \{b_q(k) = a(k - k_q) + n_q(k)\}, \quad q = 1, \ldots, Q, \]
   where \( \{b_q(k)\} \) are observed signal copies; \( a(k) \) - signal to be restored; \( \{n_q(k)\} \) - realizations of sensor noise, \( Q \) - number of copies

2. Correlational Accumulation: principle and optimality

   \[
   \hat{a}(k) = \arg \min_{\{a(k)\}} \left\{ \sum_{q=1}^{Q} \frac{1}{\sigma_q^2} \sum_{k=0}^{N-1} [b_q(k) - a(k - k_q)]^2 \right\} = \arg \min_{\{a(k)\}} \left\{ \sum_{q=1}^{Q} \frac{1}{\sigma_q^2} \sum_{k=0}^{N-1} [b_q(k)]^2 - 2 \sum_{q=1}^{Q} \frac{1}{\sigma_q^2} \sum_{k=0}^{N-1} b_q(k)a(k - k_q) + \sum_{q=1}^{Q} \frac{1}{\sigma_q^2} \sum_{k=0}^{N-1} a(k - k_q)^2 \right\} \Rightarrow
   \]

   - Signal shifts \( \{\tilde{k}_q\} \) are first estimated that maximize the correlation term
   - Signal estimation is found by averaging aligned observed signals:
     \[
     \hat{a}_k = \left( \frac{\sum_{q=1}^{Q} \frac{1}{\sigma_q^2} b_q(k - \tilde{k}_q)}{\sum_{q=1}^{Q} \frac{1}{\sigma_q^2}} \right)
     \]

3. Expected performance of the Correlational Accumulation

   Given channel noise levels \( \{\sigma_q\} \), signal estimation quality is affected by:
   - misalignment errors of noisy signal copies \( \{b_q(k - \tilde{k}_q)\} \) (normal and anomalous ones);
   - the number of signal copies available

   Normal misalignment errors limit additive noise variance reduction factor to \( Q \cdot (1 - P_{an.err}) \) and cause restored signal blur with PSF determined by variance of normal errors. Anomalous misalignment errors cause a spurious signal resulted from accumulation of noise realizations that exhibited high correlation with the signal template.

4. Iterative restoration algorithm: one of the noisy realizations is used as an initial template which is then replaced, at each new iteration, by the estimated signal obtained on the previous iteration.

5. Two-channel algorithm: correlational signal accumulation is supplemented with accumulation of the signal power Fourier spectra. The former is then used to compute phase component of the restored signal Fourier spectrum; the latter provides estimation of the magnitude of the spectrum. This allows to reduce signal blur due to misalignment errors.

References:
2. N. El Sherif, G. Turitto, eds., High Resolution Electrocardiography, Futura Mount Kisco, N.Y., 1992
Electrocardiogram signal

Examples of electron micrograph virus particles and 3D reconstruction of virus
Correlational accumulation: signal blur.
Correlational accumulation: artifacts due to anomalous alignment errors
Correlational accumulation of a piece of ECG: 1000 iterations; StDevNoise=2
Iterative correlational accumulation flow diagram

- Iteration of correlational accumulation
- Formation of the template
- Computing correlation function
- Finding signal displacement with respect to the template by means of locating position of the highest correlation peak
- Signal displacement estimate
- Signal shift by displacement found
- Signal accumulator
- Accumulated signal

Iterative correlational accumulation
Iterative correlational accumulation: threshold effect
Two channel restoration algorithm

Correlational accumulation (Iterative algorithm)

Accumulation of signal power spectra

Computing Fourier Transform phase component

Accumulated spectrum magnitude

Inverse Fourier Transform

Restored signal