

L. Yaroslavsky. Course 0510.7211 “Digital Image Processing: Applications”

Lecture 12. “Nonlinear filters for image restoration and enhancement

Local criteria of image quality: $AVLOSS(k,l) = AV \left\{ \sum_{m,n} LOC(m,n;a_{k,l}) LOSS(a_{m,n}, \hat{a}_{m,n}) \right\}$

Locality function $LOC(.)$ and loss function $LOSS(.)$ and a priori knowledge on images

Linear filters: $\hat{a}_{k,l} = \lambda_0 a_{k,l}^{inp} + \bar{a}_{k,l}$; $\bar{a}_{k,l}$ is a weighted average over a spatial neighbourhood

Nonlinear filters (Fig. 1): $\{a_{k,l}^{inp}\} \rightarrow \hat{a}_{k,l} : \hat{a}_{k,l} = ESTM(NBH(a_k))$

Rank filters: $\hat{a}_{k,l} = F_{hst}(a_{k,l}^{inp})$ where $F_{hst}(\cdot)$ is a non-linear function determined by the local histogram over a certain neighbourhood of the pixel (k,l) .

Neighbourhoods:

In terms of pixel values (V-neighbourhoods):

$$EV\{a_{k,l}\} = \{a_{m,n} : a_{k,l} - \varepsilon_v^- \leq a_{m,n} \leq a_{k,l} + \varepsilon_v^+\} \quad KNV(a_{k,l}) = \left\{ a_{m,n}^p : \sum_{p=1}^K |a_{k,l} - a_{m,n}^p| = \min_{m,n} \right\}$$

In terms of pixel rank (position in the variational row) (R-neighbourhoods):

$$ER\{a_{k,l}\} = \{a_{m,n} : R(a_{k,l}) - \varepsilon_r^- \leq R(a_{m,n}) \leq a_{k,l} + \varepsilon_r^+\}; \quad KNR(a_{k,l}) = \left\{ a_{m,n}^p : \sum_{p=1}^K |R(a_{k,l}) - R(a_{m,n}^p)| = \min_{m,n} \right\}$$

In terms of histogram features (H-neighbourhoods): **Cluster**, (CL -neighbourhood): subset of pixels that belong to the same cluster, or mode, of the histogram as that of the central pixel.

In terms of image function geometrical features.

Other possible neighbourhoods

Neighborhood operations (estimators):

MEAN(NBH) - arithmetic mean over the neighbourhood NBH ;

$$MEAN(NBH) = \arg \min_k \left(\sum_k |a_k - \hat{a}|^2 \right); \quad MEDN(NBH) = \arg \min_k \left(\sum_k |a_k - \hat{a}| \right)$$

ROS(NBH) - R -th Order Statistics over the neighbourhood; special cases:

MEDN(NBH)- median; **MAX(NBH)** - maximum; **MIN(NBH)** - minimum;

MODE(NBH) - a value that corresponds to the highest maximum of the histogram over the neighbourhood;

RAND(NBH) - pseudo-random number taken from the same distribution as the histogram over the neighbourhood;

RANK(NBH) - position of the central pixel of the window in the variational row over the neighbourhood;

SIZE(NBH) - size (in pixels or grey levels, whatever is appropriate) of the neighbourhoods .

TYPICAL ALGORITHMS:

Rank filters for smoothing additive and impulse noise and image segmentation :

$$\hat{a}_{k,l}^{(t)} = SMTH(NBH(\hat{a}_{k,l}^{(t-1)})) \quad \text{and} \quad \hat{a}_{k,l}^{(t)} = \begin{cases} \hat{a}_{k,l}^{(t-1)}, & \text{if } |\hat{a}_{k,l}^{(t-1)} - SMTH(NBH(\hat{a}_{k,l}^{(t-1)}))| \leq thr \\ SMTH(NBH(\hat{a}_{k,l}^{(t-1)})), & \text{otherwise} \end{cases}$$

where $SMTH$ is a smoothing operations (**MEAN**, **MED**, **ROS**, **MODE** or **RAND**), thr is a detection threshold t is number of the iteration.

Rank order filters for local contrast enhancement:

Unsharp masking: $\hat{a}_{k,l} = G(a_{k,l} - SMTH(NBH(a_{k,l})))$, where G is an enhancement coefficient,

Local histogram equalization: $\hat{a}_{k,l} = RANK(NBH(a_{k,l}))$; Local histogram p -equalization.

Possible generalizations and implementation issues:

Weighted histograms; nonlinearly transformed histograms; new types of neighbourhoods; extension of the set of basic operations.

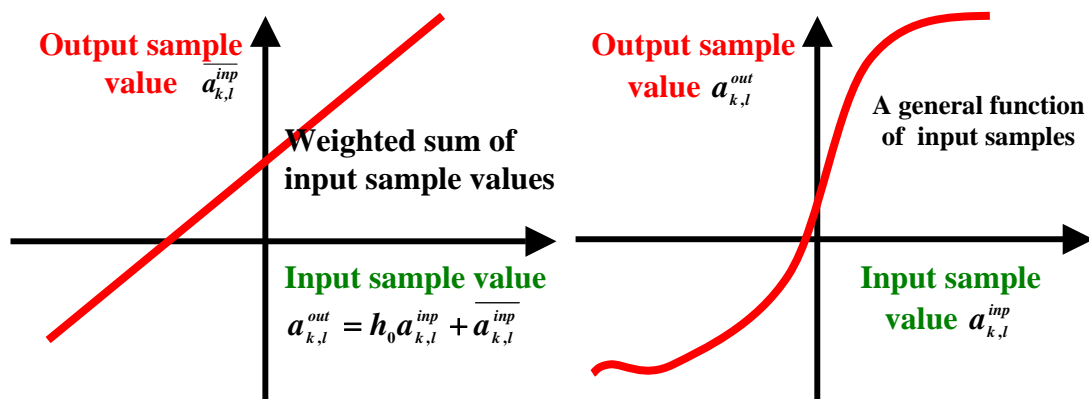
Implementation of local adaptive filters in computers, optics and neuro-morphic parallel networks.

Problems for self-testing:

1. Formulate the basic principle of nonlinear filtering
2. Explain the difference between linear and nonlinear filters
3. List types of pixel's neighborhood and explain how they are defined as a priori assumptions on image properties
4. List neighborhood estimation operations
5. Give examples of typical nonlinear filtering algorithms

Additional references:

1. J. Tukey, Exploratory Data Analysis, Addison Wesley, 1971
2. J. Serra, Image analysis and mathematical morphology, Academic Press, London, 1982
3. S.R. Sternberg, "Grayscale morphology", Computer Vision, Graphics and Image Processing, v. 35, pp.333-355, 1986
4. V. Kim, L. Yaroslavsky, Rank algorithms for picture processing, Computer Vision, Graphics and Image Processing, v. 35, 1986, p. 234-258
5. J. Astola, P. Kuosmanen, Fundamentals of Nonlinear Digital Filtering, CRC Press, Boca Raton, New York, 1997
6. I. Pitas, A. N. Venetsanopoulos, Nonlinear Digital Filters. Principles and Applications. Kluwer, 1990
7. L. Yaroslavsky, M. Eden, Fundamentals of Digital Optics, Birkhauser, Boston, 1996
8. L. Yaroslavsky, Nonlinear Signal Processing Filters: A Unification Approach. Manuscript: <http://www.eng.tau.ac.il/~yaro/recent>

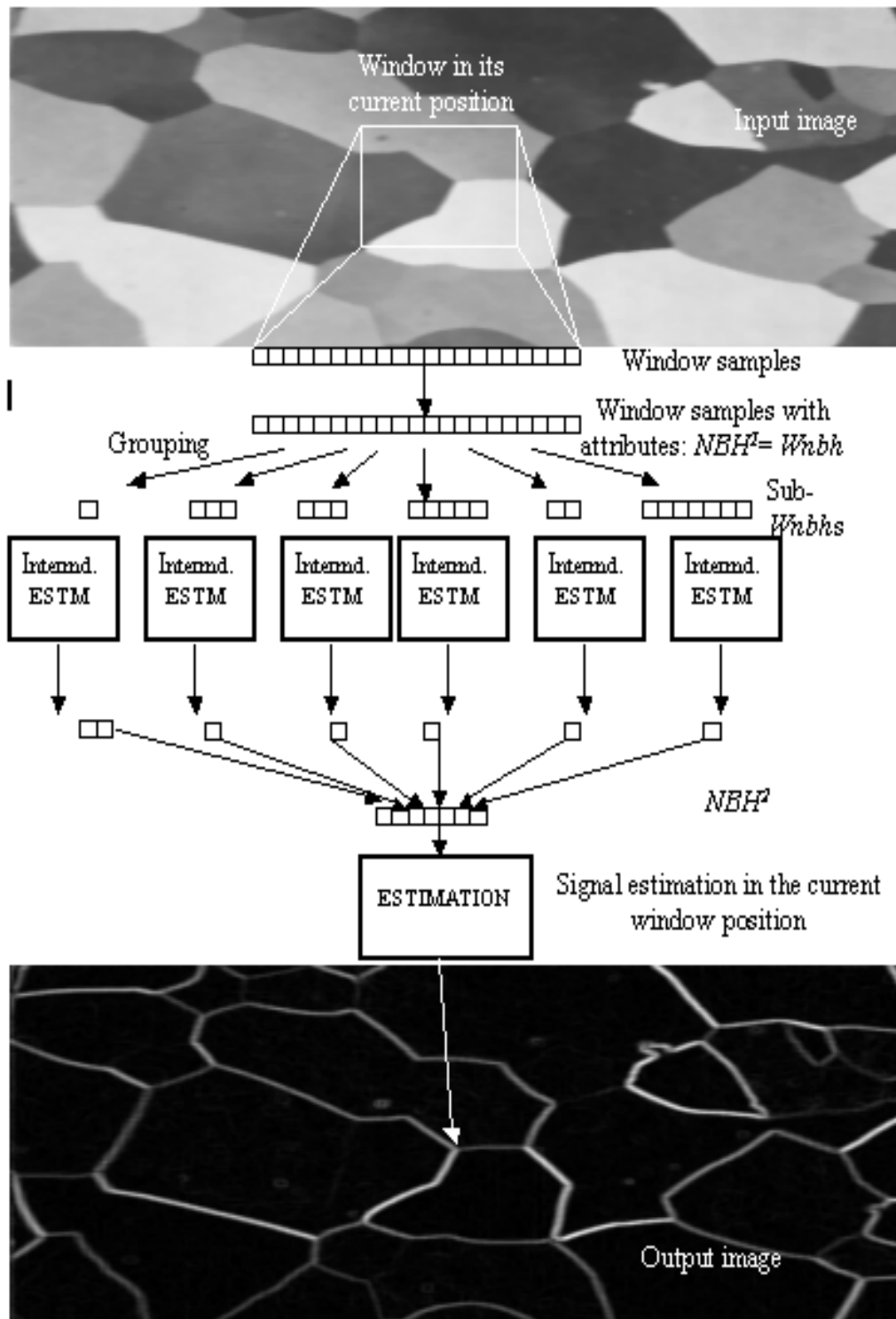


Linear and nonlinear filtering: the principle



John Wilder Tukey, 1915-2000

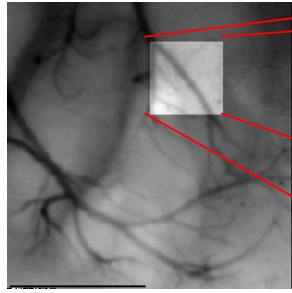
(Courtesy D. Donoho: <http://www-stat.stanford.edu/~donoho>)



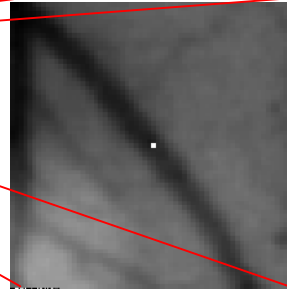
A unified representation of multi stage nonlinear filters (on the example of NBH^2 -filtering) and schematic diagram of non-linear filtering

EXAMPLES OF NEIGHBORHOODS

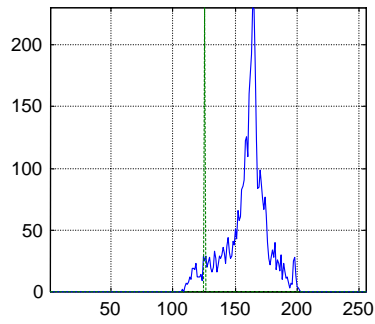
test_v.m:Input image



Input signal
window 65x65



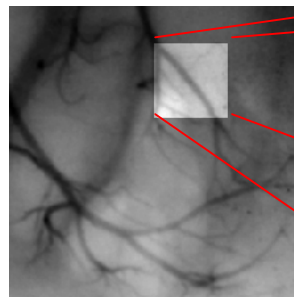
histogram of the window



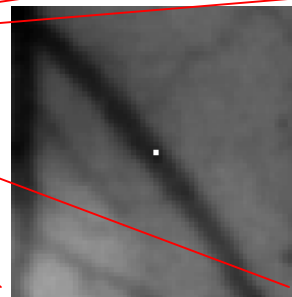
EV-neighborhood; +Ev=20; -Ev=10



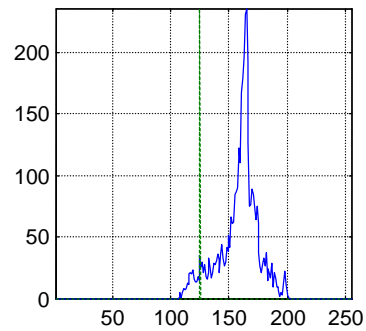
Test_k.nv:input image



window 65x65

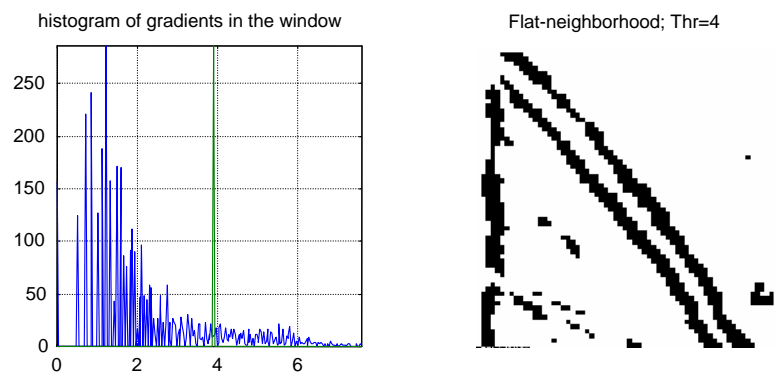
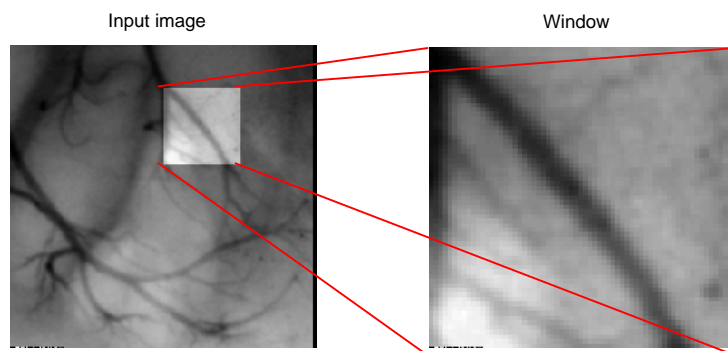
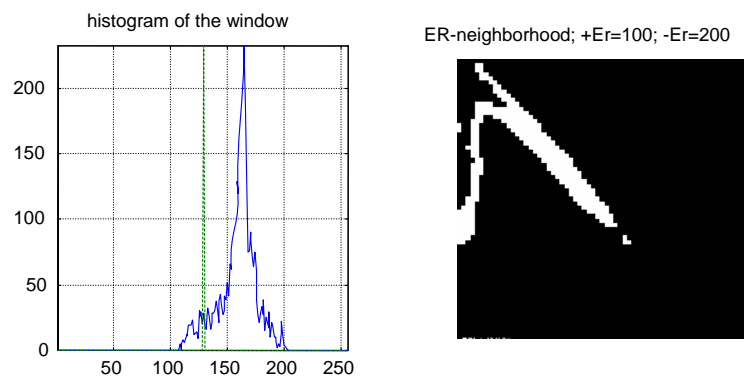
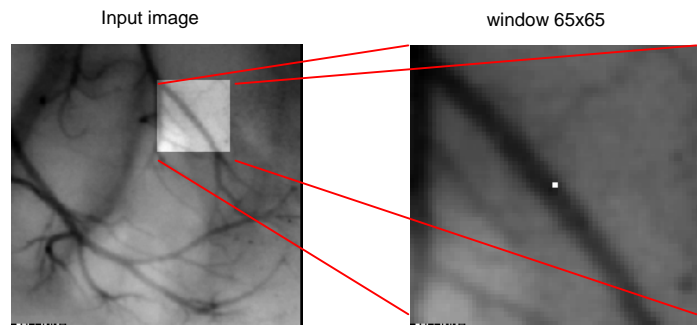


histogram of the



KNV-neighborhood, K=300





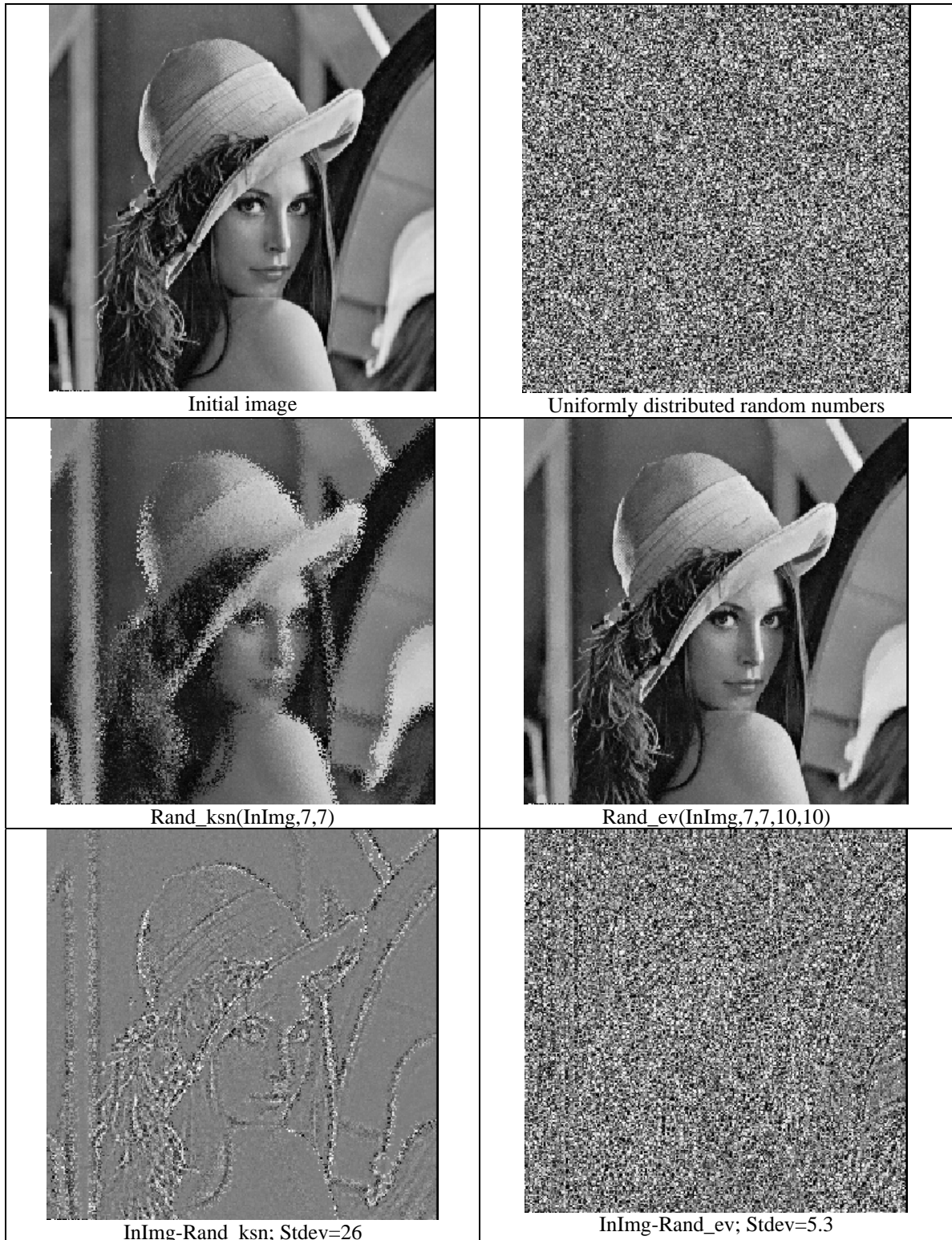
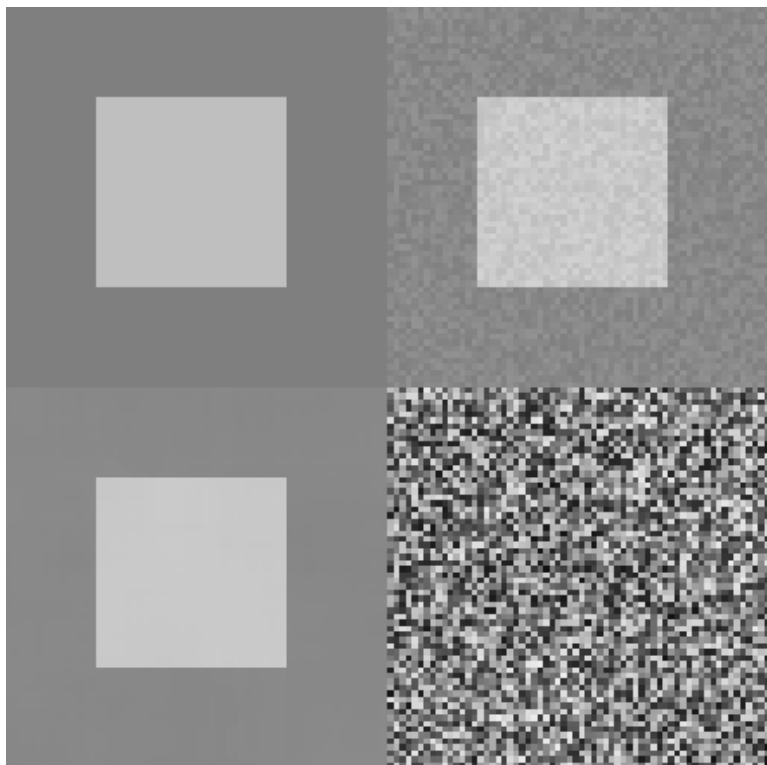
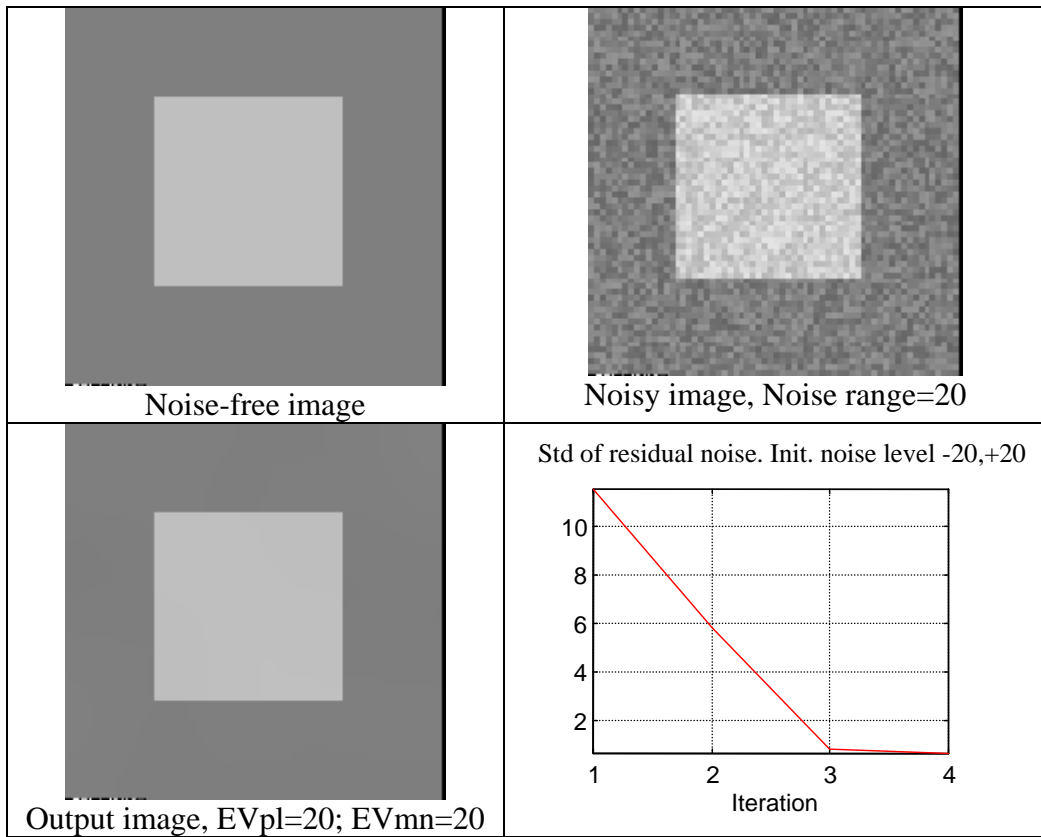


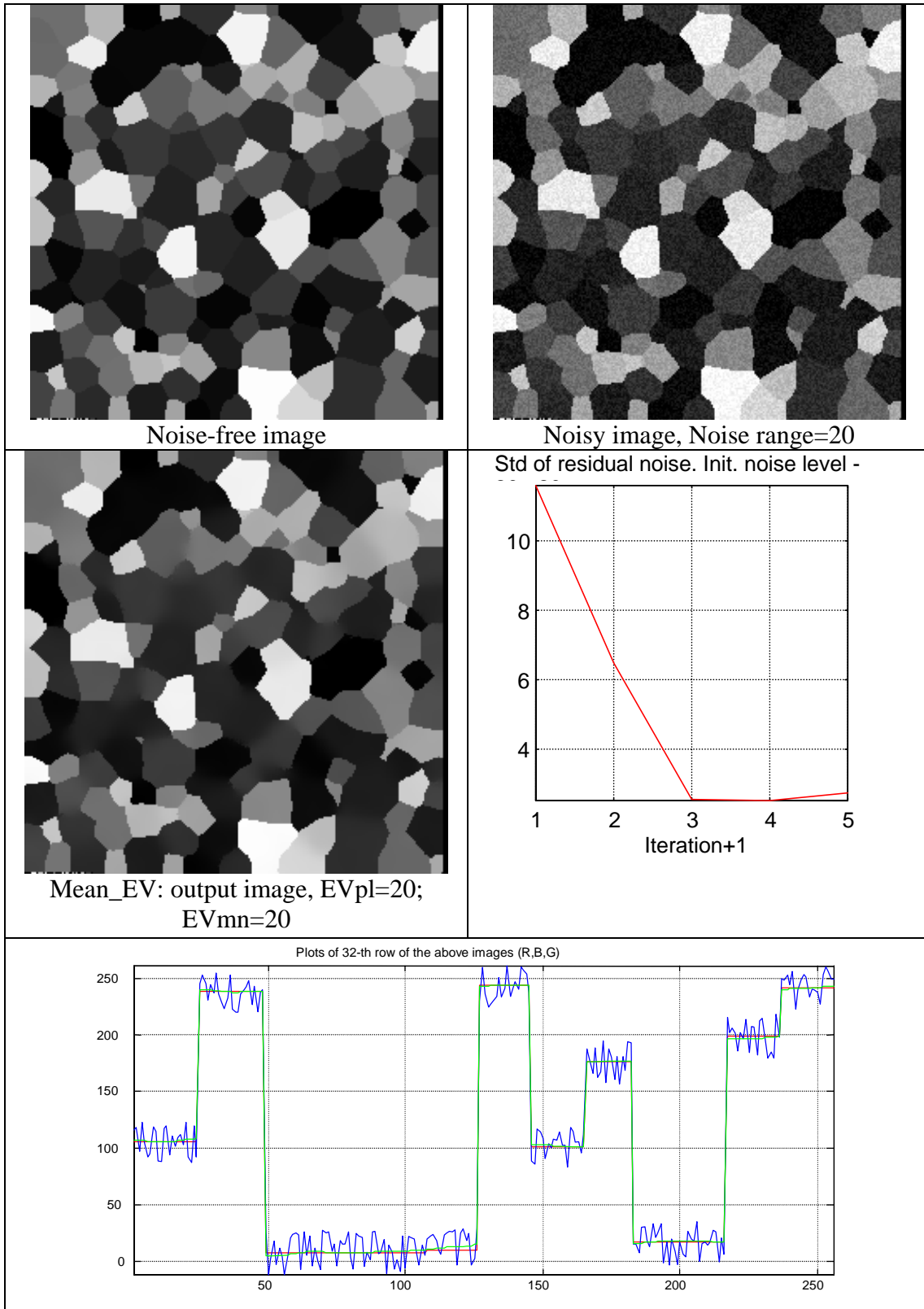
Illustration of information content of local histograms over spatial and EV neighbourhoods. Images in the second row are generated from pseudo-random numbers with the same distribution as that of pixels in spatial window of 7x7 pixels (left) and of pixels in EV-neighbourhood with $\epsilon V_{plus} = \epsilon V_{minus} = 10$

Noise filtering with iterative rank filter MEAN/MEDN($EVnbh$)

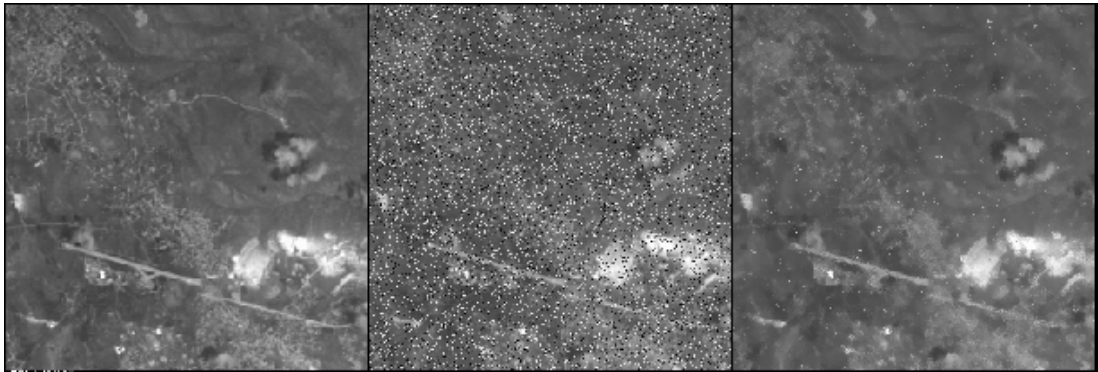


Test, noisy (20*rand), mednEV (1,5,20,20) filtered, and 10x amplified difference images

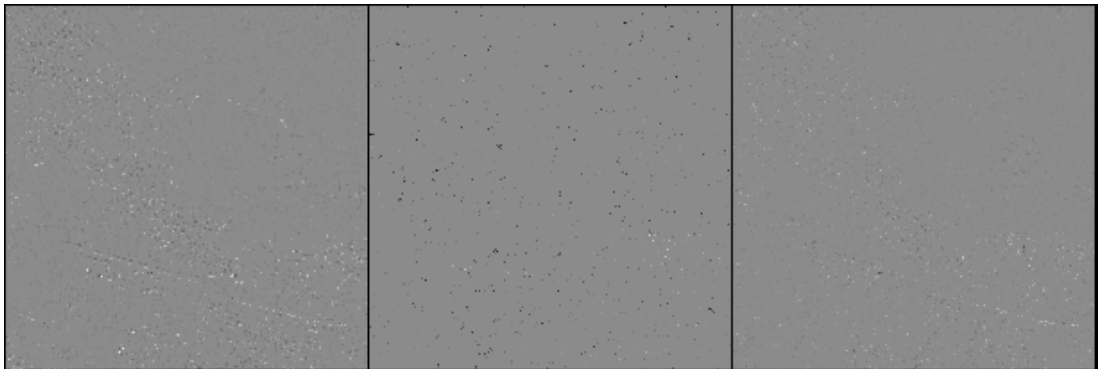
Iterative MEAN(EVnbh)filtering for additive noise suppression



Rank filtering impulse noise (rfltrimp.m)



certfimp.m:Initial, noisy and filtered images;STDprer=9.28;Pn=0.201; STDreserr=9.05



FALSdetect,MISSING and ESTIM errors;Pfd=0.183; STDfd=5.86; Pm=0.0275; SDTm=5.66; STDest=3.96

Rank smoothing for image segmentation

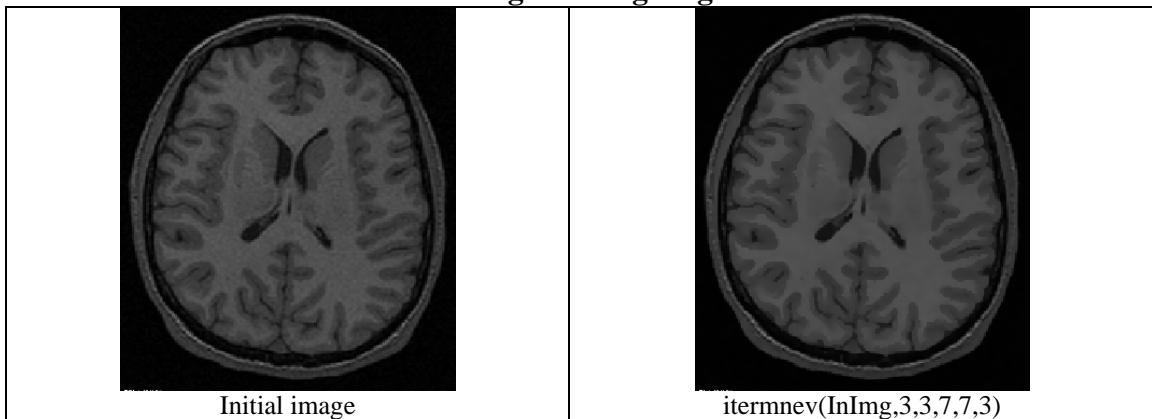
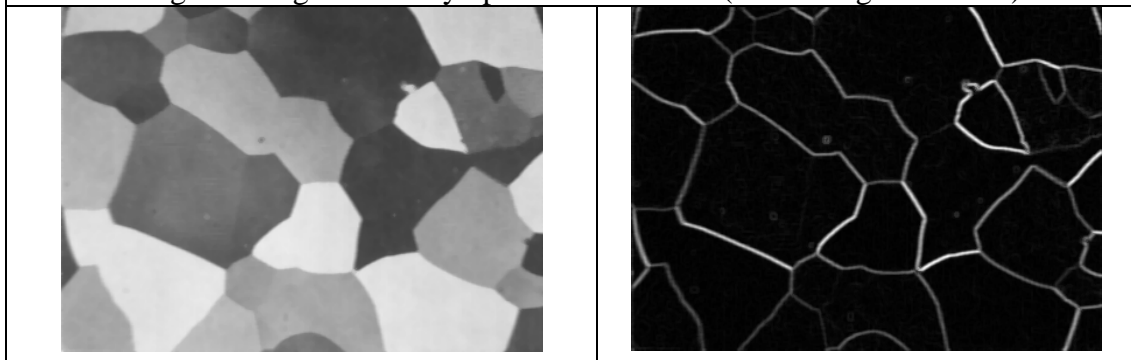


Image segmentation and boundary detection

Image and edges found by operation *Max-Min*(3x3 S-neighbourhood)



Implementation options:

In serial computers:

- On the base of recursive computing local histograms

In electro-optical devices:

- On the base of threshold decomposition

In parallel computers:

- Multi layer neuro-morphic networks

