Lect. 12. Local adaptive nonlinear filters.

Local criteria of image quality: $AVLOSS(k,l) = AV\left\{\sum_{m,n} LOC(m,n;a_{k,l}) LOSS(a_{m,n},\hat{a}_{m,n})\right\}$

Locality function ***LOC*(.)** and loss function ***LOSS* (.)** and a priori knowledge on images

Linear filters: $\hat{a}_{k,l} = \lambda_0 a^{inp}_{k,l} + \bar{a}_{k,l}$; $\bar{a}_{k,l}$ is a weighted average over a spatial neighborhood

Nonlinear filters (Fig. 1): $\left\{a^{inp}_{k,l}\right\} \rightarrow \hat{a}_{k,l} : \hat{a}_{k,l} = ESTM\left(NBH\left(a_k\right)\right)$

Rank filters: $\hat{a}_{k,l} = \mathrm{F}_{hst}\left(a^{inp}_{k,l}\right)$ where $\mathrm{F}_{hst}(\cdot)$ is a non-linear function determined by the local histogram over a certain neighborhood of the pixel (***k,l***).

Neighborhoods and pixel attributes:

In terms of pixel position: Spatial ***S***-neighborhoods

In terms of pixel values (***V***-neighborhoods):

$$EV\{a_{k,l}\} = \{a_{m,n} : a_{k,l} - \varepsilon_V^- \le a_{m,n} \le a_{k,l} + \varepsilon_V^+\} \qquad KNV(a_{k,l}) = \left\{a^p_{m,n} : \sum_{p=1}^{K}\left|a_{k,l} - a^p_{m,n}\right| = \min_{m,n}\right\}$$

In terms of pixel rank (position in the variational row) (***R***-neighborhoods),:

$$ER\{a_{k,l}\} = \{a_{m,n} : R(a_{k,l}) - \varepsilon_r^- \le R(a_{m,n}) \le a_{k,l} + \varepsilon_r^+\}; \qquad KNR(a_{k,l}) = \left\{a^p_{m,n} : \sum_{p=1}^{K}\left|R(a_{k,l}) - R(a^p_{m,n})\right| = \min_{m,n}\right\}$$

In terms of histogram features (H-neighborhoods): **Cluster**, (***CL***-neighborhood): subset of pixels that belong to the same cluster, or mode, of the histogram as that of the central pixel.

In terms of image function geometrical features.

Other possible neighborhoods

Neighborhood operations (estimators):

MEAN(***NBH***) - arithmetic mean over the neighborhood ***NBH***;

$$\mathbf{MEAN}(NBH) = \arg\min\left(\sum_k \left|a_k - \hat{a}\right|^2\right); \mathbf{MEDN}(NBH) = \arg\min\left(\sum_k \left|a_k - \hat{a}\right|\right)$$

ROS(***NBH***) - *R*-th Order Statistics over the neighborhood; special cases:

MEDN(***NBH***)- median; **MAX**(***NBH***) - maximum; **MIN**(***NBH***) - minimum;

MODE(***NBH***) - a value that corresponds to the highest maximum of the histogram over the neighborhood;

RAND(***NBH***) - pseudo-random number taken from the same distribution as the histogram over the neighborhood;

RANK(***NBH***) - position of the central pixel of the window in the variational row over the neighborhood;

SIZE(***NBH***) -size (in pixels or gray levels, whatever is appropriate) of the neighborhoods.

Iterative nature of local adaptive nonlinear filters:

$$\hat{a}^{(t)}_k = \mathbf{ESTM}\left(NBH^{(t-1)}\right)$$

TYPICAL ALGORITHMS:

Rank filters for smoothing additive and impulse noise and image segmentation :

$$\hat{a}^{(t)}_{k,l} = \mathbf{SMTH}\left(NBH\left(\hat{a}^{(t-1)}_{k,l}\right)\right) \quad \text{and} \quad \hat{a}^{(t)}_{k,l} = \begin{cases}\hat{a}^{(t-1)}_{k,l}, if \left|\hat{a}^{(t-1)}_{k,l} - SMTH\left(NBH\left(\hat{a}^{(t-1)}_{k,l}\right)\right)\right| \le thr \\ SMTH\left(NBH\left(\hat{a}^{(t-1)}_{k,l}\right)\right), otherwise\end{cases},$$

where ***SMTH*** is a smoothing operations (**MEAN**, **MED**, **ROS**, **MODE** or **RAND*)***, *thr* is a detection threshold *t* is number of the iteration.

Rank order filters for local contrast enhancement:

Unsharp masking: $\hat{a}_{k,l} = G\left(a_{k,l} - \mathbf{SMTH}\left(NBH\left(a_{k,l}\right)\right)\right)$, where ***G*** is an enhancement coefficient,

Local histogram equalization: $\hat{a}_{k,l} = \mathbf{RANK}\left(NBH\left(a_{k,l}\right)\right)$; Local histogram *p*-equalization.

Possible generalizations and implementation issues:

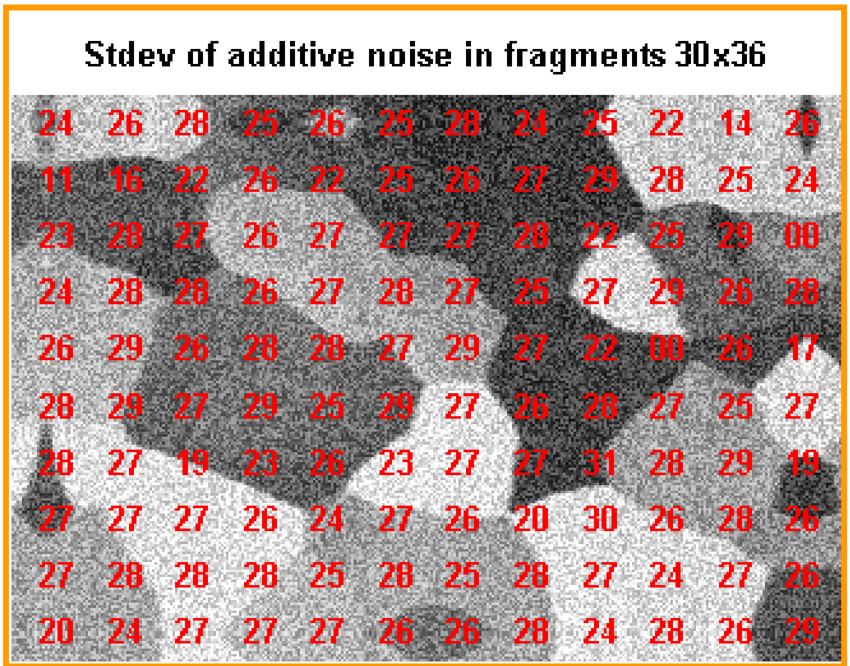
Weighted histograms; nonlinearly transformed histograms; new types of neighborhoods; extension of the set of basic operations.

Implementation of local adaptive filters in computers, optics and neuro-morphic parallel networks.

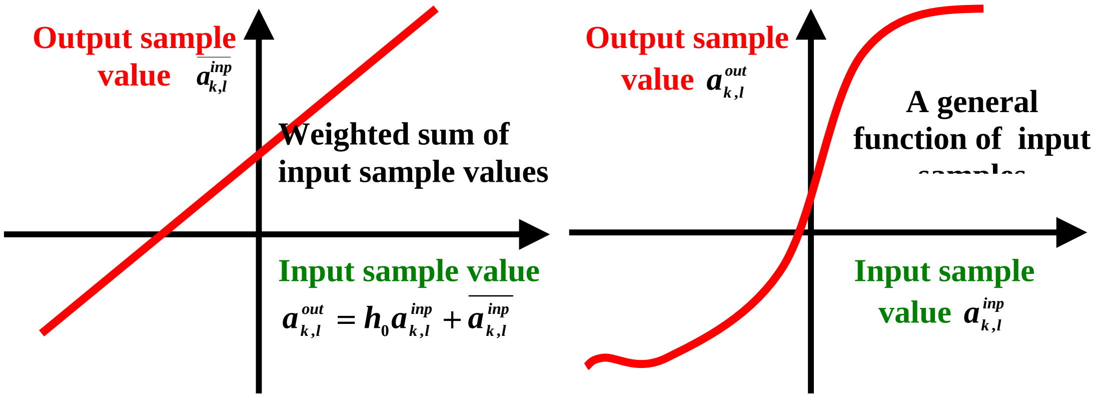
Local criteria of image processing quality

$$AVLOSS(k) = \mathrm{AV}_{\Omega_N}\left\{\sum_m LOC[k; a(k)]\, LOSS\left(a_{ref}(k-m), \hat{a}(k-m)\right)\right\}$$

- Averaging over an image ensemble
- Spatial averaging
- Reference image
- Evaluated image
- Image quality loss measure in *k*-th pixel
- Locality function
- Loss function

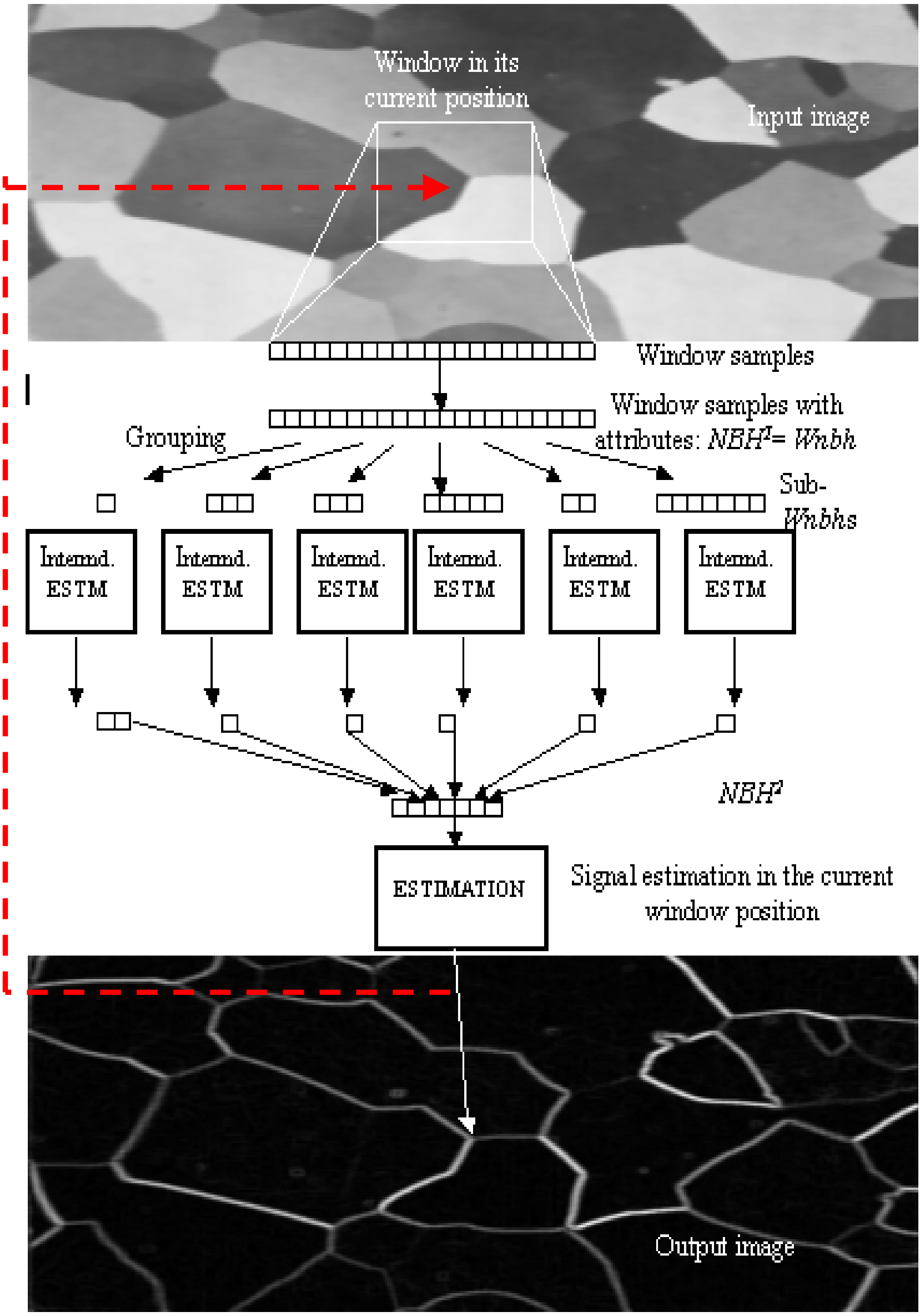


Noisevar_disp.m



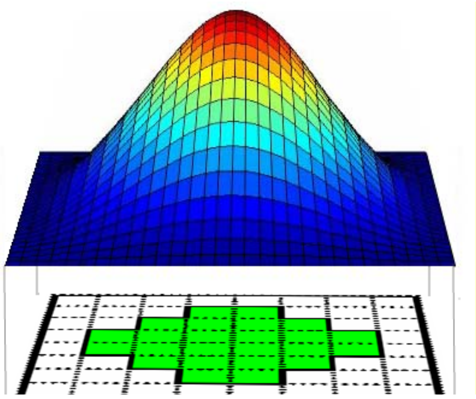
Linear and nonlinear filtering: the principle

John Wilder Tukey, 1915-2000

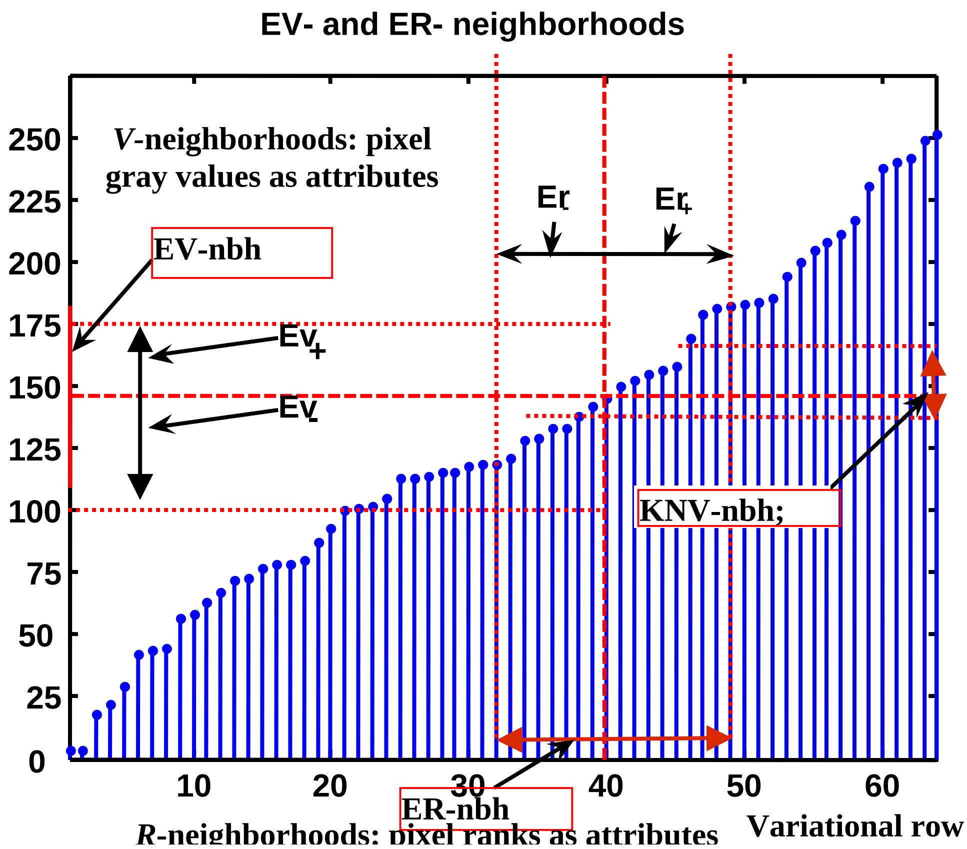


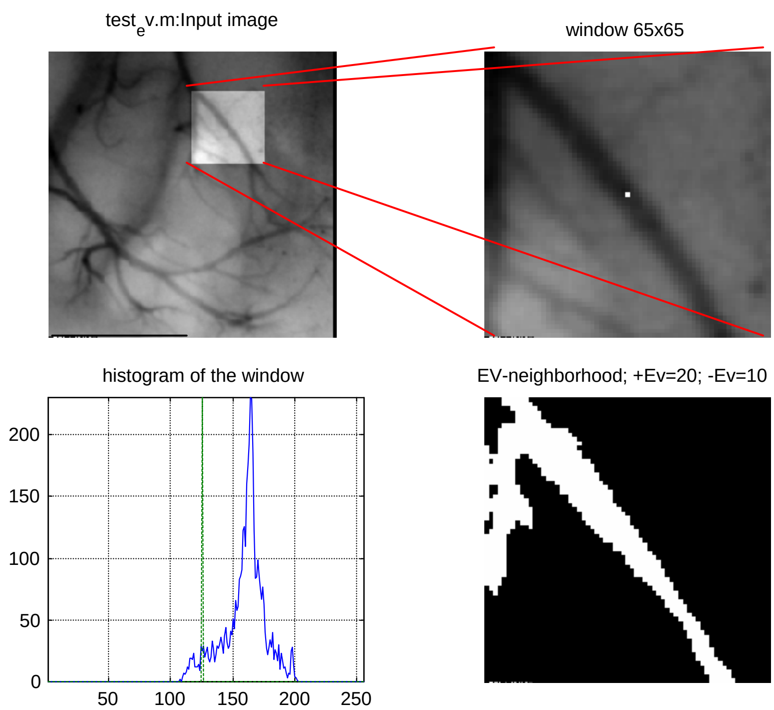
A unified representation of multi stage nonlinear filters
(on an example of *NBH*^{2} –filtering) and schematic diagram of non-linear filtering

S-neighborhoods



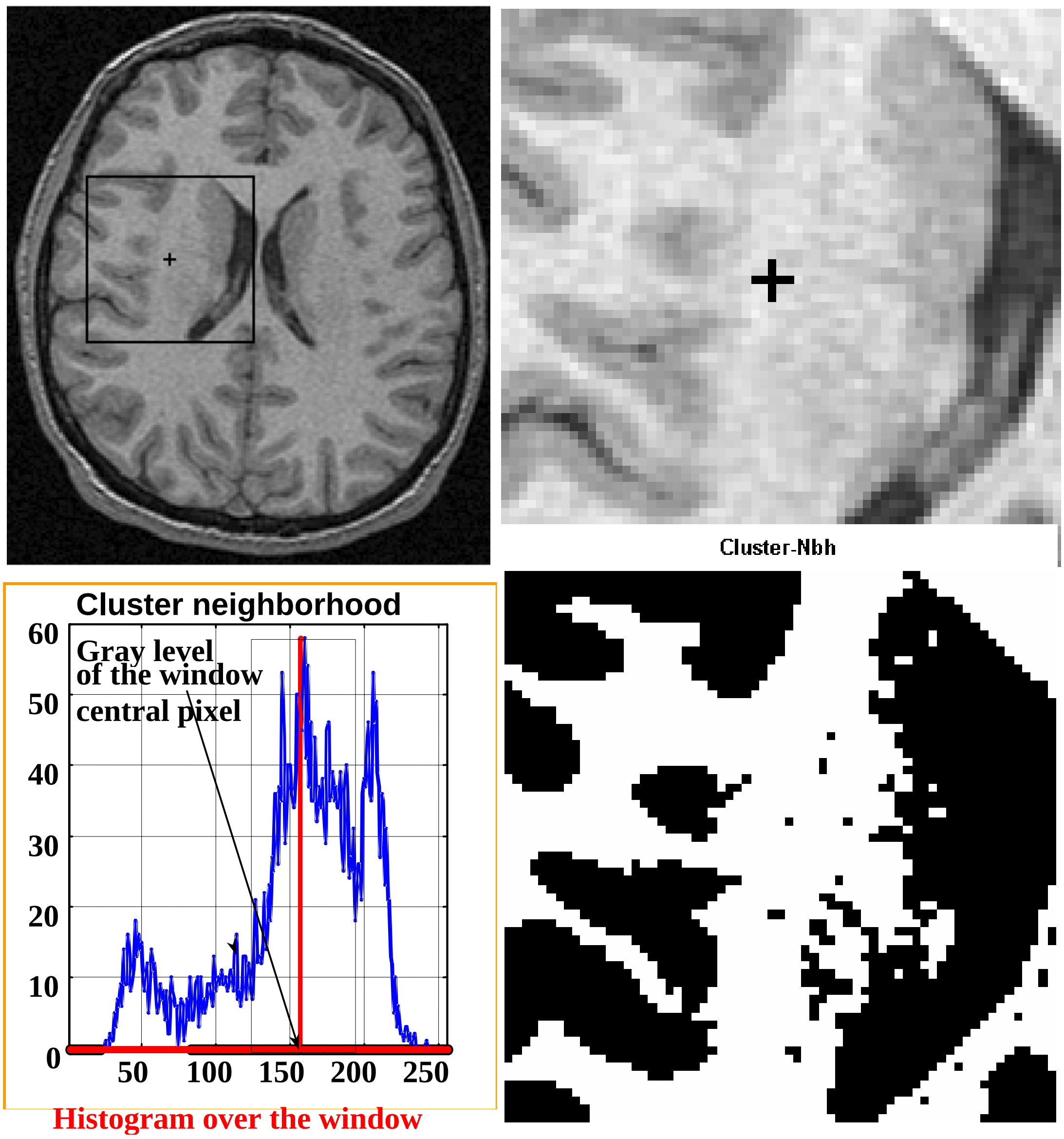
Weighted spatial neighborhood





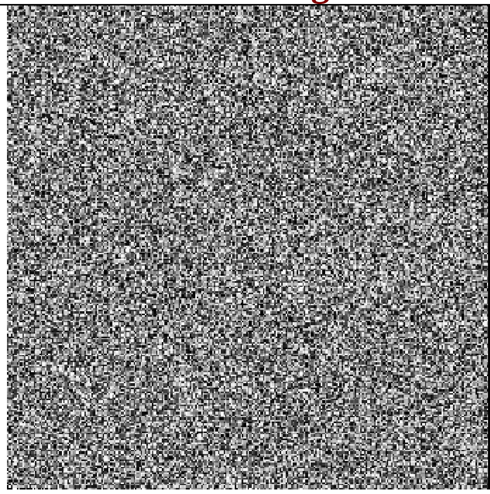
EV-neighborhood: an illustrative example

H-neighborhoods: pixel cardinality as an attribute



Information content in image global and local histograms

Initial image

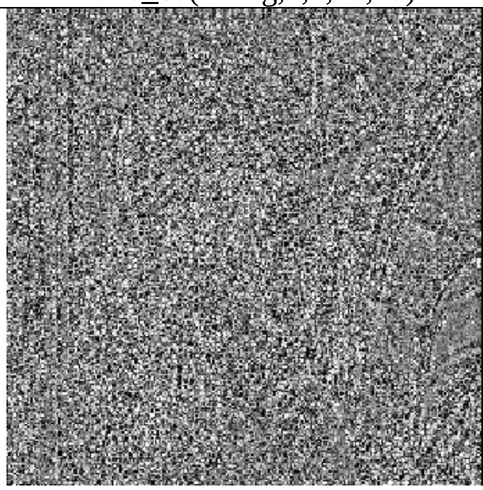


Uniformly distributed random numbers

Rand_ksn(InImg,7,7)

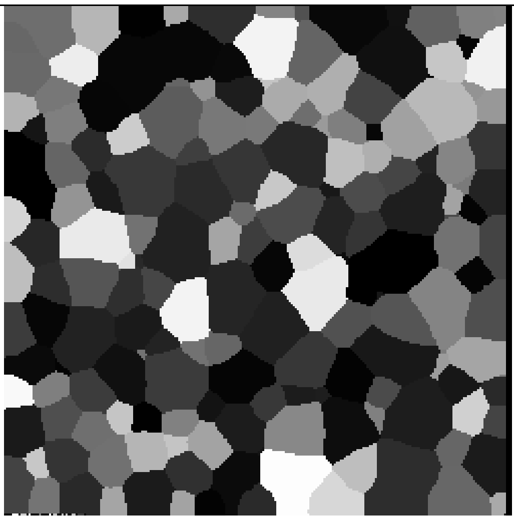
Rand_ev(InImg,7,7,10,10)

InImg-Rand_ksn; Stdev=26

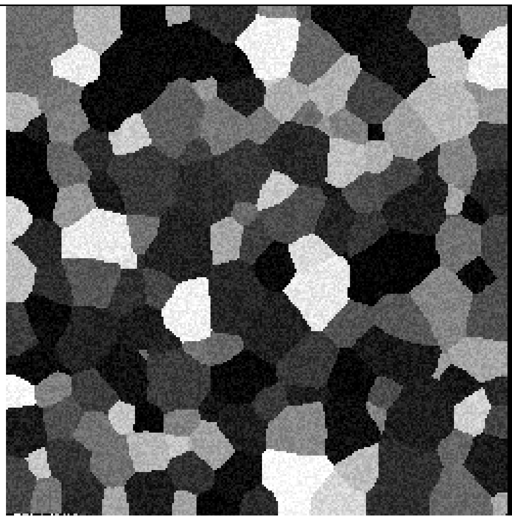


InImg-Rand_ev; Stdev=5.3

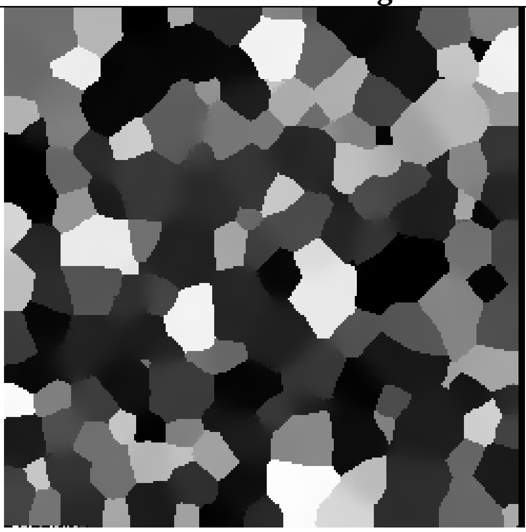
Illustration of information content of local histograms over spatial and EV neighbourhoods. Images in the second raw are generated from pseudo-random numbers with the same distribution as that of pixels in spatial window of 7x7 pixels (left) and of pixels in EV-neighbourhood with $\boldsymbol{\varepsilon Vplus = \varepsilon Vminus = 10}$



Noise-free test image

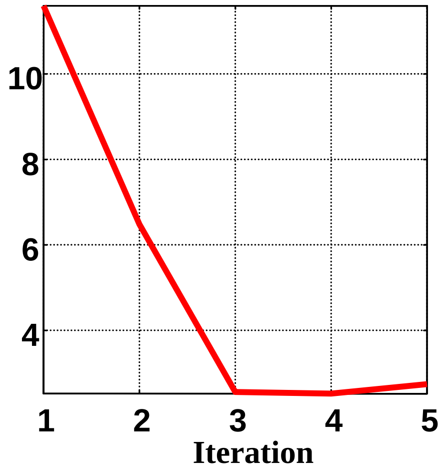


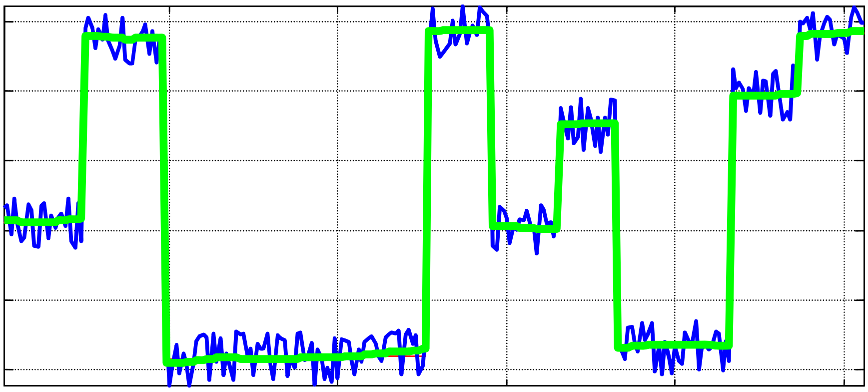
Noisy test image, Noise range=±20



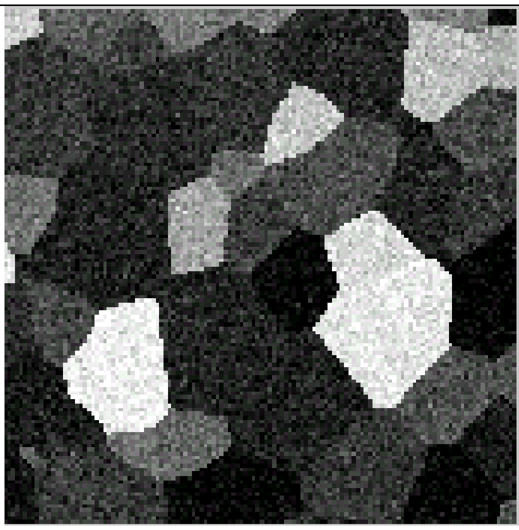
Mean(Evnbh): output image, EVpl=20; EVmn=20

Std of residual noise. Init. noise level ±20

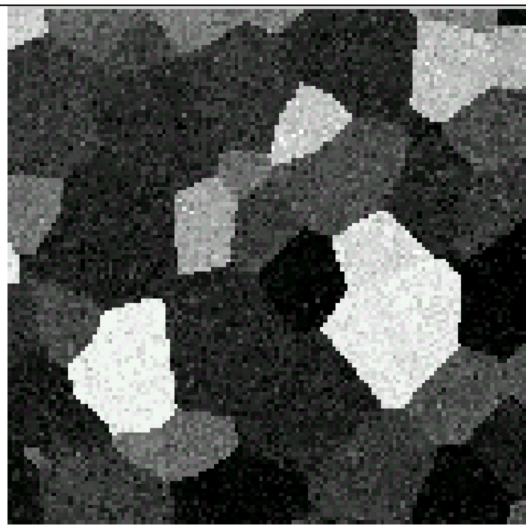




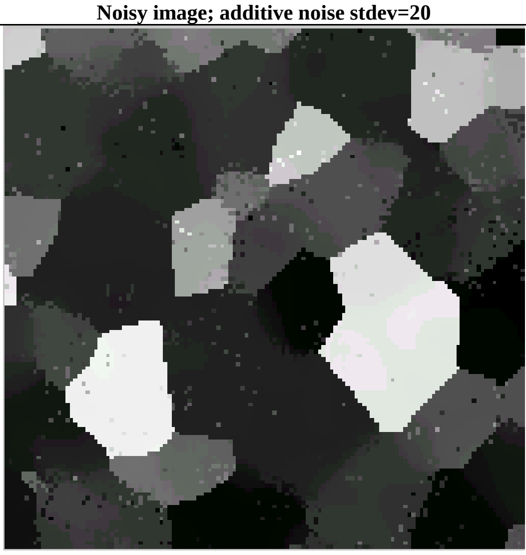
Plots of 32-th row of the above images (R: original, B: noisy, G: filtered)



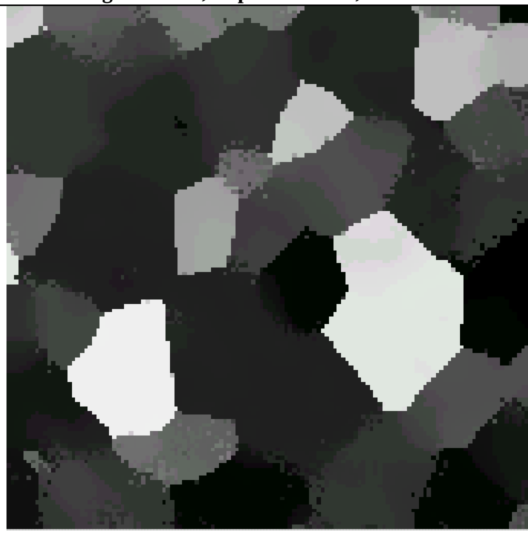
Noisy image; additive noise stdev=20



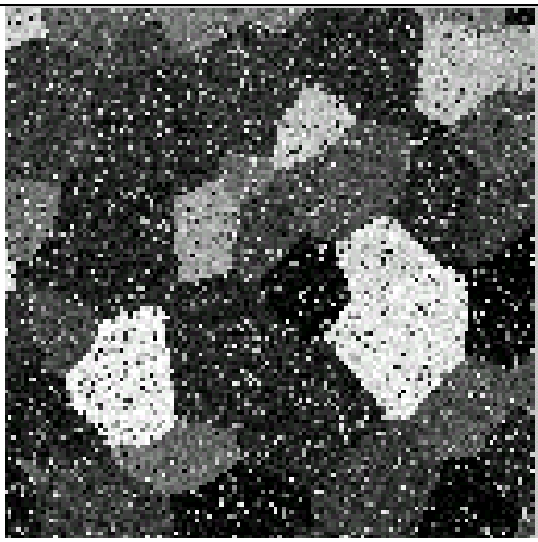
“Sigma” filter, Evpl=Evmn=20, Wnbh=5x5



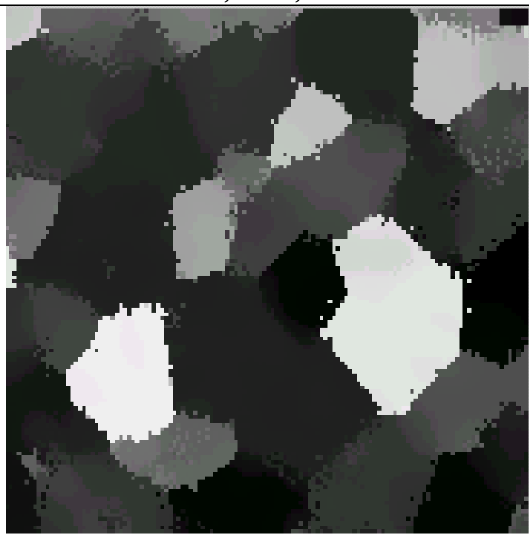
Iterative “sigma” filter, Evpl=Evmn=20, Wnbh=5x5; 5 iterations



Iterative SizeEV-contr. “Sigma” filter; Evpl=Evmn=20, Wnbh=5x5; Thr=5; 5 iterations



Noisy image; additive noise (stdev 20) and impulse noise (Perr= 0.15)



Iterative SizeEV-contr. “Sigma” filter; Evpl=Evmn=20, Wnbh=5x5; Thr=5; 5 iterations

Fig. 2 Comparison of noise suppression capability of “Sigma”- and Size-EV-controlled “Sigma”-filters

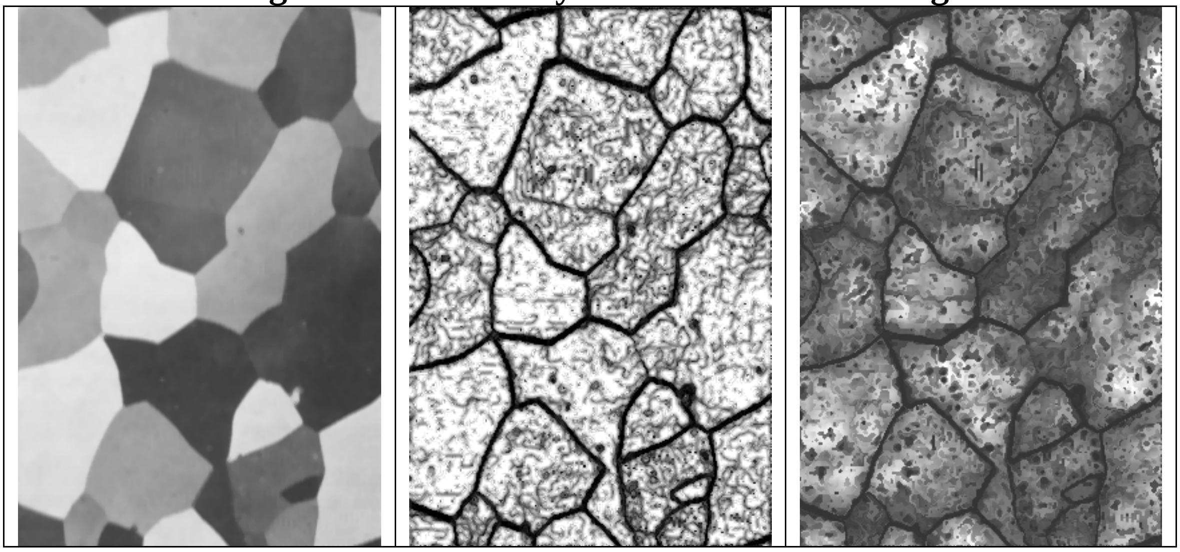
Image enhancement by means of rank filtering

Before

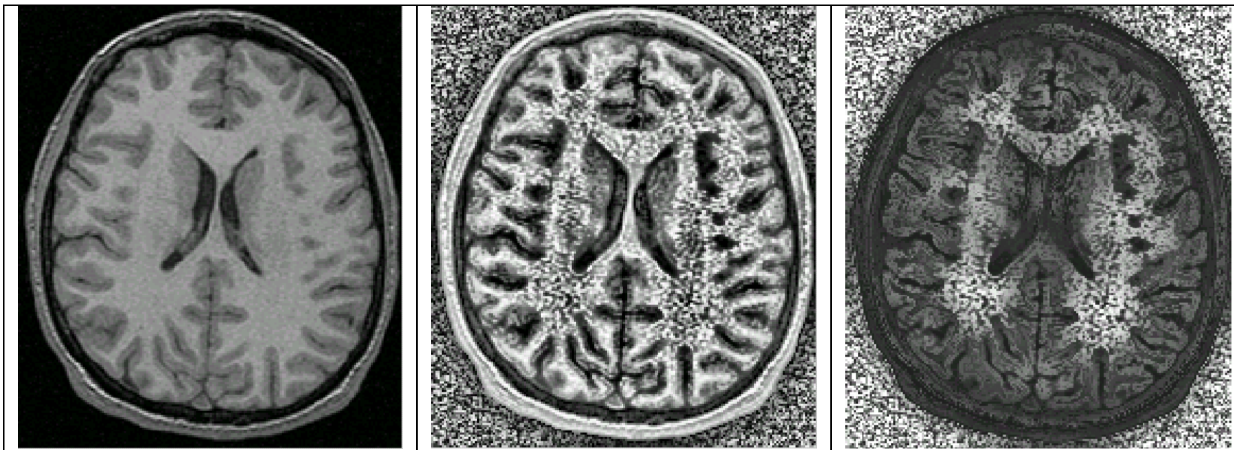
After

Component-wise P-histogram equalization

Image enhancement by means of rank filtering

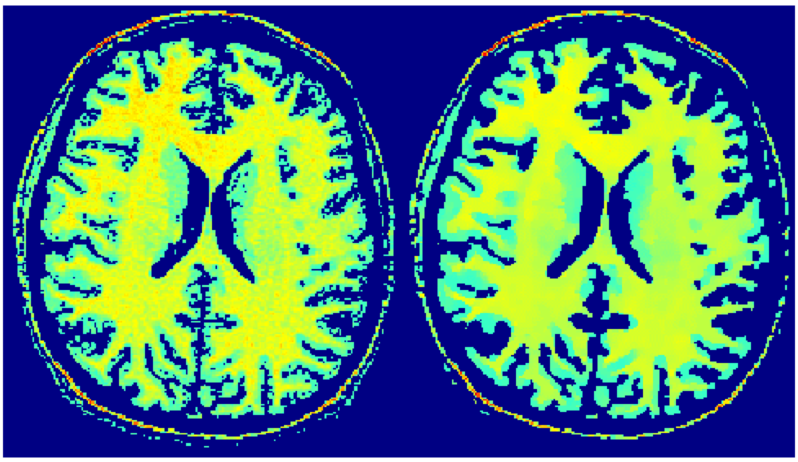


Size-EV **and** ***Cardnl*****-filtering**



Initial MRI image	RANK(*Wnbh*25x25)	RANK(*EV*(*Wnbh*25x25;7,7))

Wnbh **and** ***EV*****-neighborhood local histogram equalization**



Initial image **Segmented image**

itermnev(InImg,3,3,7,7,5); colormap jet