Lectures on Selected Topics in Image Processing.

Pt. 5. Local adaptive filters for image restoration and enhancement

Lect. 12. Local adaptive nonlinear filters.

Local criteria of image quality:
\[ \text{AVLOSS}(k,l) = \sum_{m,n} \left| \text{LOSS}(a_{m,n}, \hat{a}_{m,n}) - \text{LOSS}(a_{m,n}, a_{m,n}) \right| \]

Locality function \( \text{LOC}(.) \) and loss function \( \text{LOSS}(.) \) and a priori knowledge on images

Linear filters:
\[ \hat{a}_{k,l} = \lambda_{k,l} a_{k,l} + \alpha_{k,l} \]
where \( \lambda_{k,l} \) is a weighted average over a spatial neighborhood

Nonlinear filters (Fig. 1):
\[ \{ \} \]

Rank filters:
\[ \hat{a}_{k,l} = F_{\text{loc}}(a_{k,l}) \]
where \( F_{\text{loc}}(.) \) is a non-linear function determined by the local histogram over a certain neighborhood of the pixel \((k,l)\).

Neighborhoods and pixel attributes:

In terms of pixel position: Spatial \( S \)-neighborhoods

In terms of pixel values (\( V \)-neighborhoods):
\[ \{ \} \]

In terms of pixel rank (position in the variational row) (\( R \)-neighborhoods):
\[ \{ \} \]

In terms of histogram features (\( H \)-neighborhoods): Cluster, (\( CL \)-neighborhood): subset of pixels that belong to the same cluster, or mode, of the histogram as that of the central pixel.

In terms of image function geometrical features.

Other possible neighborhoods

Neighborhood operations (estimators):

- \( \text{MEAN}(\text{NBH}) \) - arithmetic mean over the neighborhood \( \text{NBH} \);
- \( \text{MEDN}(\text{NBH}) \) - median;
- \( \text{MAX}(\text{NBH}) \) - maximum;
- \( \text{MIN}(\text{NBH}) \) - minimum;
- \( \text{MODE}(\text{NBH}) \) - a value that corresponds to the highest maximum of the histogram over the neighborhood;
- \( \text{RAND}(\text{NBH}) \) - pseudo-random number taken from the same distribution as the histogram over the neighborhood;
- \( \text{RANK}(\text{NBH}) \) - position of the central pixel of the window in the variational row over the neighborhood;
- \( \text{SIZE}(\text{NBH}) \) - size (in pixels or gray levels, whatever is appropriate) of the neighborhoods.

Iterative nature of local adaptive nonlinear filters:
\[ \hat{a}_{k,l}^{(t)} = \text{ESTM}(\text{NBH}^{(t-1)}) \]

Typical algorithms:

Rank filters for smoothing additive and impulse noise and image segmentation:
\[ \hat{a}_{k,l}^{(t)} = \text{SMTH}(\text{NBH}(\hat{a}_{k,l}^{(t-1)})) \]
and
\[ \hat{a}_{k,l}^{(t)} = \begin{cases} \hat{a}_{k,l}^{(t-1)}, & \text{if } \left| \hat{a}_{k,l}^{(t-1)} - \text{SMTH}(\text{NBH}(\hat{a}_{k,l}^{(t-1)})) \right| < \text{thr} \\ \text{SMTH}(\text{NBH}(\hat{a}_{k,l}^{(t-1)})), & \text{otherwise} \end{cases} \]

where \( \text{SMTH} \) is a smoothing operations (\( \text{MEAN}, \text{MED}, \text{ROS}, \text{MODE} \) or \( \text{RAND} \)), \( \text{thr} \) is a detection threshold \( t \) is number of the iteration.

Rank order filters for local contrast enhancement:

Unsharp masking:
\[ \hat{a}_{k,l} = G(\hat{a}_{k,l} - \text{SMTH}(\text{NBH}(\hat{a}_{k,l}))) \]

where \( G \) is an enhancement coefficient.

Local histogram equalization:
\[ \hat{a}_{k,l} = \text{RANK}(\text{NBH}(\hat{a}_{k,l})) \]

Local histogram p-equalization.

Possible generalizations and implementation issues:

- Weighted histograms; nonlinearly transformed histograms; new types of neighborhoods; extension of the set of basic operations.
Local criteria of image processing quality

\[
AVLOSS(k) = AV_{\Omega_K} \left\{ \sum_m LOC[k; a(k)] LOSS(a_{\text{ref}}(k - m), a(k - m)) \right\}
\]

- Averaging over an image ensemble
- Spatial averaging
- Evaluated image
- Reference image
- Image quality loss measure in \(k\)-th pixel
- Locality function
- Loss function

**Stdev of additive noise in fragments 30x36**

![Noisevar Disp.m](attachment:Noisevar_disp.m)
Linear and nonlinear filtering: the principle

\[ a_{\text{out},k,j} = h a_{\text{inp},k,j} + a_{\text{inp},k,j} \]

John Wilder Tukey, 1915-2000
A unified representation of multi stage nonlinear filters
(on an example of $NBH^2$–filtering) and schematic diagram of non-linear filtering
S-neighborhoods

Weighted spatial neighborhood
EV-neighborhood: an illustrative example

EV- and ER- neighborhoods

*V*-neighborhoods: pixel gray values as attributes

EV-nbh

Ev

Ev-

ER-nbh

Er-

Er+

KNV-nbh;

R-neighborhoods: pixel ranks as attributes

Variational row

test_v.m: Input image

window 65x65

histogram of the window

EV-neighborhood: +Ev=20; -Ev=10

EV-neighborhood: an illustrative example
H-neighborhoods: pixel cardinality as an attribute

Cluster neighborhood

Histogram over the window

Cluster Nbh
### Information content in image global and local histograms

<table>
<thead>
<tr>
<th>Image Description</th>
<th>Histogram Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial image</td>
<td>Uniformly distributed random numbers</td>
</tr>
<tr>
<td>Rand_ksn(InImg,7,7)</td>
<td>Rand_ev(InImg,7,7,10,10)</td>
</tr>
<tr>
<td>InImg-Rand_ksn; Stdev=26</td>
<td>InImg-Rand_ev; Stdev=5.3</td>
</tr>
</tbody>
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Illustration of information content of local histograms over spatial and EV neighbourhoods. Images in the second raw are generated from pseudo-random numbers with the same distribution as that of pixels in spatial window of 7x7 pixels (left) and of pixels in EV-neighbourhood with $\epsilon_{\text{V plus}} = \epsilon_{\text{V minus}} = 10$. 
Fig. 2 Comparison of noise suppression capability of “Sigma”- and Size-EV-controlled “Sigma”-filters
Image enhancement by means of rank filtering

Component-wise P-histogram equalization
Image enhancement by means of rank filtering

$\text{Size-EV and Cardnl-filtering}$

$\text{Wnbh and EV}-\text{neighborhood local histogram equalization}$

Initial image           Segmented image

$\text{itermnev(Img,3,3,7,7,5); colormap jet}$