Lecture 1. Imaging transforms

Imaging systems and the linearity assumption. $b(x,y)=\int\limits_X\int\limits_Y a(\xi,\eta)h(x,y;\xi,\eta)d\xi d\eta$

Point spread function of imaging systems.
Direct imaging: PSF $h(x,y;\xi,\eta)$ is a function concentrated around point $x=\xi; y=\eta$
Space invariance assumption: $h(x,y;\xi,\eta)=h(x-\xi,y-\eta)$

Convolution transform. $b(x,y)=\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}h(\xi,\eta)a(x-\xi,y-\eta)d\xi d\eta$

Resolving power of imaging systems. Characterization of the imaging system resolving power through the minimal distance between two point sources that the system is capable to resolve. "Reyleigh's criterion" of the system resolving power.
System noise and resolving power of imaging systems. Characterization of system resolving power through variance of the error of localization of a point source.

For separable PSF, $h(x,y)=h_x(x)h_y(y)$: $\sigma_x^2 \propto \left(\int\limits_{-\infty}^{\infty}\left|\partial h_x(x)/\partial x\right|^2\right)^{-1}$; $\sigma_y^2 \propto \left(\int\left|\partial h_y(y)/\partial y\right|^2\right)^{-1}$

Converting space variant to space-invariant system by means of image geometrical transform.
Transform imaging. $b(x,y)=\int\limits_X\int\limits_Y a(\xi,\eta)h(x,y;\xi,\eta)d\xi d\eta$

Free space wave propagation PSF: $h(x,f)=\exp\left(i2\pi\sqrt{D^2+(x-f)^2}/\lambda\right)\Big/\left[D^2+(x-f)^2\right]$

Kirchhoff - Sommerfeld's integral: $\alpha(f)=\int\limits_{-\infty}^{\infty}a(x)\exp\left(i2\pi\sqrt{D^2+(x-f)^2}/\lambda\right)\Big/\left[D^2+(x-f)^2\right]dx$

Near diffraction zone approximation: $(x-f)^2 << D^2$: Fresnel integral transforms

$$\alpha(\vec{f})=\int\limits_{-\infty}^{\infty}a(\vec{x})\exp\left[i\pi(\vec{x}-\vec{f})^2/\lambda D\right]d\vec{x} \qquad a(\vec{x})=\int\limits_{-\infty}^{\infty}\alpha(\vec{f})\exp\left[-i\pi(\vec{x}-\vec{f})^2/\lambda D\right]d\vec{f}$$

Fraunhofer's (far diffraction zone) approximation: direct and inverse Fourier integral transforms

$$\alpha(\vec{f})=\int\limits_{-\infty}^{\infty}a(\vec{x})\exp\left[-i2\pi(\vec{x}\cdot\vec{f})/\lambda D\right]d\vec{x} \qquad a(x)=\int\limits_{-\infty}^{\infty}\alpha(f)\exp\left[i2\pi(\vec{x}\cdot\vec{f})/\lambda D\right]d\vec{f}$$

Special cases of Fourier transforms:

Integral cosine transform: $\alpha(f)=\int\limits_0^{\infty}a(x)\cos(2\pi xf)dx$

Inseparable 2-D cosine transform $\alpha(f_x,f_y)=\int\limits_0^{\infty}\int\limits_0^{\infty}a(x,y)\cos[2\pi(f_x x+f_y y)]dxdy$

Separable 2-D cosine transform $\alpha(f_x,f_y)=\int\limits_0^{\infty}\int\limits_0^{\infty}a(x,y)\cos(2\pi f_x x)\cos(2\pi f_y y)dxdy$

Radon transform. $\alpha(\xi_1,\theta)=\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}a(x_1,x_2)\delta(x_1\cos\theta+x_2\sin\theta-\xi_1)dx_1dx_2$

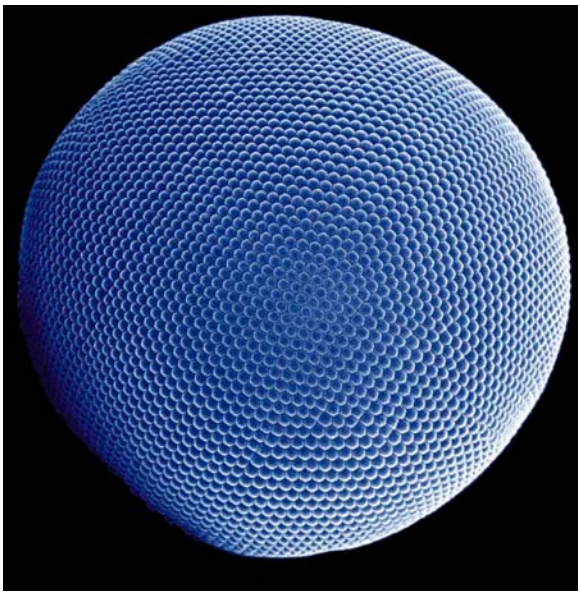
Projection theorem:

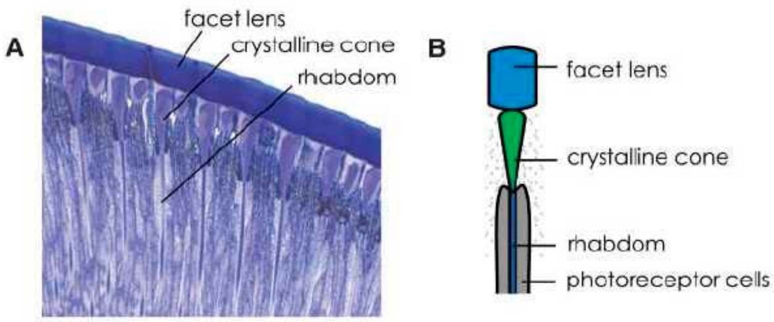
$$\int\limits_{-\infty}^{\infty}\alpha(\xi_1,\theta)\exp(i2\pi f_\theta\xi_1)d\xi_1=\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}a(x_1,x_2)\exp[i2\pi(f_\theta x_1\cos\theta+f_\theta x_2\sin\theta)]dx_1dx_2$$

Filtered back projection reconstruction:

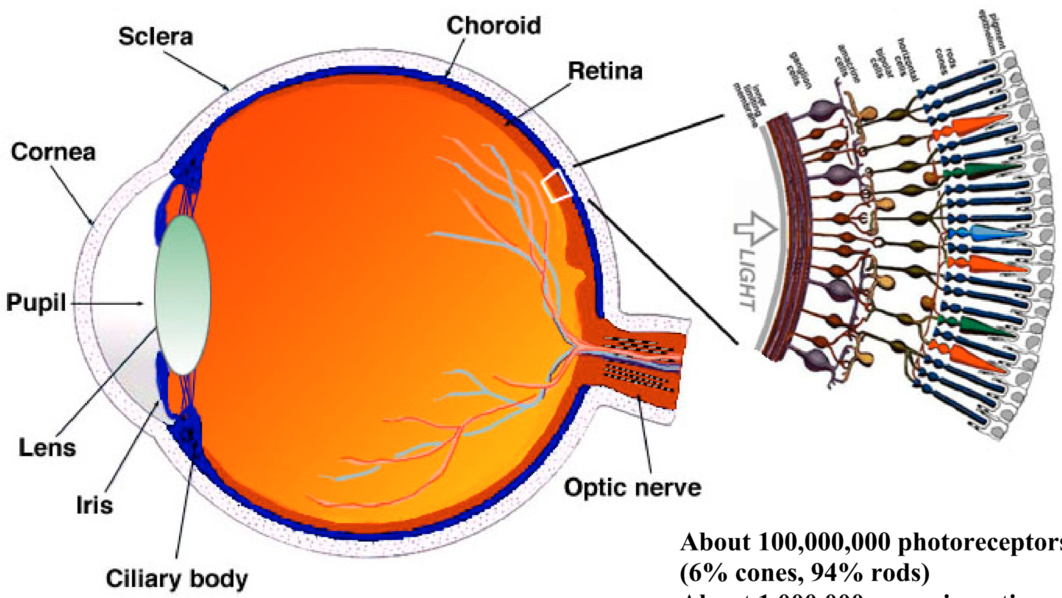
$$a(x,y)=\int\limits_0^{\pi}d\theta\int\limits_{-\infty}^{\infty}|f_\theta|\alpha(f_\theta,\theta)\exp[-i2\pi f_\theta(x\cos\theta+y\sin\theta)]df_\theta$$

Direct imaging and animal eyes





Compound eye of insects

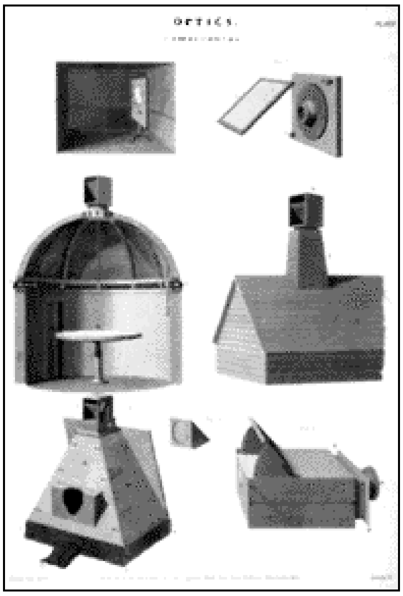


About 100,000,000 photoreceptors
(6% cones, 94% rods)
About 1,000,000 axons in optic nerve

Human eye

Direct imaging: man made devices

Camera-obscura



An 1817 encyclopedia page from the Wilgus Collection

The earliest mention of this type of device was by the **Chinese philosopher Mo-Ti (5th century BC)**. He formally recorded the creation of an inverted image formed by light rays passing through a pinhole into a darkened room. He called this darkened room a "collecting place" or the "locked treasure room."

Aristotle (384-322 BC) understood the optical principle of the camera obscura. He viewed the crescent shape of a partially eclipsed sun projected on the ground through the holes in a sieve, and the gaps between leaves of a plane tree.

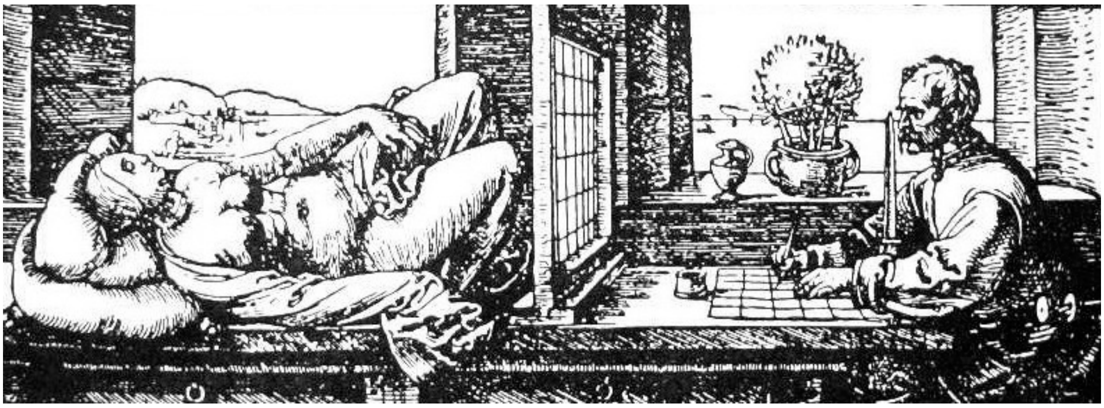
The Islamic scholar and scientist **Alhazen (Abu Ali al-Hasan Ibn al-Haitham) (c.965 - 1039)** gave a full account of the principle including experiments with five lanterns outside a room with a small hole.

In **1490 Leonardo Da Vinci** gave two clear descriptions of the camera obscura in his notebooks. Many of the first camera obscuras were large rooms like that illustrated by the Dutch scientist Reinerus Gemma-Frisius in 1544 for use in observing a solar eclipse.

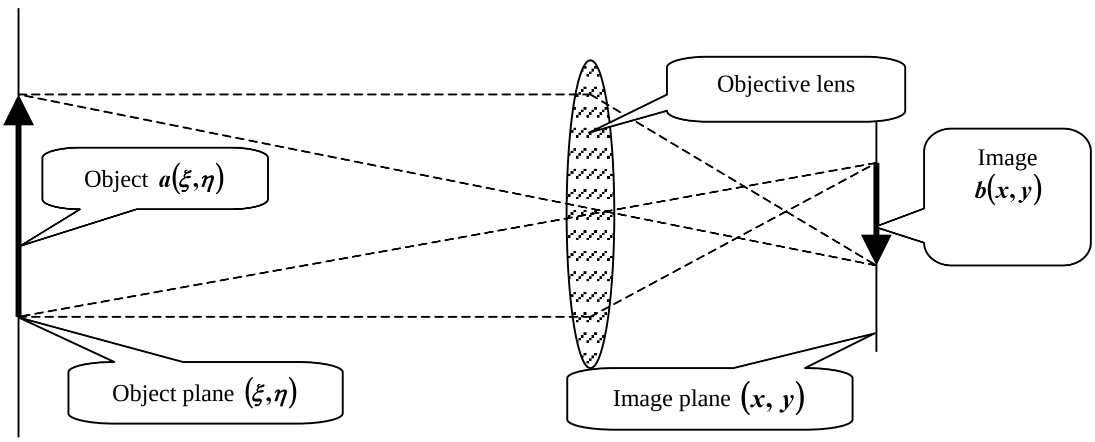
The image quality was improved with the addition of a convex lens into the aperture in the 16th century and the later addition of a mirror to reflect the image down onto a viewing surface. **Giovanni Battista Della Porta in his 1558** book “Magiae Naturalis” recommended the use of this device as an aid for drawing for artists.

The term "camera obscura" was first used by the German astronomer **Johannes Kepler in the early 17th** century. He used it for astronomical applications and had a portable tent camera for surveying in Upper Austria.

Adopted from http://brightbytes.com/cosite/what.html

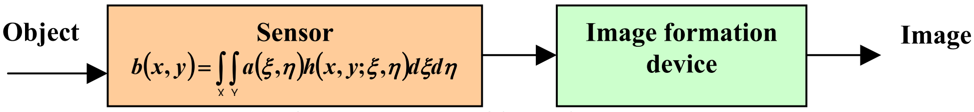


A woodcut by Albrecht Dürer showing the relationship between the light distribution on an object and image plane (adopted form R. Bracewell, Two-dimensional imaging, Prentice Hall Int. 1995)

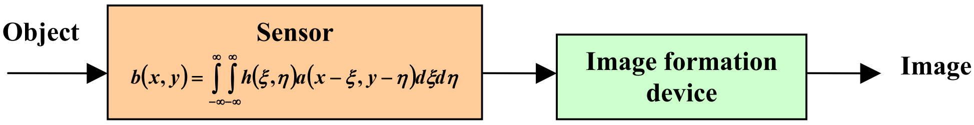


Schematic diagram of optics of photographic and TV cameras

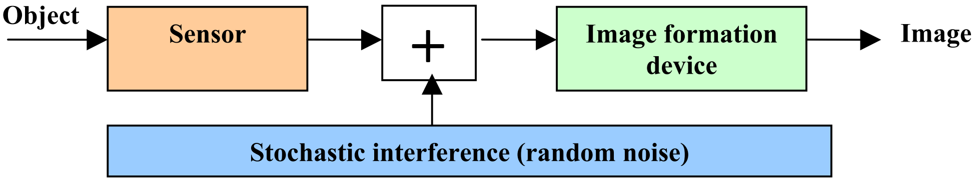
Mathematical models of imaging systems



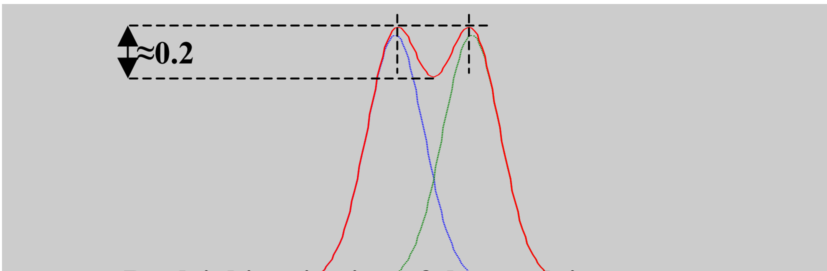
A linear model of imaging systems



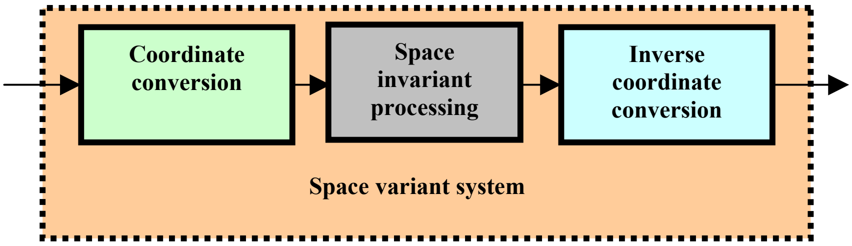
A space invariant linear model of imaging systems



A linear model of imaging systems with additive stochastic interference



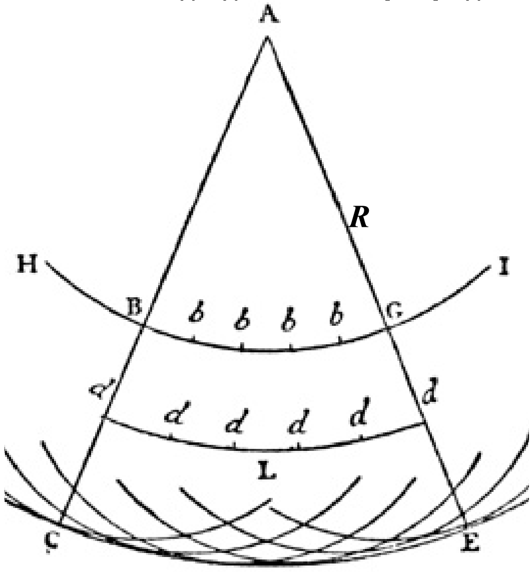
Rayleigh's criterion of the resolving power



Space variant to space invariant system conversion

Transform imaging and wave propagation

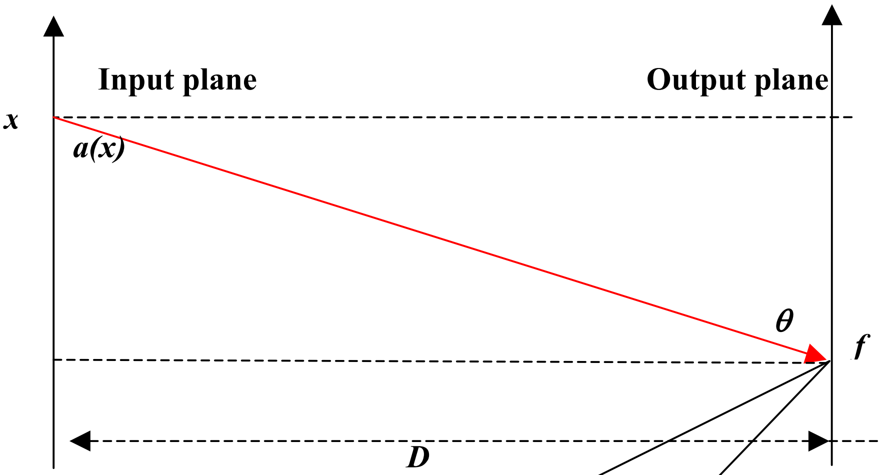
C. *Huygens* (1629-95), Dutch physicist



A. J. Fresnel (1788-1827), French physicist

$$\frac{A}{R}\exp\left(i2\pi\frac{R}{\lambda}\right)$$

Huygens-Fresnel principle.



$$\frac{a(x)\exp\left[i2\pi\left(\frac{\sqrt{D^2+(x-f)^2}}{\lambda}\right)\right]}{\sqrt{D^2+(x-f)^2}}\cos\theta =$$

$$a(x)\frac{\exp\left[i2\pi\left(\frac{\sqrt{D^2+(x-f)^2}}{\lambda}\right)\right]}{\sqrt{D^2+(x-f)^2}}\,\frac{D}{\sqrt{D^2+(x-f)^2}}$$

Free space wave propagation point spread function

$$h(x,y;f_x,f_y)=\frac{\exp\left[i2\pi\left(\frac{\sqrt{D^2+(x-f_x)^2+(y-f_y)^2}}{\lambda}\right)\right]}{D^2+(x-f_x)^2+(y-f_y)^2}$$

Kirchhoff-Rayleigh-Sommerfeld integral transform

$$\alpha(f)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}a(x,y)\frac{\exp\left(i2\pi\frac{D\sqrt{1+(x-f_x)^2/D^2+(y-f_y)^2/D^2}}{\lambda}\right)}{1+(x-f_x)^2/D^2+(y-f_y)^2/D^2}dxdy$$

In "near zone" approximation: $D^2 >> (x-f_x)^2+(y-f_y)^2$,

Fresnel integral transform:

$$\alpha(f_x,f_y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}a(x,y)\exp\left[i\pi\frac{(x-f_x)^2+(y-f_y)^2}{\lambda D}\right]dxdy\,.$$

In "far zone" approximation: $\pi(f_x^2+f_y^2)/\lambda D\approx 0$ and $\pi(x^2+y^2)/\lambda D\approx 0$,

Fourier integral transform:

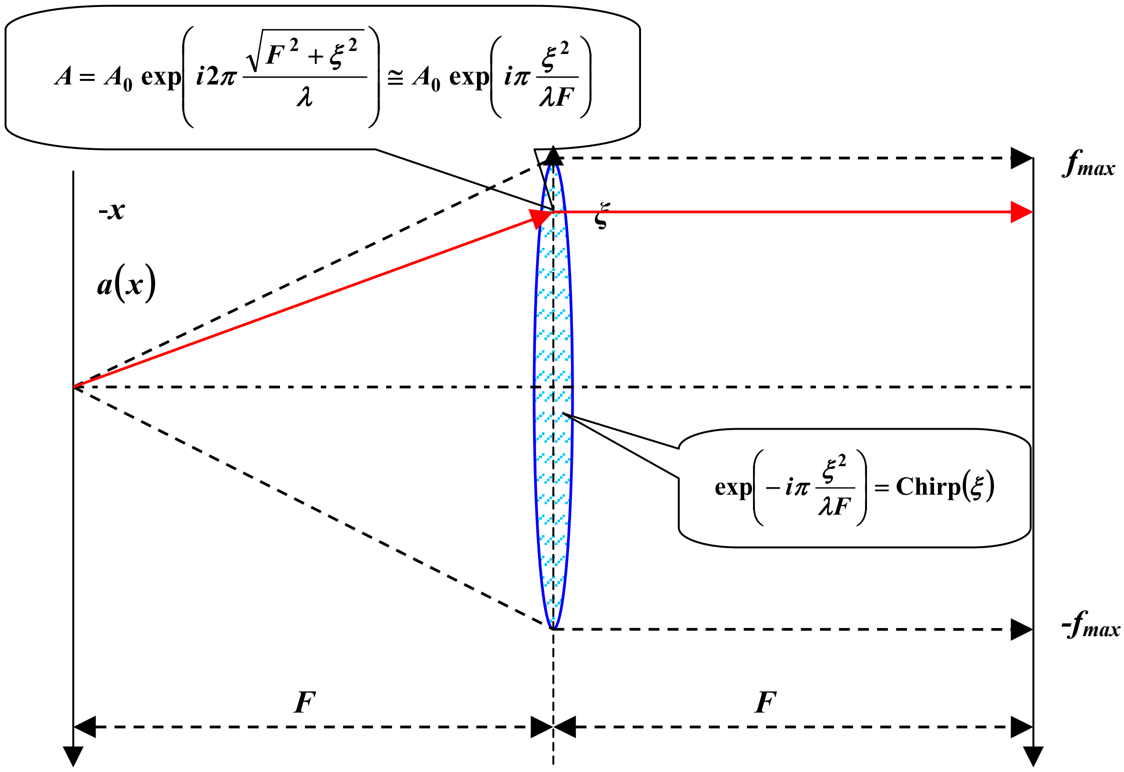
$$\alpha(f_x,f_y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}a(x,y)\exp\left[-i2\pi\frac{xf_x+yf_y}{\lambda D}\right]dxdy\,.$$

Inseparable 2-D cosine transform: $\alpha(f_x,f_y)=\int_0^{\infty}\int_0^{\infty}a(x,y)\cos[2\pi(f_x x+f_y y)]dxdy$

Separable 2-D cosine transform: $\alpha(f_x,f_y)=\int_0^{\infty}\int_0^{\infty}a(x,y)\cos(2\pi f_x x)\cos(2\pi f_y y)dxdy$

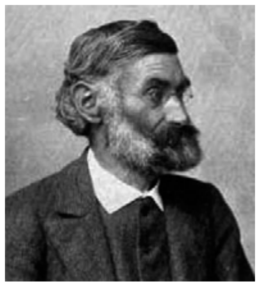
Two types of image even symmetry that correspond to two types of Cosine integral transforms (Images are reconstructed from computer generated Fourier holograms; adopted from L. Yaroslavsky, N. Merzlyakov, Methods of Digital Holography, Consultance Bureau, N.Y., 1980)

Lens a "chirp"-spatial light modulator



Lenses as a Fourier transformers

Jean Baptiste Joseph Fourier (1768-1830)



Ernst Abbe, 1840-1905

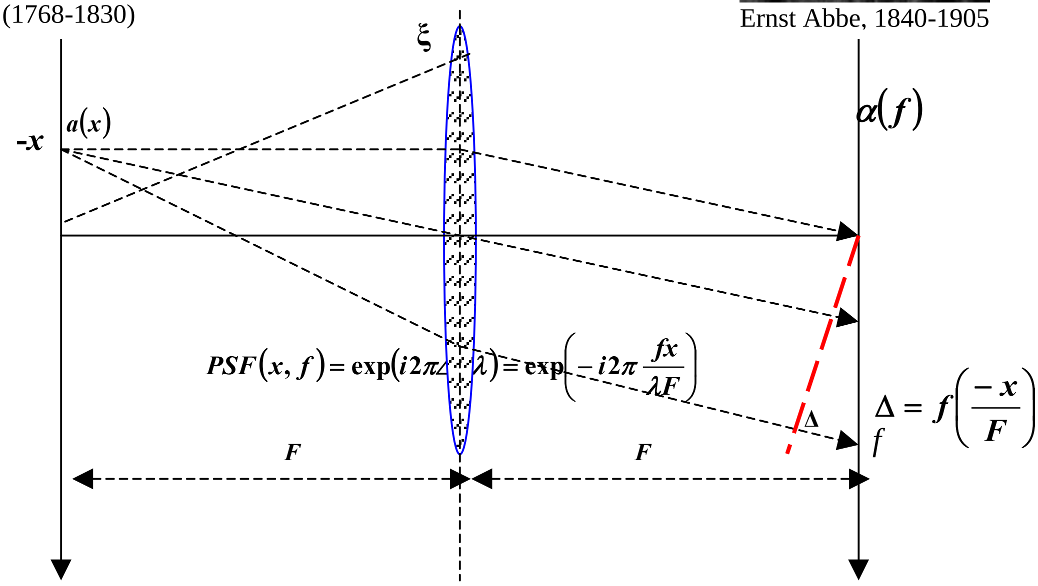


Image formation from projections: Radon transform

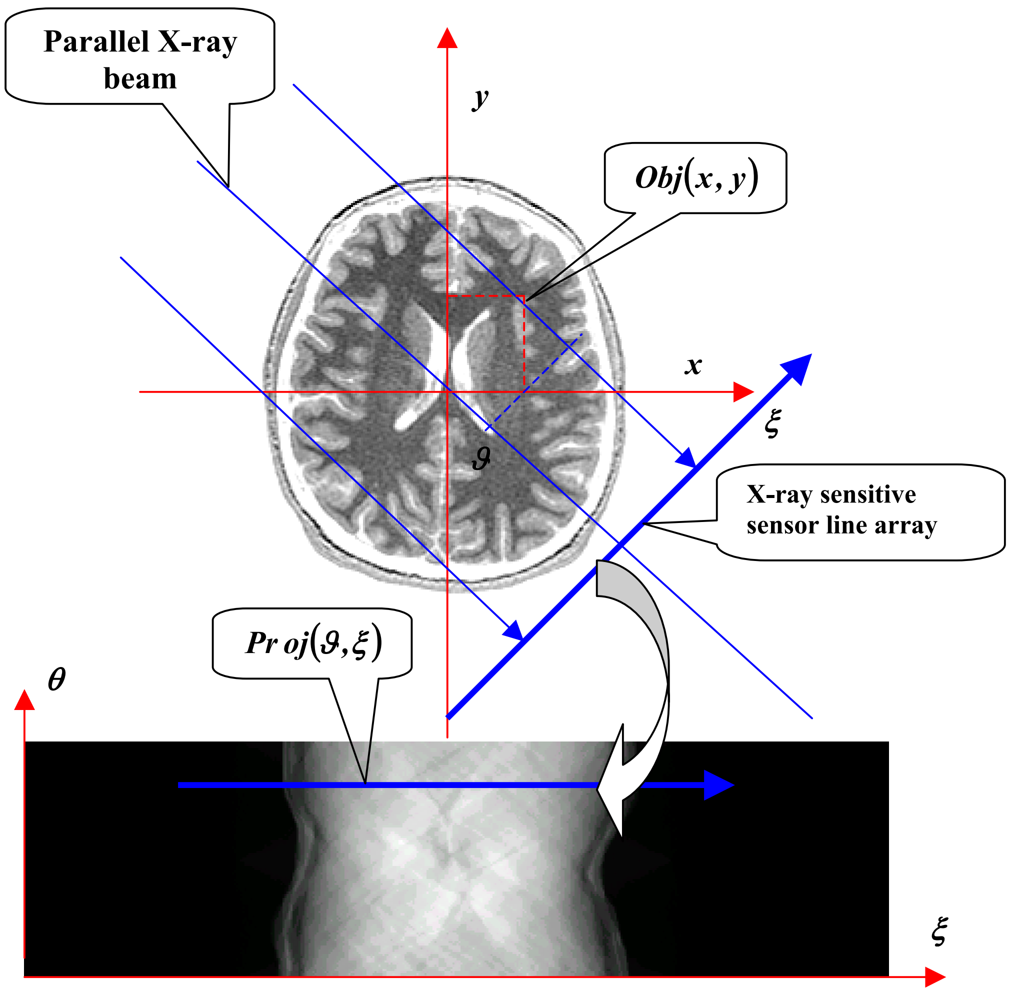


Image parallel projection geometry and Radon transform

Radon transform: $$\alpha(\xi_1,\theta)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}a(x_1,x_2)\delta(x_1\cos\theta+x_2\sin\theta-\xi_1)dx_1dx_2$$