L. Yaroslavsky. ***Course 0510.7211 "Digital Image Processing: Applications"***

Lect. 3. Image Quantization in Image and Transform Domains

Optimal element-wise quantization.

$$\varepsilon(q)=\alpha-\hat{\alpha}^{(q)}; \qquad E_I=\sum_{q=1}^{Q-2}\int_{\alpha^{(q-1)}}^{\alpha^{(q)}} p(\alpha)D(\varepsilon_q)d\alpha;\ E_L=\int_{-\infty}^{\alpha^{(0)}} p(\alpha)D(\varepsilon_0)d\alpha;\ E_R=\int_{\alpha^{(Q)}}^{\infty} p(\alpha)D(\varepsilon_{Q-1})d\alpha;$$

"Max-Lloyd" quantizer: $\left\{\hat{\alpha}^{(q)},\alpha^{(q)}\right\}=\underset{\{\hat{\alpha}^{(q)},\alpha^{(q)}\}}{\arg\min}\sum_{q=1}^{Q-2}\int_{\alpha^{(q-1)}}^{\alpha^{(q)}} p(\alpha)D(\varepsilon_q)d\alpha \Rightarrow; D\left(\alpha^{(q)}-\hat{\alpha}^{(q)}\right)=D\left(\alpha^{(q)}-\hat{\alpha}^{(q-1)}\right)$

Compander-expander quantizer.

Modified quantization quality criterion: $E_I\approx\sum_{q=1}^{Q-2}\int_{\alpha^{(q-1)}}^{\alpha^{(q)}} p(\alpha)D\left(\alpha^{(q-1)}-\hat{\alpha}^{(q)}\right)d\alpha\approx\int_{\alpha^{(0)}}^{\alpha^{(Q)}} p(\alpha)D(\Delta_u/w'(\alpha))d\alpha;$

Optimal predistortion function $w_{opt}(\alpha)=\underset{w(\alpha)}{\arg\min}\left\{\int_{\alpha^{(0)}}^{\alpha^{(Q)}} p(\alpha)D(\Delta_u/w'(\alpha))d\alpha;\right\}$

are found from Euler-Lagrange equation: $\frac{\partial}{\partial w'}\left\{p(\alpha)D(\Delta_u/w'(\alpha))\right\}=const$

Examples:

1. Threshold criterion: $D\left(\alpha^{(q)}-\alpha^{(q+1)}\right)=\begin{cases}0, \left|\alpha^{(q)}-\alpha^{(q+1)}\right|<\Delta_{thr}\\ 1, \left|\alpha^{(q)}-\alpha^{(q+1)}\right|>\Delta_{thr}\end{cases}\Rightarrow$ uniform quantization

2. Threshold criterion:

$$D\left(\alpha^{(q)}-\alpha^{(q+1)}\right)=\begin{cases}0, \left|\alpha^{(q)}-\alpha^{(q+1)}\right|<\Delta_{thr}=\delta_0\alpha\\ 1, \left|\alpha^{(q)}-\alpha^{(q+1)}\right|>\Delta_{thr}=\delta_0\alpha\end{cases}\Rightarrow \frac{w(\alpha)-w(\alpha_{\min})}{w(\alpha_{\max})-w(\alpha_{\min})}=\frac{\ln(\alpha/\alpha_{\min})}{\ln(\alpha_{\max}/\alpha_{\min})}; Q=\left(\ln(\alpha_{\max}/\alpha_{\min})\right)/\delta_0$$

<u>Image quantization and false contours. Why 256 quantization levels?</u>

3. $D\left(\alpha^{(r)}-\alpha^{(r+1)}\right)=\left(\alpha^{(r)}-\alpha^{(r+1)}\right)^{2n}\Rightarrow \frac{w(\alpha)-w(\alpha_{\min})}{w(\alpha_{\max})-w(\alpha_{\min})}=\frac{\int_{\alpha_{\min}}^{\alpha}(p(\alpha))^{1/(2n+1)}d\alpha}{\int_{\alpha_{\min}}^{\alpha_{\max}}(p(\alpha))^{1/(2n+1)}d\alpha}$

When $n=1$, $\frac{w(\alpha)-w(\alpha_{\min})}{w(\alpha_{\max})-w(\alpha_{\min})}=\frac{\int_{\alpha_{\min}}^{\alpha}(p(\alpha))^{1/3}d\alpha}{\int_{\alpha_{\min}}^{\alpha_{\max}}(p(\alpha))^{1/3}d\alpha}$

For $p(\alpha)=c\exp\left\{-\frac{(\alpha-\bar{\alpha})^2}{2\sigma_\alpha^2}\right\}$, $\frac{w(\alpha)-w(\alpha_{\min})}{w(\alpha_{\max})-w(\alpha_{\min})}=\frac{\Phi\left(\frac{\alpha-\bar{\alpha}}{\sqrt{3}\sigma_\alpha}\right)-\Phi\left(\frac{\alpha_{\min}-\bar{\alpha}}{\sqrt{3}\sigma_\alpha}\right)}{\Phi\left(\frac{\alpha_{\max}-\bar{\alpha}}{\sqrt{3}\sigma_\alpha}\right)-\Phi\left(\frac{\alpha_{\min}-\bar{\alpha}}{\sqrt{3}\sigma_\alpha}\right)}$, where $\Phi(\alpha)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left(-\frac{x^2}{2}\right)dx$

4. $D\left(\alpha^{(r)}-\alpha^{(r+1)}\right)=\left(\left(\alpha^{(r)}-\alpha^{(r+1)}\right)/\alpha\right)^{2n}\Rightarrow \frac{w(\alpha)-w(\alpha_{\min})}{w(\alpha_{\max})-w(\alpha_{\min})}=\frac{\int_{\alpha_{\min}}^{\alpha}\left(p(\alpha)/\alpha^{2n}\right)^{1/(2n+1)}d\alpha}{\int_{\alpha_{\min}}^{\alpha_{\max}}\left(p(\alpha)/\alpha^{2n}\right)^{1/(2n+1)}d\alpha}$

When $p(\alpha)=const$, $\frac{w(\alpha)-w(\alpha_{\min})}{w(\alpha_{\max})-w(\alpha_{\min})}=\frac{\alpha^{1/(2n+1)}-\alpha_{\min}^{1/(2n+1)}}{\alpha_{\max}^{1/(2n+1)}-\alpha_{\min}^{1/(2n+1)}}$

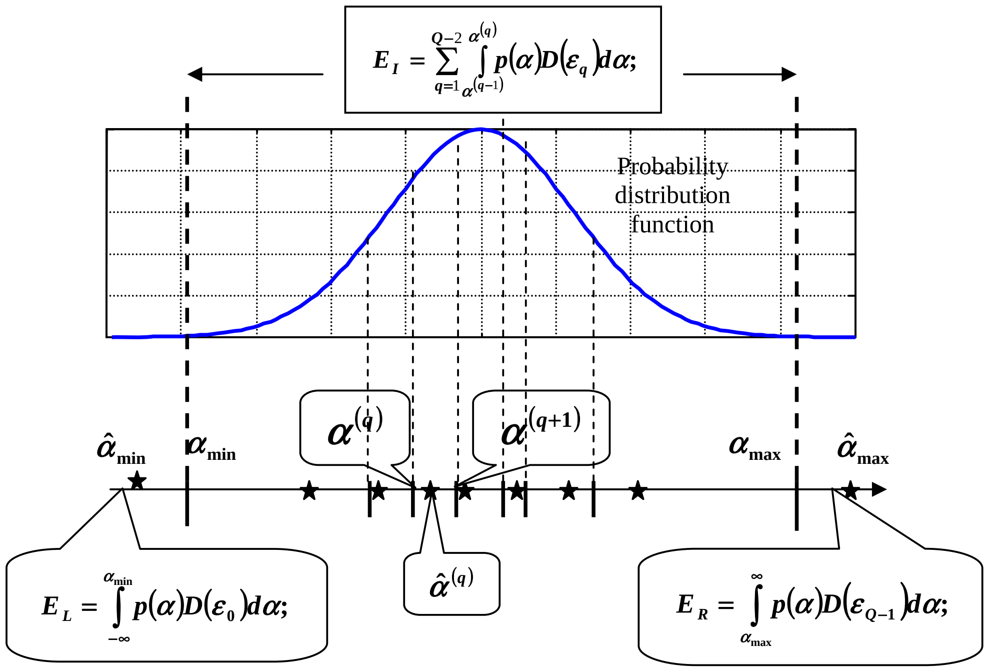
Practical aspects of element-wise quantization.

P-th law quantization: $w(\alpha)=|\alpha|^P \operatorname{sign}\alpha$. Quantization in the presence of noise. Quantization and correcting image dynamic range. Quantization of holograms. Speckle noise in coherent radiation based imaging systems. Quantization in computer tomography

Problems for self-examination

1. What role loss function and signal probability density play in quantization optimization
2. Compare Max-Lloyd and compander-expander quantization. What is ***P***-th law quantization?
3. Why 256 quantization levels in logarithmic scale were selected for image quantization?
4. Describe quantization artefacts for quantization in transform domain.
5. How additive noise affects quantization?

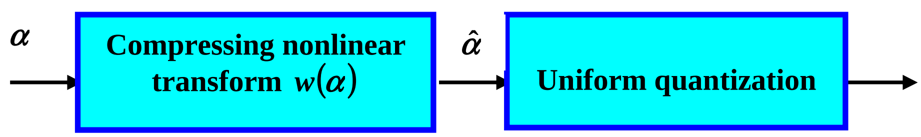
The principle of element-wise quantization



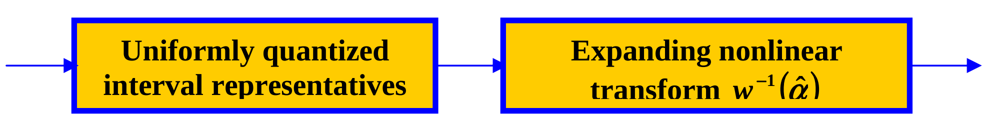
"Max-Lloyd" quantizer:

$$\left\{\hat{\alpha}^{(q)}, \alpha^{(q)}\right\} = \underset{\left\{\hat{\alpha}^{(q)}, \alpha^{(q)}\right\}}{\arg\min} \sum_{q=1}^{Q-2} \int_{\alpha^{(q-1)}}^{\alpha^{(q)}} p(\alpha) D(\varepsilon_q) d\alpha \Rightarrow; D\left(\alpha^{(q)} - \hat{\alpha}^{(q)}\right) = D\left(\alpha^{(q)} - \hat{\alpha}^{(q-1)}\right)$$

Compander-expander quantizer.



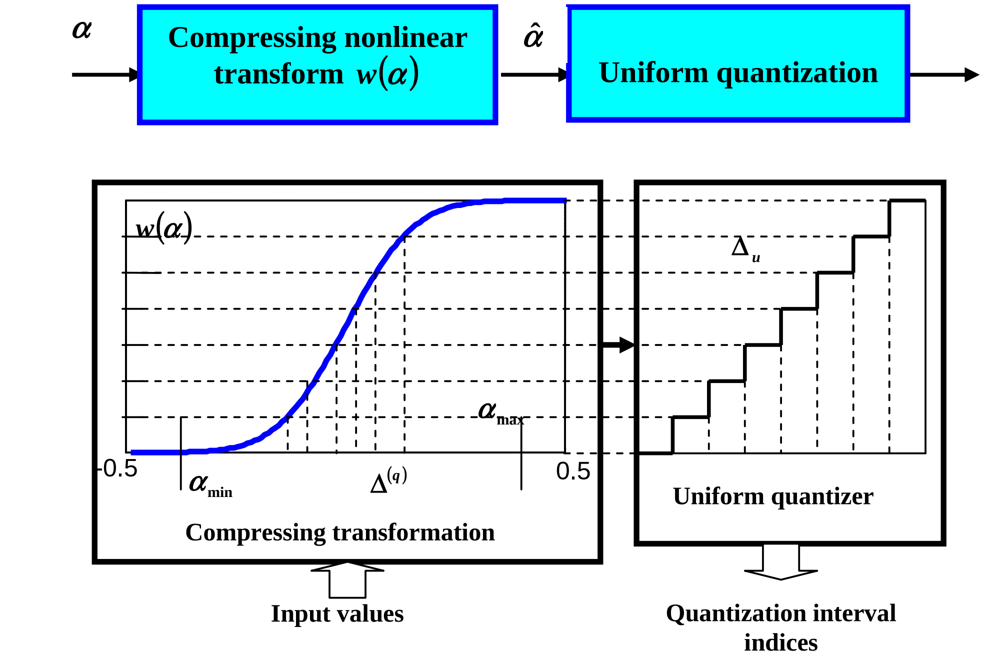
Restoration:



Optimal predistortion function $w_{opt}(\alpha) = \underset{w(\alpha)}{\arg\min} \left\{ \int_{\alpha^{(0)}}^{\alpha^{(Q)}} p(\alpha) D(\Delta_u / w'(\alpha)) d\alpha; \right\}$

Element-wise quantization: compander-expander method

Quantization:



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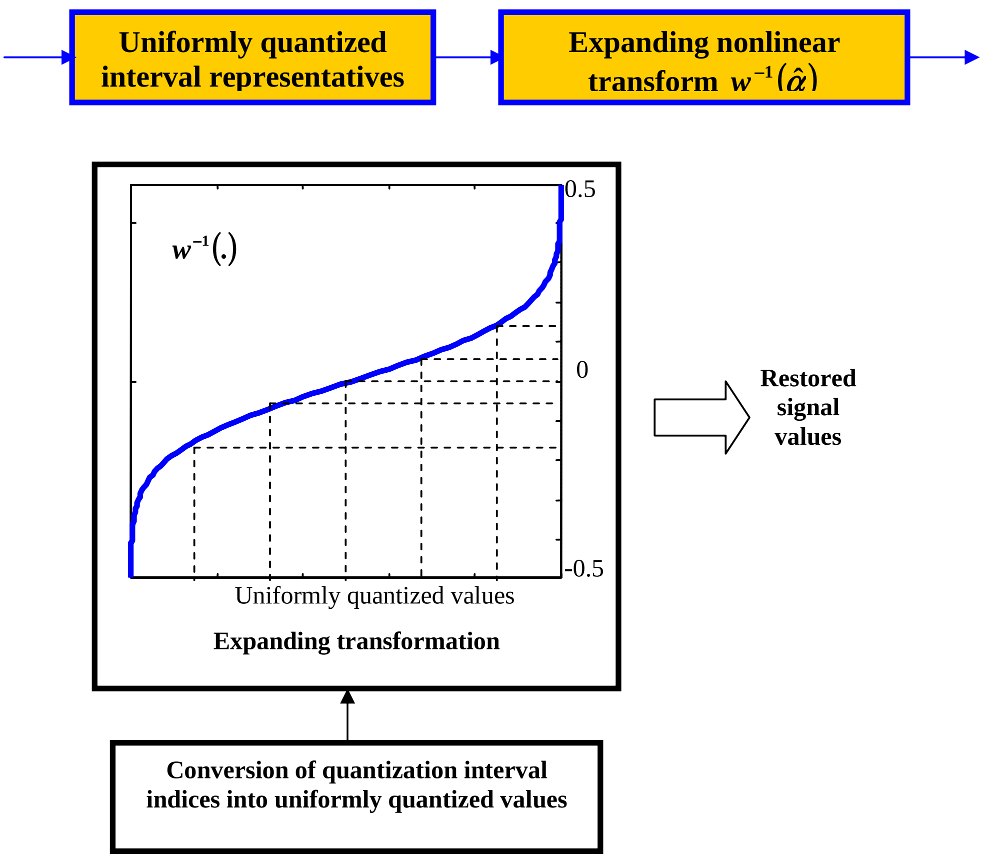
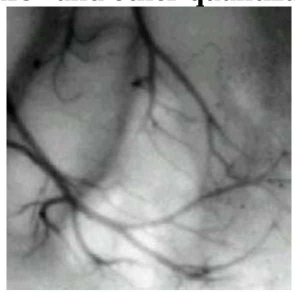
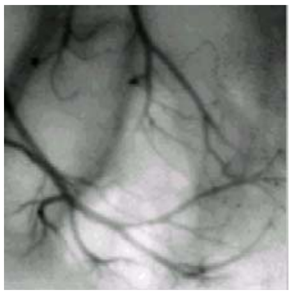


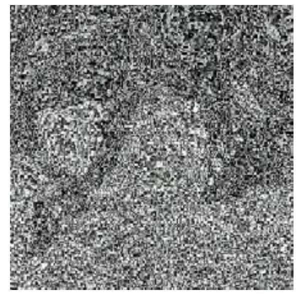
"False contours" and other quantization artefacts



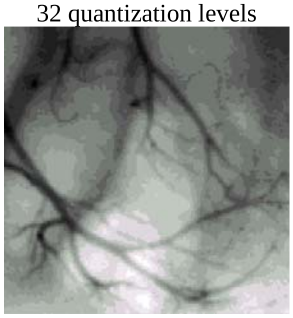
256 quantization levels



32 quantization levels

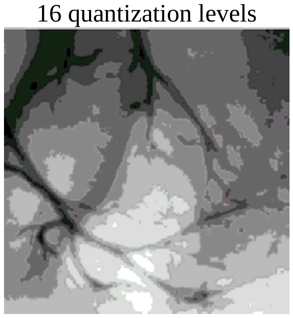


Quantization error: StDev=1.46

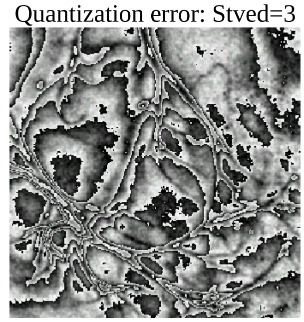


16 quantization levels

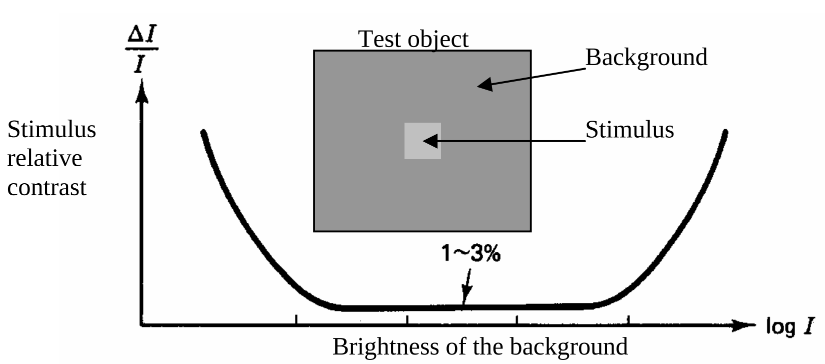
Quantization error: Stved=3



8 quantization levels

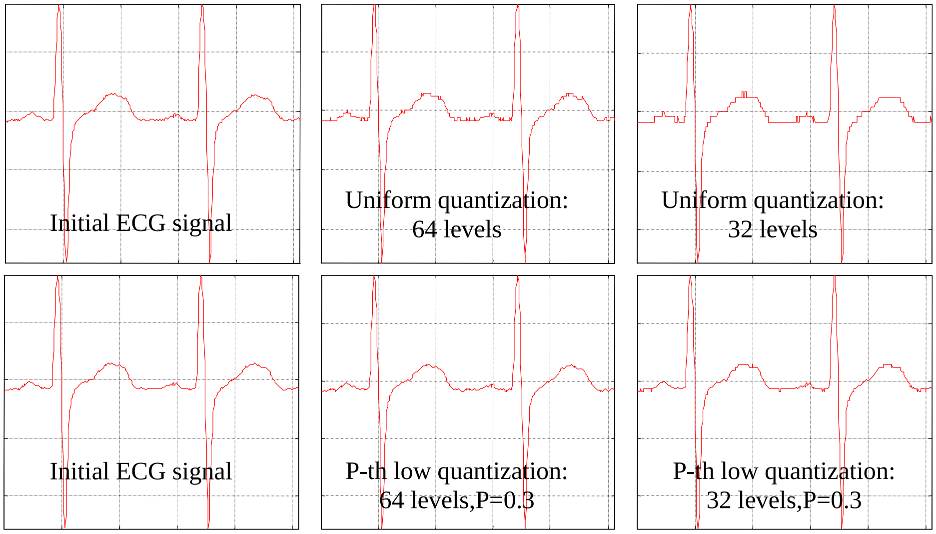


Quantization error: (St. Dev 6.4)



Weber-Fechner law of eye sensitivity to object's contrast

Quantization with *P*-th law nonlinearity



Uniform and non-uniform (*P*-th law) quantization of orthogonal components of image Fourier spectra


Initial image

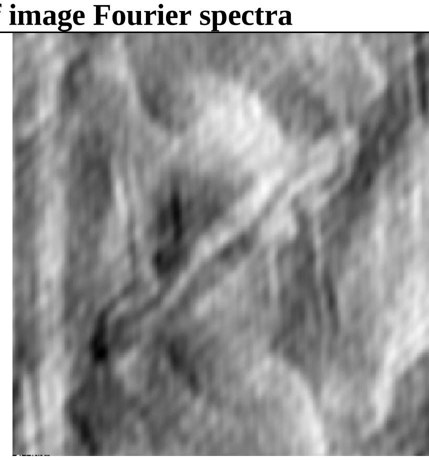


Image restored after uniform quantization (31 levels) of its spectrum orthogonal components

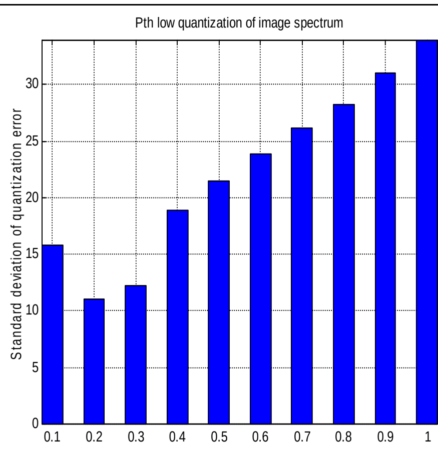


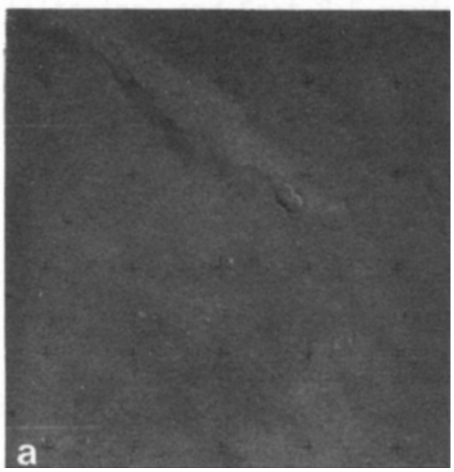

Image restored after optimal *P*-th law quantization of its spectrum orthogonal components (P=0.2, 31 quantization levels)

Quantization with noise

Image quantized to 16 levels. Observe false contours.

Image with additive noise (std=4) quantized to 16 levels

Correcting image contrast and quantization with noise



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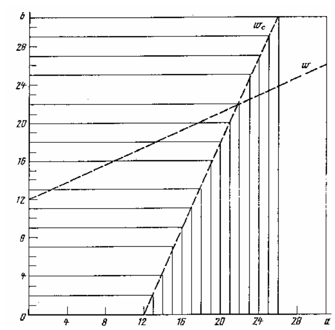
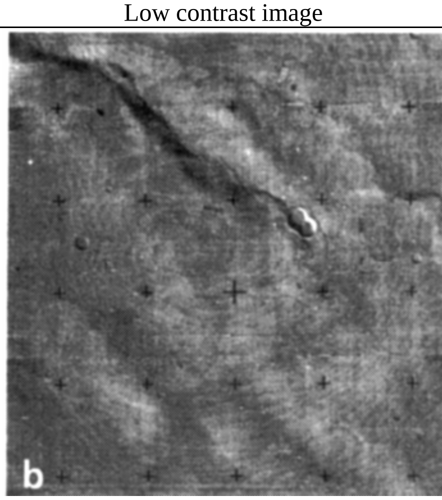
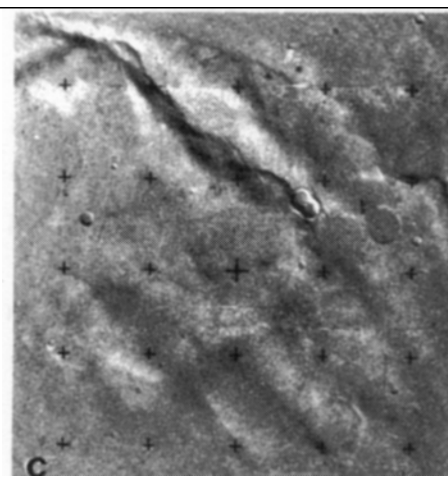
Low contrast image



Contrast correcting look-up table and course quantization effects

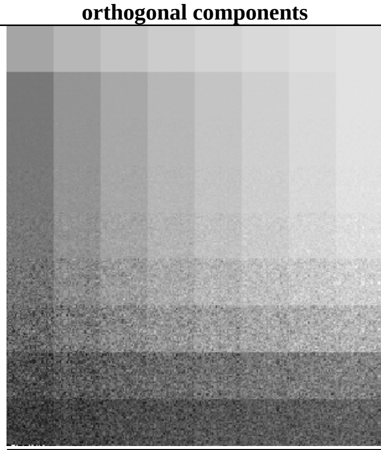
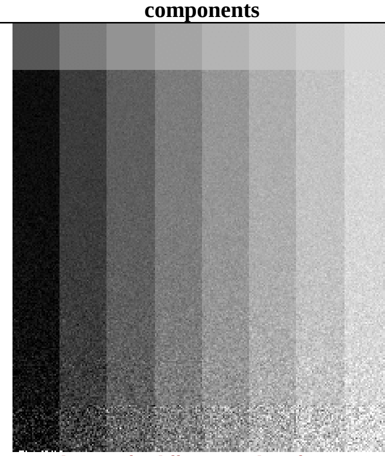
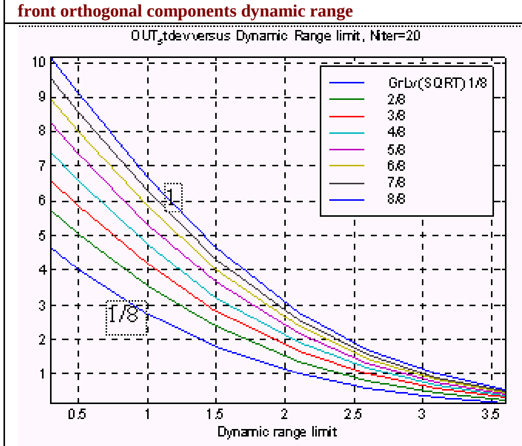


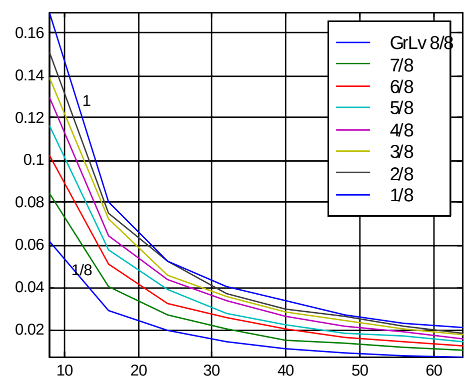
Contrast corrected image and false contours



Destroying false contours by quantization with additive noise

Speckle noise phenomena in imaging by means of coherent radiation

<table>
<tr><th>Limitation of the dynamic range of the hologram orthogonal components</th><th>Quantization of the hologram orthogonal components</th></tr>
<tr><td>
Reconstructed images for different degree of limitation of the wave front orthogonal components dynamic range</td><td>
Reconstructed images for different number of quantization levels (64 to 8, from top to bottom</td></tr>
<tr><td>

Standard deviation of speckle versus the threshold parameter of limitation of the wave front orthogonal components dynamic range</td><td>

Standard deviation of speckle noise as a function of the number of quantization levels *Q*</td></tr>
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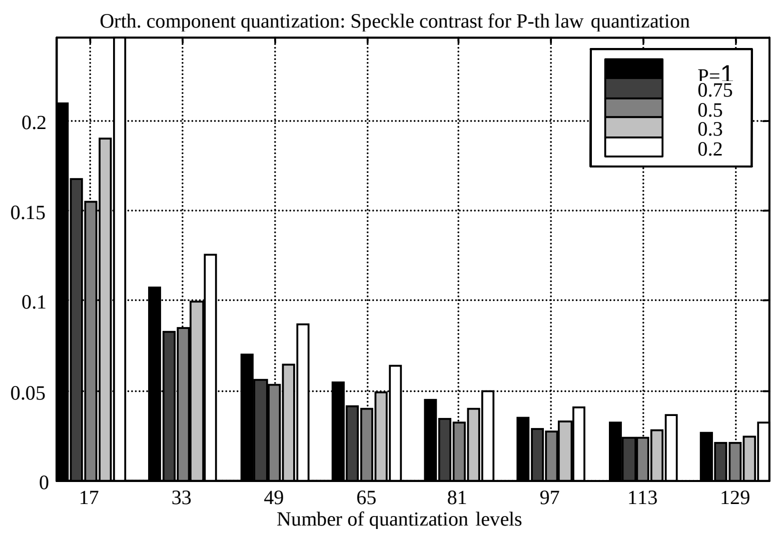

Speckle contrast in images reconstructed from a hologram of a diffusely reflecting object versus number of quantization levels in *P*-th law quantizing hologram orthogonal components for different values of nonlinearity index *P*. One can see that speckle contrast due to the quantization is minimized when *P* is about 0.3-0,5.