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FROM PHOTOGRAPHY TO *.GRAPHIES: UNCONVENTIONAL IMAGING TECHNIQUES

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Lecture 7.

SPECKLE NOISE IN COHERENT IMAGING SYSTEMS

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SPECKLE NOISE IN COHERENT IMAGING SYSTEMS

Phenomenon of ***speckle noise*** is characteristic for coherent imaging systems such as holographic ones, synthetic aperture radar and ultrasound imaging systems. Speckle noise originates from property of objects to diffusely scatter irradiation and is caused by distortions introduced by wavefront sensors. Conventionally it is associated with finite resolution power of sensors, or, in holographic systems, by the limitation of the area in which wave front is measured by sensors or recorded on holograms. Let $a(\xi,\eta)\exp(i\theta(\xi,\eta))$ is complex amplitude of wavefront in the object plane that describes object reflectance/transmittance properties. For describing property of objects to diffusely scatter irradiation, $\theta(\xi,\eta)$ can be modeled as a random process. Diffuse uniform in all direction scattering correspond to spatially noncorrelated process $\theta(\xi,\eta)$. Specular scattering takes place if $\theta(\xi,\eta)$ is highly spatially correlated process.

Consider coherent imaging system with point spread function $h(x,y;\xi,\eta)$. Then, for input object $a(\xi,\eta)\exp(i\theta(\xi,\eta))$, output image formed by a sensor that is sensitive to squared magnitude of the output wavefront is

$$b^2(x,y)=\left|\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}a(\xi,\eta)\exp[i\theta(\xi,\eta)]h(x,y;\xi,\eta)d\xi d\eta\right|^2 . \qquad (1)$$

Let the object has uniformly painted surface such that $a(\xi,\eta)=A_0$. Then:

$$b^2(x,y)=\left|A_0\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp[i\theta(\xi,\eta)]h(x,y;\xi,\eta)d\xi d\eta\right|^2=\left|b^{re}\right|^2+\left|b^{im}\right|^2 \qquad (2)$$

where

$$b^{re}=A_0\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cos[\theta(\xi,\eta)]h(x,y;\xi,\eta)d\xi d\eta , \qquad (3)$$

$$b^{im}=A_0\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sin[\theta(\xi,\eta)]h(x,y;\xi,\eta)d\xi d\eta \qquad (4)$$

If functions $\cos[\theta(\xi,\eta)]$ and $\sin[\theta(\xi,\eta)]$ are changing much faster then point spread function $h(x,y;\xi,\eta)$, or, in other words, many uncorrelated values of $\exp[i\theta(\xi,\eta)]$ are observed within aperture of the imaging system, the central limit theorem of the probability theory can be used to assert that b^{re} and b^{im} are normally distributed with zero mean:

$$\overline{b^{re}} = AV_\theta\left(b^{re}\right) = A_0 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} AV_\theta\left(\cos[\theta(\xi,\eta)]\right) h(x,y;\xi,\eta)\, d\xi d\eta = 0;$$
$$\overline{b^{im}} = AV_\theta\left(b^{im}\right) = A_0 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} AV_\theta\left(\sin[\theta(\xi,\eta)]\right) h(x,y;\xi,\eta)\, d\xi d\eta = 0 \quad . \tag{5}$$

Find correlation function of orthogonal components b^{re} and b^{im}. For b^{re},

$$R_{b^{re}}(\mathbf{x}_1,\mathbf{x}_2) = AV_\theta\left\{ b^{re}(\mathbf{x}_1)\left[b^{re}(\mathbf{x}_2)\right]^* \right\} =$$
$$A_0^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} AV_\theta\left(\cos[\theta(\boldsymbol{\xi}_1)]\cos[\theta(\boldsymbol{\xi}_2)]\right) h(\mathbf{x}_1;\boldsymbol{\xi}_1)h^*(\mathbf{x}_2;\boldsymbol{\xi}_2)\, d\boldsymbol{\xi}_1 d\boldsymbol{\xi}_2 = , \tag{6}$$
$$A_0^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_\theta\left((\boldsymbol{\xi}_1,\boldsymbol{\xi}_2)\right) h(\mathbf{x}_1;\boldsymbol{\xi}_1)h^*(\mathbf{x}_2;\boldsymbol{\xi}_2)\, d\boldsymbol{\xi}_1 d\boldsymbol{\xi}_2$$

where $\mathbf{x} = (x,y)$, $\boldsymbol{\xi} = (\xi,\eta)$ and $R_\theta\left((\boldsymbol{\xi}_1,\boldsymbol{\xi}_2)\right)$ is correlation function of $\cos[\theta(\xi,\eta)]$. In the above accepted assumption of applicability of the central limit theorem, $R_\theta\left((\boldsymbol{\xi}_1,\boldsymbol{\xi}_2)\right)$ can be regarded, with respect to integration with $h(x,y;\xi,\eta)$, delta function which implies that

$$R_{b^{re}}(\mathbf{x}_1,\mathbf{x}_2) = \frac{1}{2}A_0^2 \int_{-\infty}^{\infty} h(\mathbf{x}_1;\boldsymbol{\xi})h^*(\mathbf{x}_2;\boldsymbol{\xi})\, d\boldsymbol{\xi} \tag{7}$$

In the same way one can obtain that:

$$R_{b^{im}}(\mathbf{x}_1,\mathbf{x}_2) = R_{b^{re}}(\mathbf{x}_1,\mathbf{x}_2) = \frac{1}{2}A_0^2 \int_{-\infty}^{\infty} h(\mathbf{x}_1;\boldsymbol{\xi})h^*(\mathbf{x}_2;\boldsymbol{\xi})\, d\boldsymbol{\xi} \quad (2.8.26)$$

From Eqs. 2.8.25 and 26 it follows that variances of orthogonal components b^{re} and b^{im} are

$$\sigma_b^2 = \overline{\left(b^{re}\right)^2} = \overline{\left(b^{im}\right)^2} = \frac{1}{2}A_0^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left|h(x,y;\xi,\eta)\right|^2 d\xi d\eta \ . \tag{8}$$

In general, they are functions of coordinate (x,y) in image plane. For space invariant imaging system ($h(x,y;\xi,\eta) = h(x-\xi, y-\eta)$) they are coordinate independent:

$$\sigma_b^2 = \frac{1}{2}A_0^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left|h(x,y)\right|^2 dxdy \tag{9}$$

Having defined probability density function of orthogonal components of $b^2(x,y)$ one can now find probability density of $b^2(x,y)$ as the probability density of sum of squared independent variables b^{re} and b^{im} with normal distribution. To this end, introduce random variables R and ϑ such that

$$b^{re} = R\cos\vartheta\,; \quad b^{im} = R\sin\vartheta\,; \quad b(x,y) = R^2\,. \tag{10}$$

Probability that b^{re} and b^{im} take values within a rectangle $d(b^{re})\mathrm{d}(b^{im})$ is

$$\begin{gathered}\frac{1}{2\pi\sigma_b^2}\exp\left[-\frac{\left(b^{re}\right)^2+\left(b^{im}\right)^2}{2\sigma_b^2}\right]d\left(b^{re}\right)d\left(b^{im}\right)= \\ \frac{d\theta}{2\pi}\frac{R}{\sigma_b^2}\exp\left[-\frac{R^2}{2\sigma_b^2}\right]dR=\frac{d\theta}{2\pi}\frac{1}{2\sigma_b^2}\exp\left[-\frac{R^2}{2\sigma_b^2}\right]d\left(R^2\right)\end{gathered}\;. \tag{11}$$

It is joint probability of variables $\left\{R^2,\vartheta\right\}$. It follows, therefore, that $\left\{R^2 = b(x,y)\right\}$ and ϑ are statistically independent, ϑ is uniformly distributed in the range $\{0, 2\pi\}$ and $b(x,y)$ has exponential distribution density:

$$P(b) = \frac{1}{2\sigma_b^2}\exp\left(-\frac{b}{2\sigma_b^2}\right) \tag{12}$$

where σ_b^2 is defined by Eq. (9).

From Eqs.2 and 10 one can immediately see that mean value of $b^2(x,y)$ is:

$$\overline{\mathrm{b}^2(\mathrm{x},\mathrm{y})} = 2\sigma_b^2 = A_0^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left|h(x,y)\right|^2 dxdy \tag{13}$$

It is a property of the exponential distribution that its standard deviation is equal to its mean value

$$\sigma_{b^2} = \left(\overline{\left(\mathrm{b}^2(\mathrm{x},\mathrm{y})\right)^2}\right)^{1/2} = A_0^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left|h(x,y)\right|^2 dxdy\;. \tag{14}$$

Fluctuation of $b^2(x,y)$ around its mean value are called speckle noise. It is customary to characterize intensity of speckle noise by ration of its standard deviation to mean value called "***speckle contrast***":

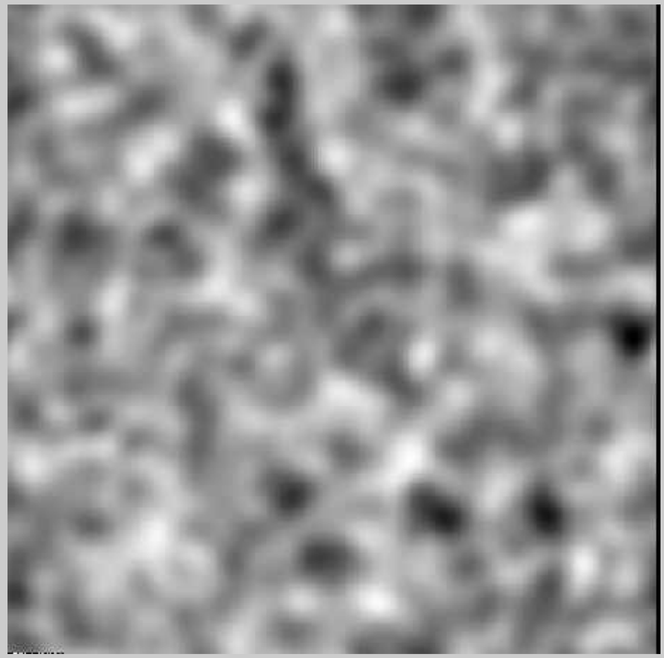
$$Spckl\_contrast = \frac{\sigma_{b^2}}{\overline{b^2(x,y)}}\;. \tag{15}$$

It follows from Eqs. (13) and (14) that, for objects that scatter irradiation almost uniformly in the space, speckle contrast of their image obtained in coherent imaging systems is unity:

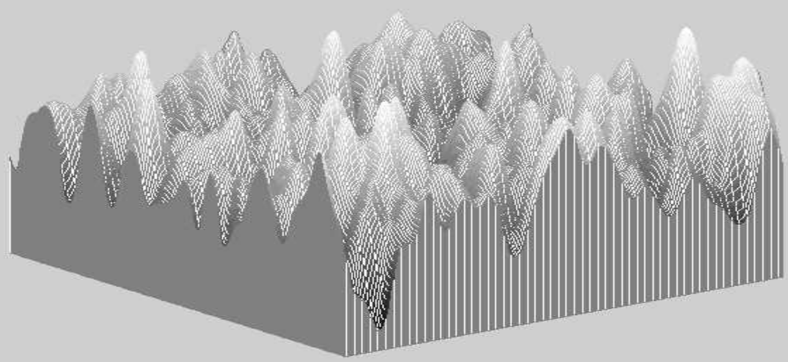
$$Spckl\_contrast = 1 . \qquad (16)$$

Therefore speckle noise can be regarded as multiplicative with respect to image mean value.

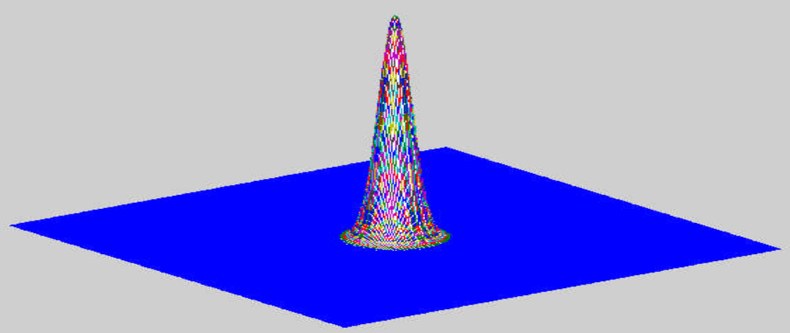
In general, speckle noise in coherent imaging systems appears not only due to insufficient resolution power of the systems. In fact, any distortions of the wave field that may happen in the sensor such as limitation of the signal dynamic range or signal quantization for its conversion into a digital signal for signal processing also cause appearance of speckle noise.

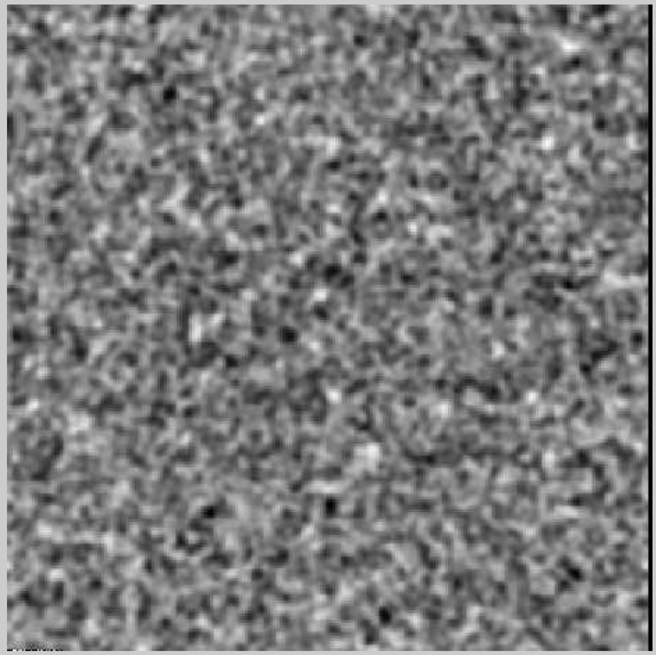


Rough surface

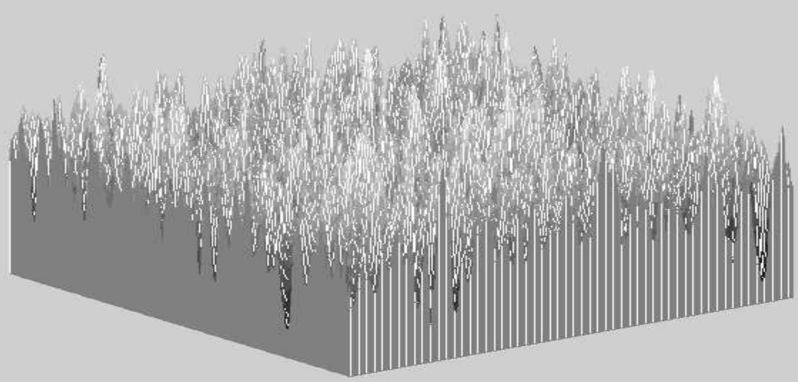


Its “directivity” function

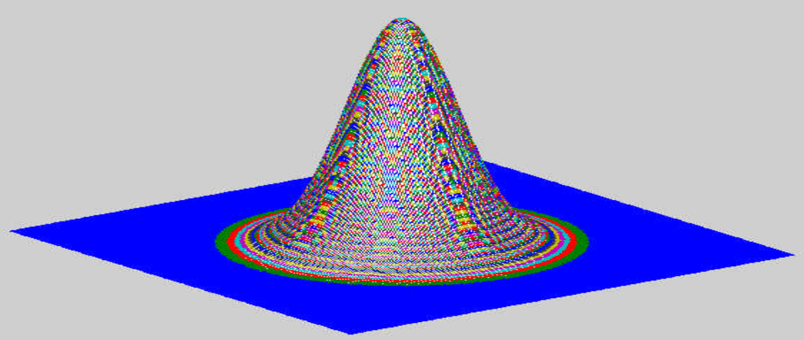




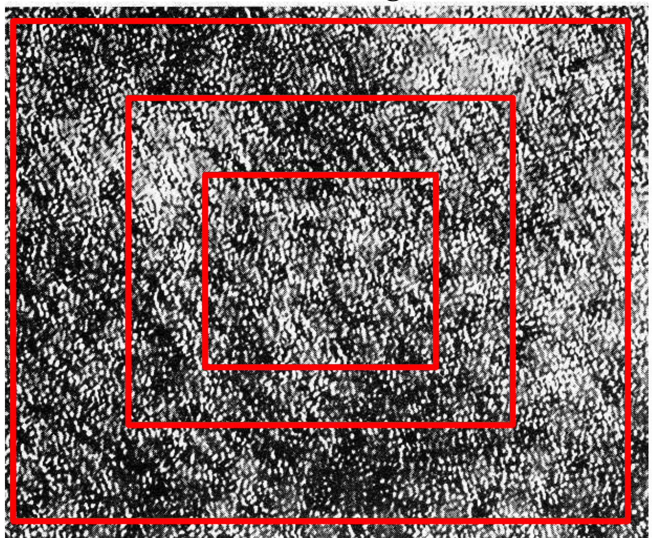
Rough surface

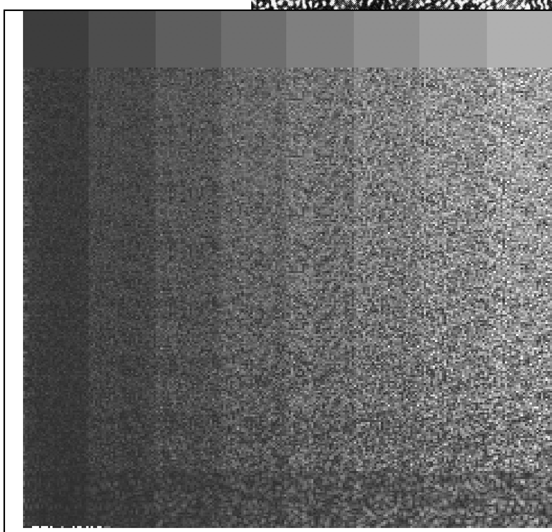


Its directivity function

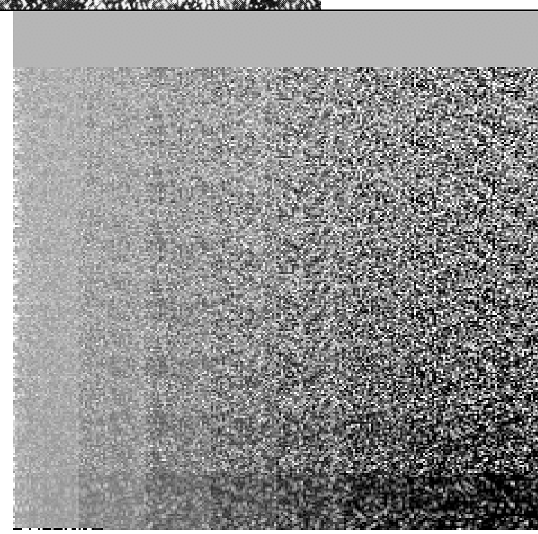


Limitation of hologram size

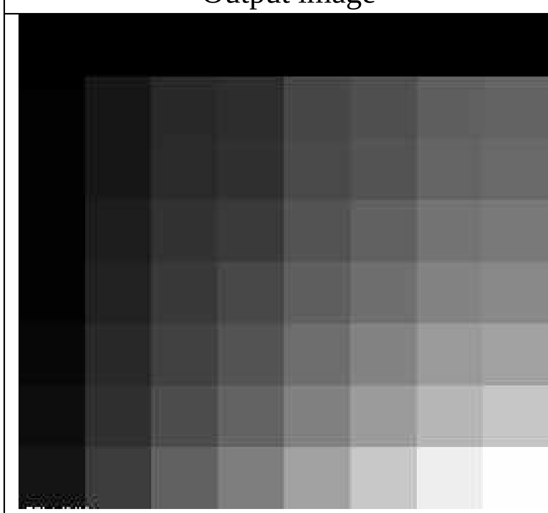




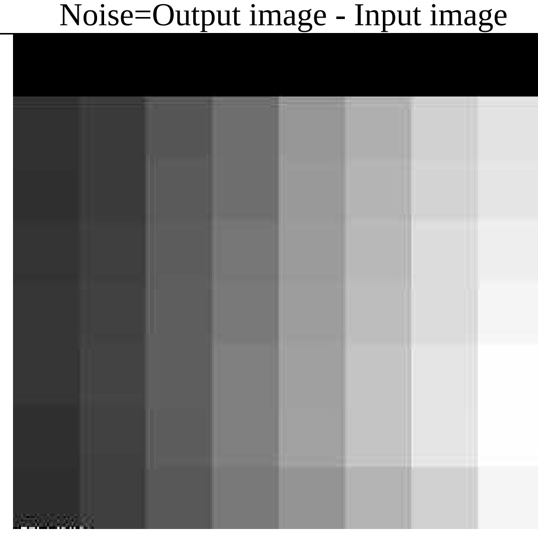
Output image



Noise=Output image - Input image



Abs. Mean value of the noise

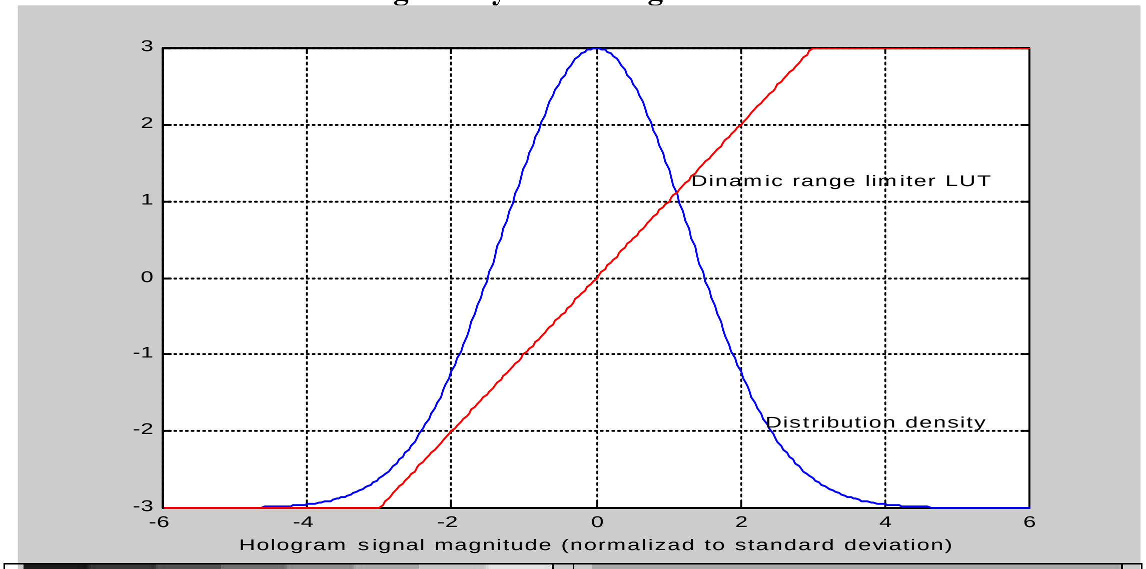


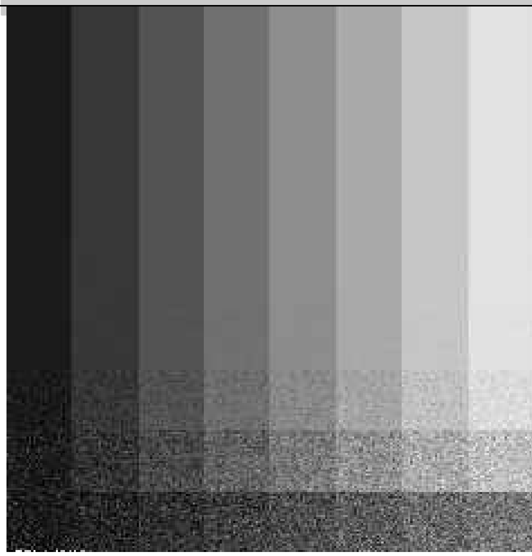
Noise standard deviation

Horizontal axis: image brightness (8 levels)
Vertical axis: fraction of hologram area (top-bottom: 1;8/9;7/8;6/7,5/6;4/5;3/4;2/3;1/2)

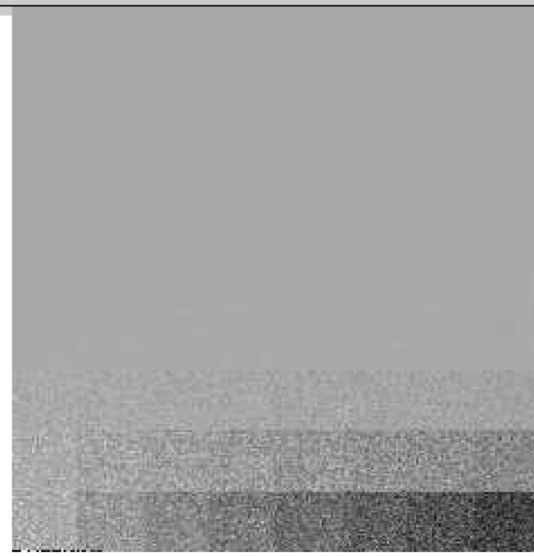
Speckle noise due to the finite resolution of the imaging system (limitation of hologram size)

Hologram dynamic range limitation

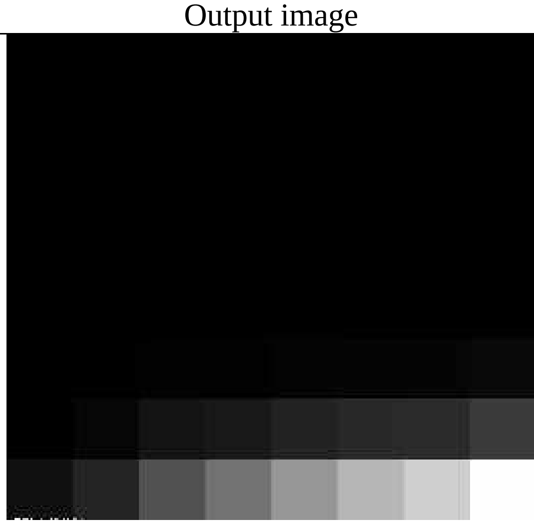




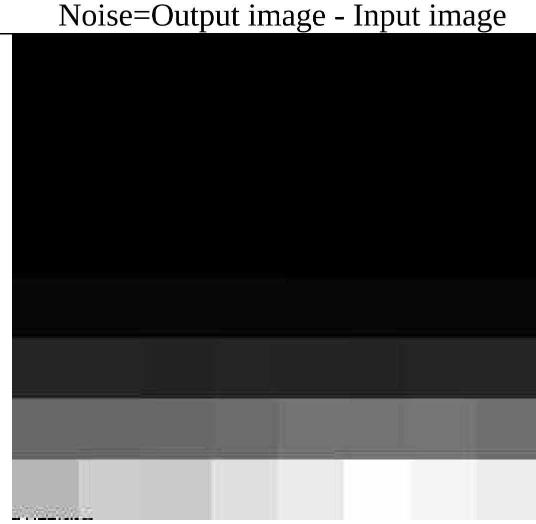
Output image



Noise=Output image - Input image



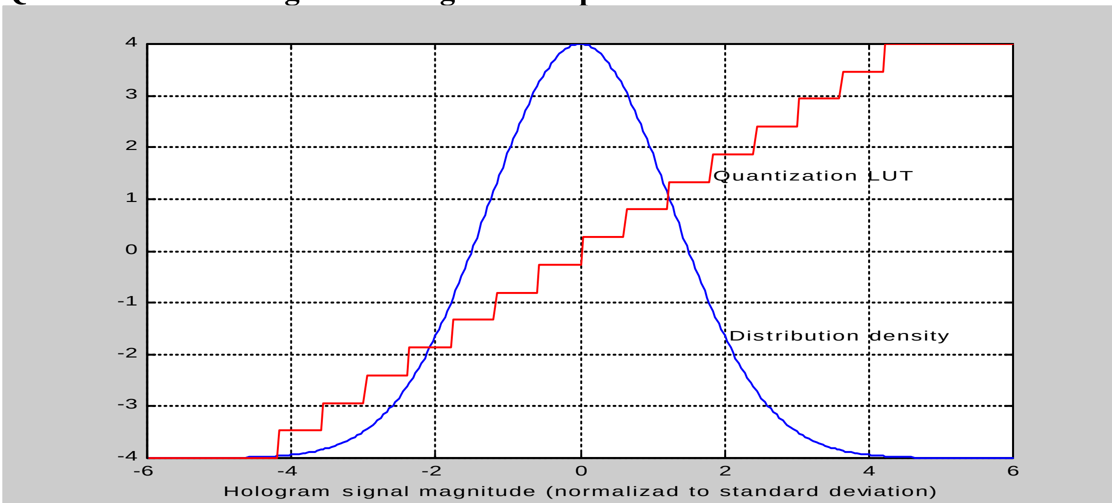
Abs. Mean value of the noise

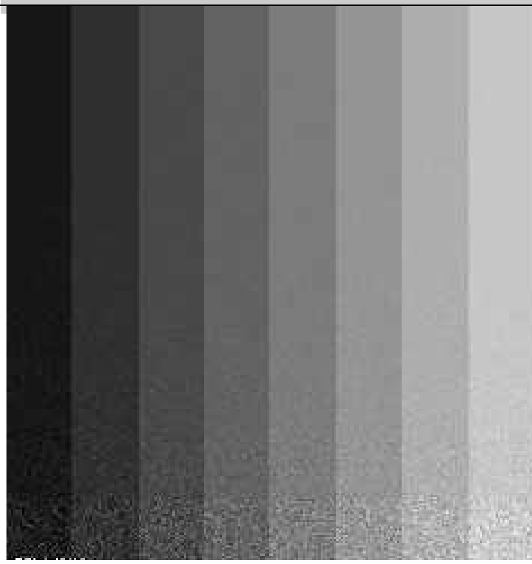


Noise standard deviation

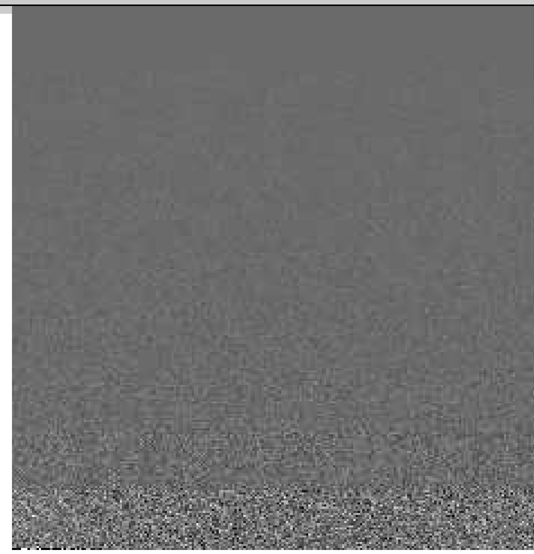
Horizontal axis: image brightness (8 levels)
Dynamic range in units of hologram st.dev (top-bottom: 1;8/9;7/8;6/7,5/6;4/5;3/4;2/3;1/2)

Quantization of hologram orthogonal components





Output image



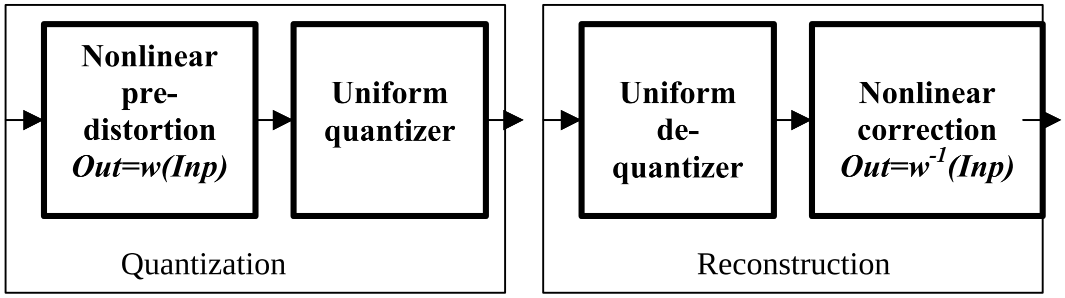
Noise=Output image - Input image

Abs. Mean value of the noise
(25 iter; (Max=0.3)

Noise standard deviation
(25 iter, Max=0.73)

Horizontal axis: image brightness (8 levels)
Number of quantization levels (top-bottom: n/q;129;103;97,81;65;49;33,17)

Nonuniform quantization for minimization of speckle noise caused by hologram quantization



Schematic diagram of compander-expander quantization

<u>Compander-expander quantizer.</u>

Quantization quality criterion:

$$E_I \approx \sum_{q=1}^{Q-2} \int_{a^{(q-1)}}^{a^{(q)}} p(a) D\left(a^{(q-1)} - \hat{a}^{(q)}\right) da \approx \int_{a^{(0)}}^{a^{(Q)}} p(a) D\left(\Delta_u / w'(a)\right) da;$$

Optimal predistortion functions

$$w_{opt}(a) = \arg\min_{w(a)} \left\{ \int_{a^{(0)}}^{a^{(Q)}} p(a) D\left(\Delta_u / w'(a)\right) da; \right\}$$

are found from Euler-LaGrange equation:

$$\frac{\partial}{\partial w'} \left\{ p(a) D\left(\Delta_u / w'(a)\right) \right\} = const$$

<u>Examples:</u> 1. Threshold criterion:

$$D\left(a^{(q)} - a^{(q+1)}\right) = \begin{cases} 0, \left|a^{(q)} - a^{(q+1)}\right| < \Delta_{thr} \\ 1, \left|a^{(q)} - a^{(q+1)}\right| > \Delta_{thr} \end{cases} \Rightarrow \text{ uniform quantization}$$

2. Threshold criterion:

$$D\left(a^{(q)} - a^{(q+1)}\right) = \begin{cases} 0, \left|a^{(q)} - a^{(q+1)}\right| < \Delta_{thr} = d_0 a \\ 1, \left|a^{(q)} - a^{(q+1)}\right| > \Delta_{thr} = d_0 a \end{cases} \Rightarrow$$

$$\frac{w(a) - w(a_{\min})}{w(a_{\max}) - w(a_{\min})} = \frac{\ln(a / a_{\min})}{\ln(a_{\max} / a_{\min})};$$

$$Q = \left(\ln(a_{\max} / a_{\min})\right) / d_0$$

3. $D\left(a^{(r)} - a^{(r+1)}\right) = \left(a^{(r)} - a^{(r+1)}\right)^{2n} \Rightarrow$

$$\frac{w(a) - w(a_{\min})}{w(a_{\max}) - w(a_{\min})} = \frac{\int_{a_{\min}}^{a} \left(p(a)\right)^{1/(2n+1)} da}{\int_{a_{\min}}^{a_{\max}} \left(p(a)\right)^{1/(2n+1)} da}$$

When $n = 1$, $\dfrac{w(a) - w(a_{\min})}{w(a_{\max}) - w(a_{\min})} = \dfrac{\int_{a_{\min}}^{a} \left(p(a)\right)^{1/3} da}{\int_{a_{\min}}^{a_{\max}} \left(p(a)\right)^{1/3} da}$

When $p(a) = c\exp\left\{-\frac{(a-\bar{a})^2}{2\sigma_a^2}\right\}$,

$$\frac{w(a)-w(a_{\min})}{w(a_{\max})-w(a_{\min})}=\frac{\Phi\left(\frac{a-\bar{a}}{\sqrt{3}\sigma_a}\right)-\Phi\left(\frac{a_{\min}-\bar{a}}{\sqrt{3}\sigma_a}\right)}{\Phi\left(\frac{a_{\max}-\bar{a}}{\sqrt{3}\sigma_a}\right)-\Phi\left(\frac{a_{\min}-\bar{a}}{\sqrt{3}\sigma_a}\right)},$$

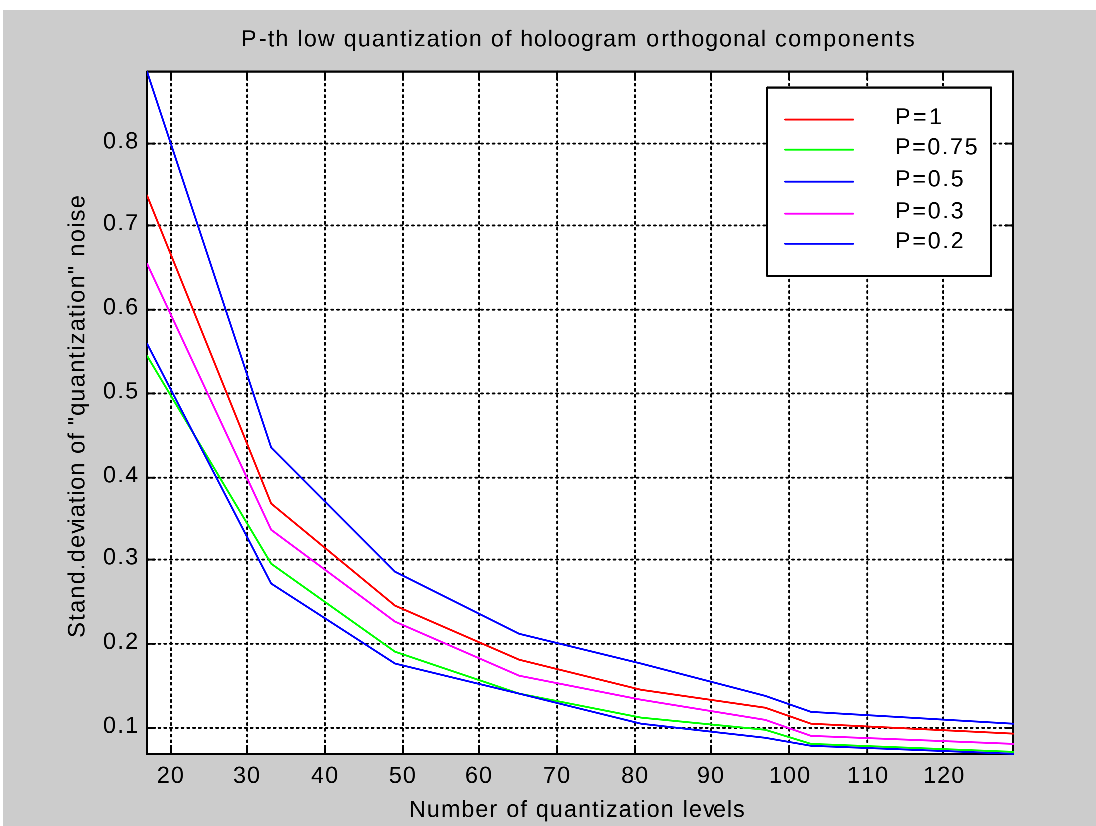
where $\Phi(a)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left(-\frac{x^2}{2}\right)dx$

4. $D\left(a^{(r)}-a^{(r+1)}\right)=\left(\left(a^{(r)}-a^{(r+1)}\right)/a\right)^{2n}\ \Rightarrow$

$$\frac{w(a)-w(a_{\min})}{w(a_{\max})-w(a_{\min})}=\frac{\int_{a_{\min}}^{a}\left(p(a)/a^{2n}\right)^{1/(2n+1)}da}{\int_{a_{\min}}^{a_{\max}}\left(p(a)/a^{2n}\right)^{1/(2n+1)}da}$$

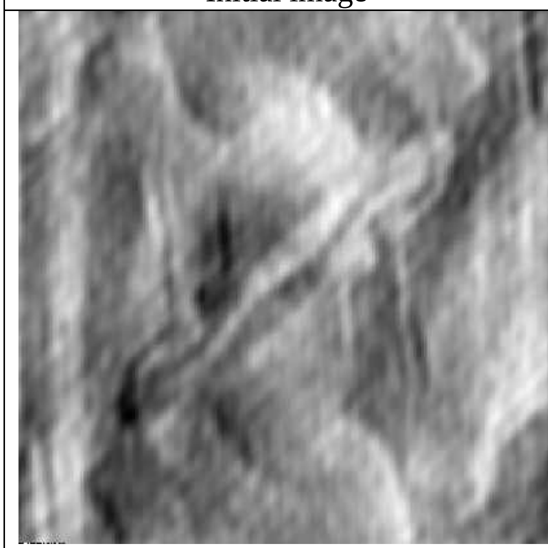
when $p(a)=const$, $\frac{w(a)-w(a_{\min})}{w(a_{\max})-w(a_{\min})}=\frac{a^{1/(2n+1)}-a_{\min}^{1/(2n+1)}}{a_{\max}^{1/(2n+1)}-a_{\min}^{1/(2n+1)}}$

P-th law quantizer: ***w*(*Inp*)=|*Inp*|^{P}*sign*(*Inp*)**



Reducing, by non-linear ***P***-th law quantization, speckle noise caused by quantization of hologram orthogonal components

Quantization in transform domain

 Initial image	 Image uniform quantization, Q=16
 Image restored after uniform quantization (31 levels) of its spectrum orthogonal components	 Image restored after optimal *P*-th law quantization of its spectrum orthogonal components (P=0.2, 31 quantization levels)

Test questions

1. How diffuse or specular scattering objects can be modeled?
2. In what sense speckle is regarded as noise?
3. What reasons cause speckle noise in coherent imaging systems?
4. What is speckle contrast?
5. Why, for uniformly scattering objects, speckle contrast is equal to 1?
6. How correlation function of speckle is connected with imaging system frequency response or point spread function?