Lect. 8. Target location as a parameter estimation task.

Problem formulation: Given signal $s(x,\rho)$ determine ρ. Statistical formulation. MAP estimation

Additive white Gaussian noise (AWGN) discrete model: $\{r_k = s_k(\rho) + n_k\}$

$$p(\rho / r(x)) = \frac{p(r(x)/\rho)\,p(\rho)}{p(r(x))} \propto p_n(n_k = r_k - s_k(\rho))\,p(\rho) =$$

$$= \exp(-\frac{1}{2\sigma_n^2}\sum_{k=0}^{N-1}|r_k - s_k(\rho)|^2 + \ln(p(\rho))) \propto \exp(-\frac{1}{2\sigma_n^2}\sum_{k=0}^{N-1}|s_k(\rho)|^2 + \frac{1}{\sigma_n^2}\sum_{k=0}^{N-1} r_k s_k(\rho) + \ln(p(\rho)))$$

MAP-estimate: $\rho_{MAP}^{opt} = \arg\max_{\rho}\left\{\exp\left[-\frac{1}{2\sigma_n^2}\sum_{k=0}^{N-1}|s_k(\rho)|^2 + \frac{1}{\sigma_n^2}\sum_{k=0}^{N-1} r_k s_k(\rho) + \ln(p(\rho))\right]\right\}$

ML-estimate: $\rho_{ML}^{opt} = \arg\max_{\rho}\left\{\exp\left[-\frac{1}{2\sigma_n^2}\sum_{k=0}^{N-1}|s_k(\rho)|^2 + \frac{1}{\sigma_n^2}\sum_{k=0}^{N-1} r_k s_k(\rho)\right]\right\}$

Multi component images: a model with statistically independent noise components

$$r_k^{(m)} = s_k^{(m)}(\rho) + n_k^{(m)};\quad m = 1,\dots,M$$

MAP-estimate: $\rho_{MAP}^{opt} = \arg\max_{\rho}\left\{\exp\left[-\sum_{m=1}^{M}\frac{1}{2\sigma_m^2}\sum_{k=0}^{N-1}|s_k^{(m)}(\rho)|^2 + \sum_{m=1}^{M}\frac{1}{\sigma_m^2}\sum_{k=0}^{N-1} r_n^{(m)} s_k^{(m)}(\rho) + \ln(p(\rho))\right]\right\}$

Estimation of signal position (target location). AWGN-model:

$$(\hat{x}_0, \hat{y}_0) = \arg\max_{(x_0,y_0)}\left\{\frac{1}{\sigma_n^2}\sum_{k=0}^{N-1} r_k s_k(x_0,y_0) + \ln(p(x_0,y_0))\right\}$$

$$(\hat{x}_0, \hat{y}_0) = \arg\max_{(x_0,y_0)}\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} r(x,y)\,s(x-x_0, y-y_0) + N_0 \ln(p(x_0,y_0))\right\}$$

Correlation as matched filtering in frequency domain: $H_{opt}(f_x, f_y) = \alpha^{*}(f_x, f_y)$.

Non white noise model: arbitrary noise spectral density $\mathrm{N}_n(f_x, f_y)$

Signal whitening (decorrelation) $H_w(f) = 1/(\mathrm{N}_n(f))^{1/2}$

"Optimal filter" $H_{opt}(f_x, f_y) = \alpha^{*}(f_x, f_y)/N_n(f_x, f_y)$

Optimality of the filter:

- computing a posteriori parameter probability distribution for AGN model
- providing maximum to (signal peak)-to-(noise standard deviation) ratio at the filter output

$$H_{opt}(\vec{f}) = \arg\max(PSNR) = \arg\max_{H}\left(\frac{\int \alpha(\vec{f})H(\vec{f})d\vec{f}}{\left(\int \mathrm{N}_n(\vec{f})|H(\vec{f})|^2 d\vec{f}\right)^{1/2}}\right) = \frac{\alpha^{*}(\vec{f})}{\mathrm{N}_n(\vec{f})},$$

which follows from Swartz inequality

$$\int \alpha(\vec{f})H(\vec{f})d\vec{f} = \int \frac{\alpha(\vec{f})}{|\mathrm{N}_n(\vec{f})|^{1/2}}\left[|\mathrm{N}_n(f)|^{1/2} H(\vec{f})\right]d\vec{f} \le \left[\int \frac{\alpha(\vec{f})}{\mathrm{N}_n(f)}d\vec{f}\right]^{1/2}\left[\int \mathrm{N}_n(\vec{f})|H(\vec{f})|^2 d\vec{f}\right]^{1/2}$$

Multi component model with correlated noise: component-wise decorrelation (whitening)

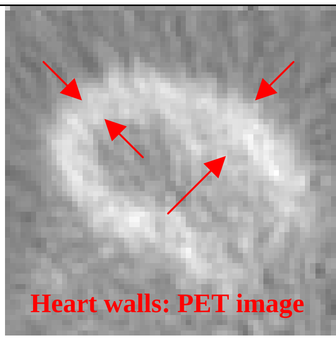
Application examples.

- Image recognition and similarity measures.
- Recognition of objects known to the accuracy of some unknown parameters (scale, rotation, etc.).
- Object tracking in image sequences. Fitting geometrical models to image data.
- Image segmentation as parameter estimation

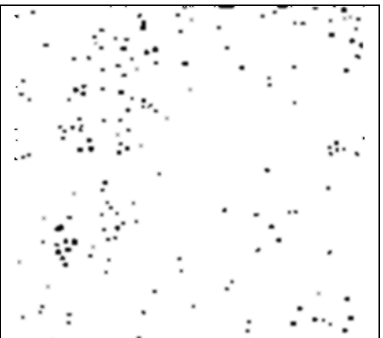
Imaging as parameter estimation

Parameter estimation as an image processing task: Examples

Celestial navigation

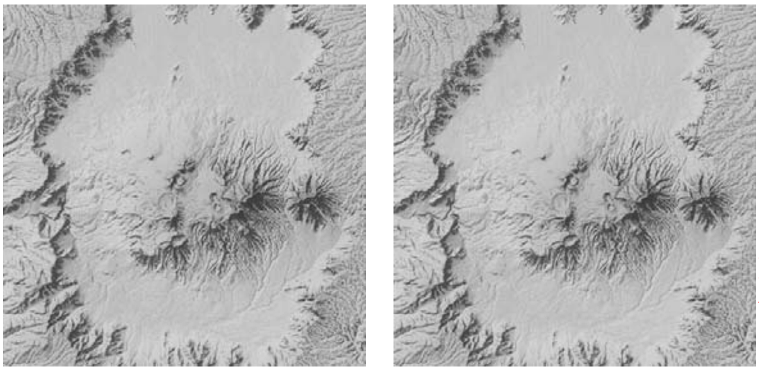


Measuring object dimensions and shape



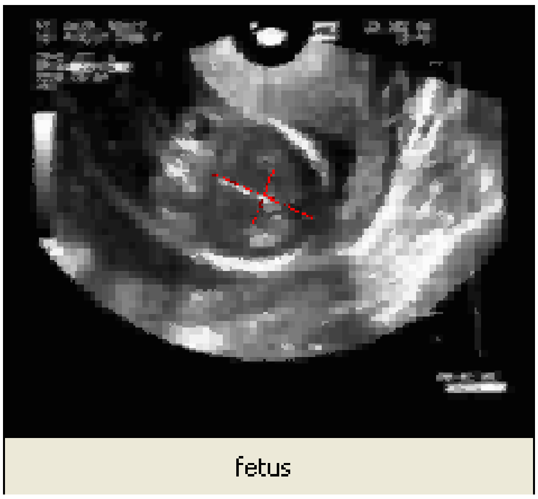
Counting and measuring particles

Face detection and recognition

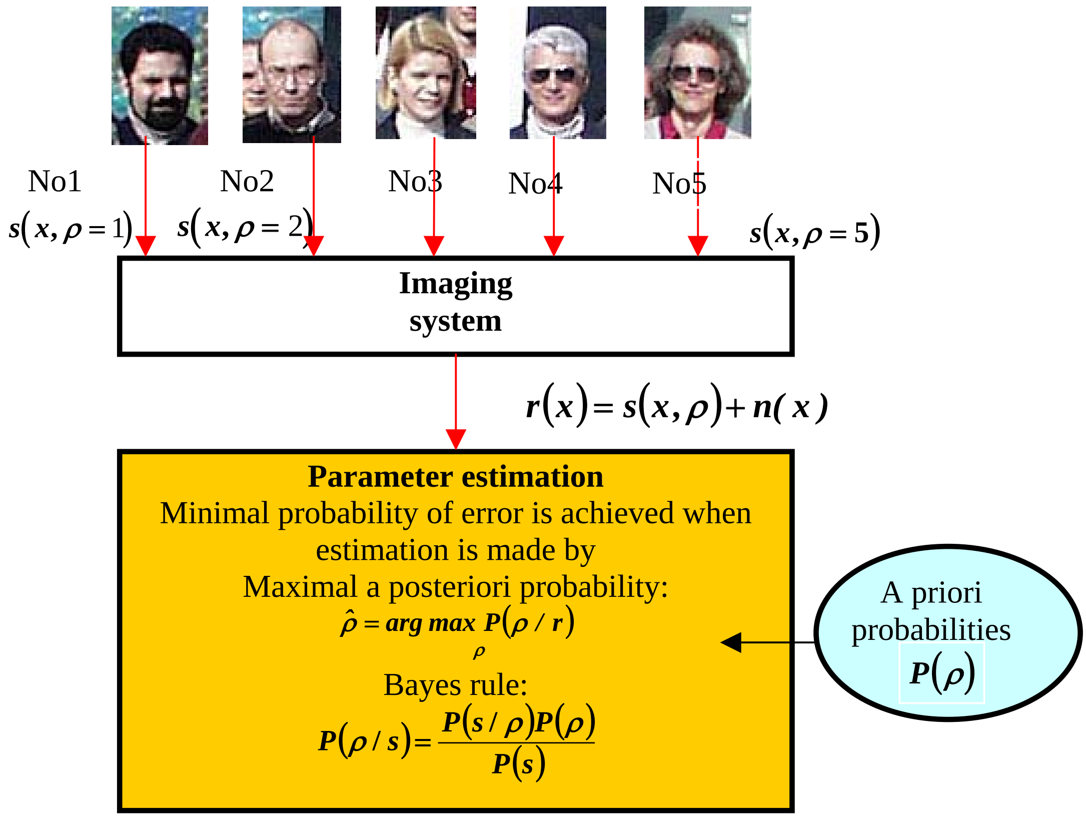


Depth from stereo

Object detection



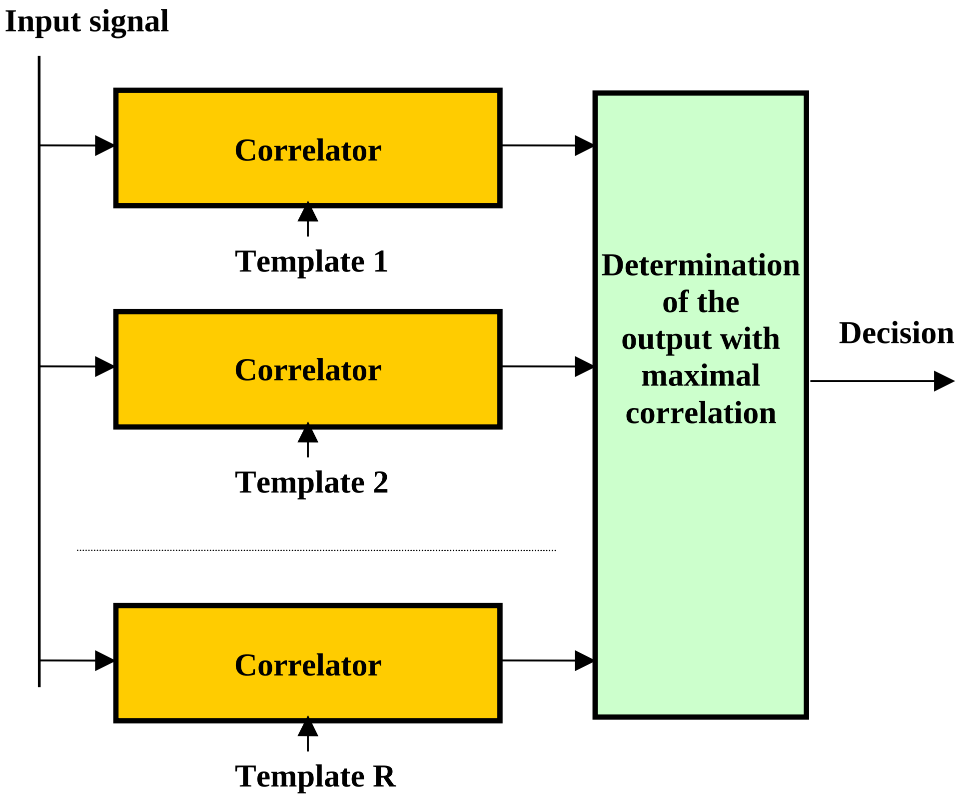
Object tracking



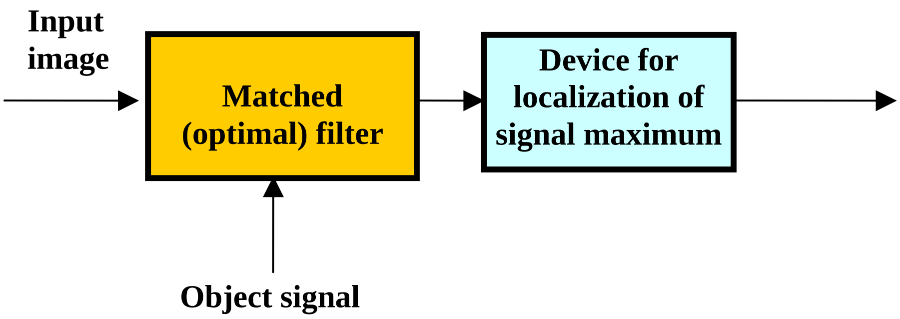
Statistical formulation of the image parameter estimation problem

Bayes' rule is actually a theorem, due to the Reverend Thomas Bayes of England, 1702-1761. This discovery was published posthumously in 1763. Supposedly Bayes did not publish his discovery because he thought it was hubris for humans to investigate the will of God. (see http://www.cs.ucsd.edu/users/elkan/250A/mar12.html)

Bayes, a noncomformist minister, developed his theory as a formal means of arguing for the existence of God. The role of symptoms in this argument is taken by the occurrence of miracles and other manifestations of God's good works, and the two hypotheses are affirmation and denial of God. (P. Szolovits, Uncertainty and decisions in Medical Informatics, Methods of Information in Medicine, 34:11-121, 1995)



Optimal “recognition” device



Optimal localization device

OPTIMAL LOCALIZATION IN COLOR (MULTI COMPONENT) IMAGES

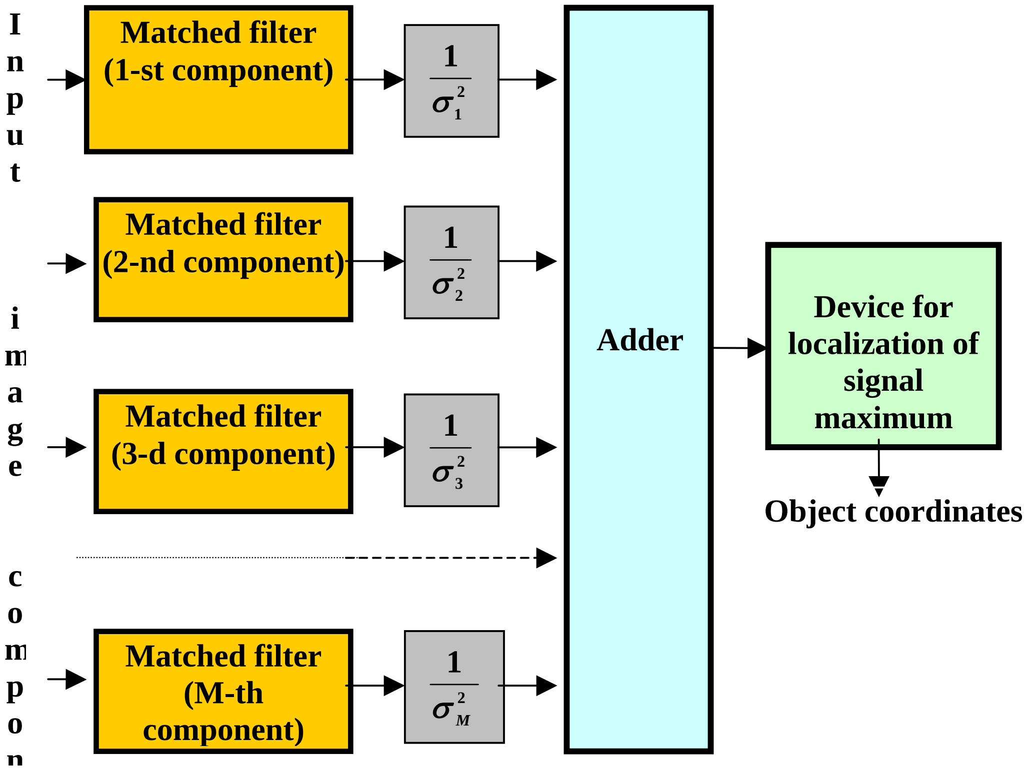
Signal model:

$$b_k^{(m)} = a_k^{(m)}(x_0, y_0) + n_k^{(m)}, \quad m = 1,2,\ldots,M;\ k = 0,1,\ldots,N-1$$

The optimal MAP- and ML-estimations:

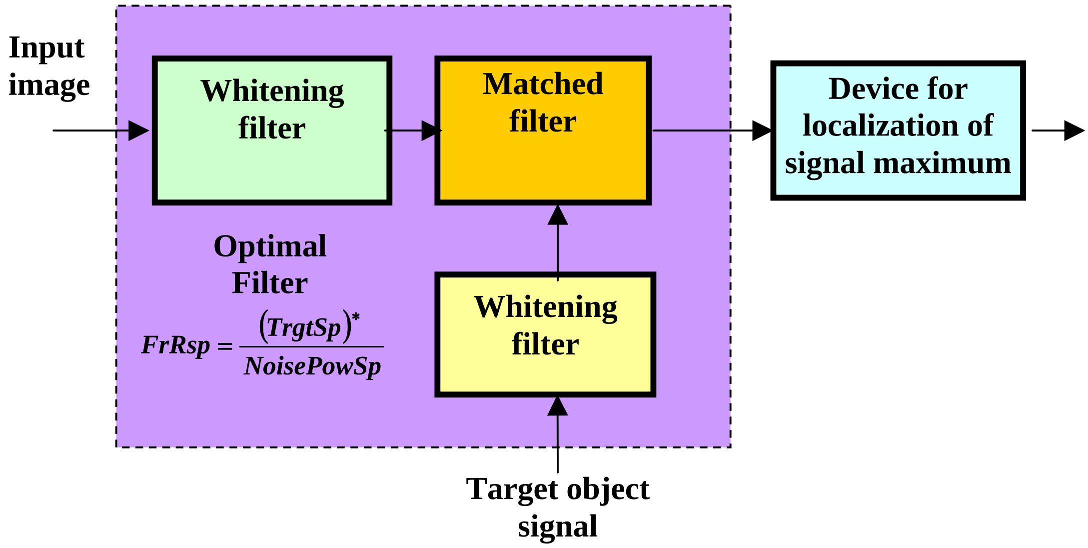
$$\{(\hat{x}_0, \hat{y}_0)\} = \underset{(x_0, y_0)}{\arg\max} \left\{ \sum_{m=1}^{M} \sum_{k=0}^{N-1} b_k^{(m)} a_k^{(m)}(x_0, y_0) - \sum_{m=1}^{M} \sigma_m^2 \ln P(x_0, y_0) \right\}$$

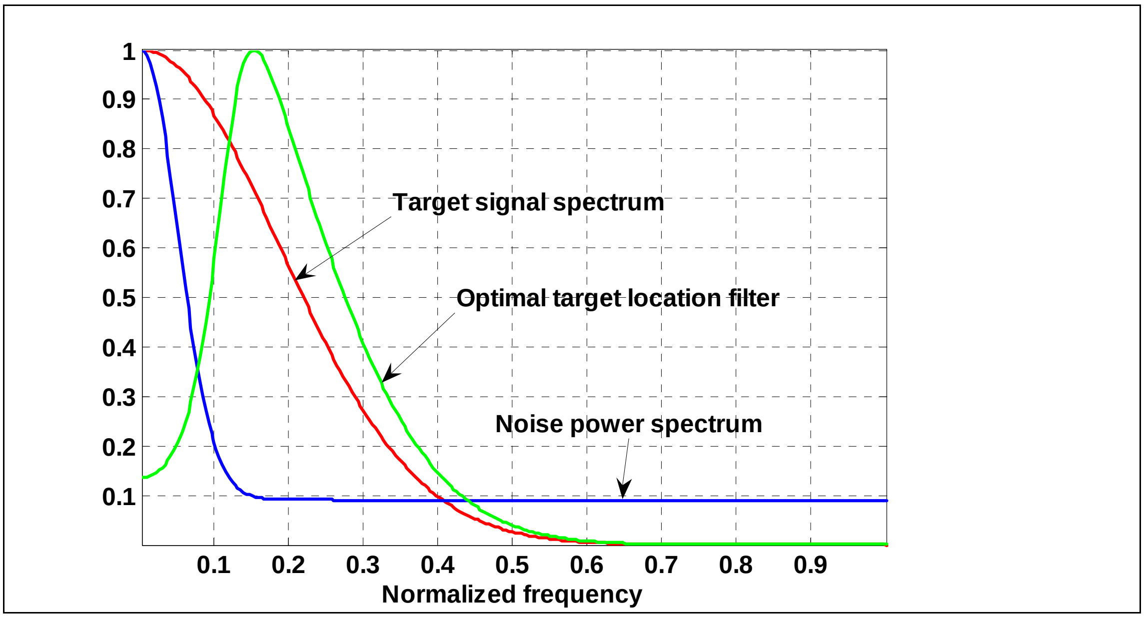
$$\{(\hat{x}_0, \hat{y}_0)\} = \underset{(x_0, y_0)}{\arg\max} \left\{ \sum_{m=1}^{M} \frac{1}{\sigma_m^2} \sum_{k=0}^{N-1} b_k^{(m)} a_k^{(m)}(x_0, y_0) \right\}$$



Schematic-diagram of the optimal device for localizing objects in multicomponent images with additive white Gaussian noise in each channel with no inter-channel correlations

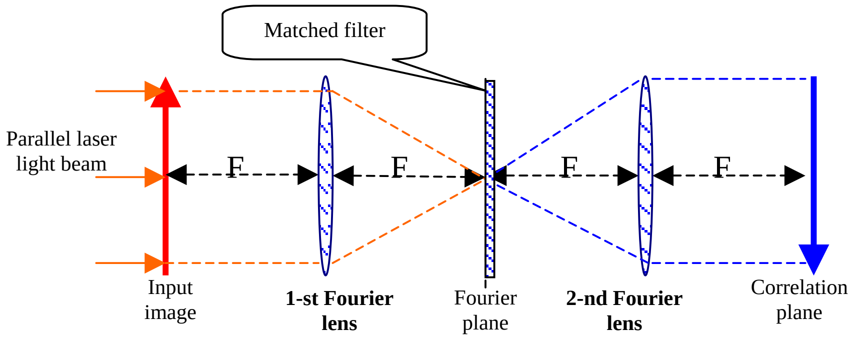
“OPTIMAL” FILTER FOR NON “WHITE” NOISE



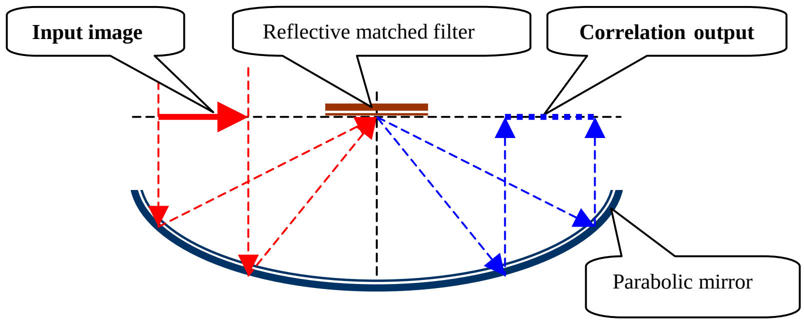


An example of frequency response of the optimal target location filter

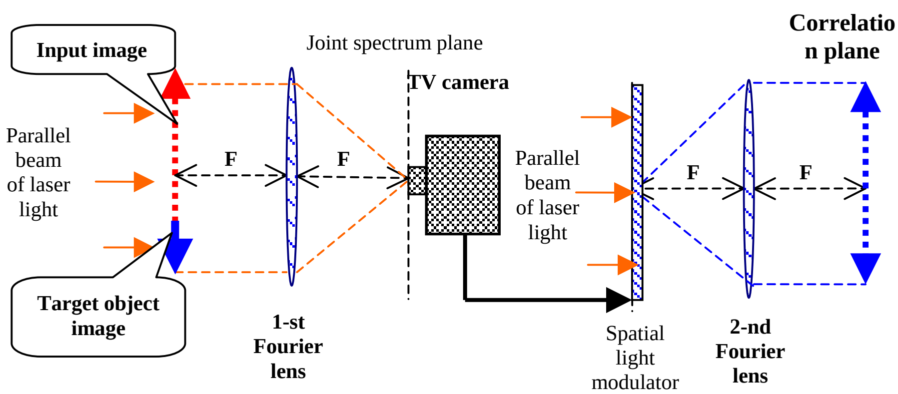
Electro-optical correlators



4-F optical correlator



Parabolic mirror optical correlators



Joint Transform Electro-Optical Correlator