L. Yaroslavsky. Fundamentals of Digital Image Processing. Course 0555.3230

Lecture 9 Statistical Image and Noise Models

Statistical versus deterministic treatment of signals. Signals are treated as being deterministic if they are treated individually. Signals are treated as being "random" when they are treated as representatives of a certain set, or ensemble, of signals. In this case, only certain "statistical characteristics" of the ensemble are considered and derived from the signals rather then individual characteristics of individual signals.

Basic statistical characteristics of random signals.

Probability distribution P_a =probability that $a \le A$

Probability density $p(a = A) = \left.\frac{dP_a}{da}\right|_{a=A}$

Probability density moments: $m_a^n = \int_{\Omega_a} a^n p(a)da$.

Distribution histogram of digital signals $\{a_k\}$: $h_a(m) = \frac{1}{N}\sum_{k=0}^{N-1}\delta(m - a_k)$

Mean value $m_a = \bar{a} = \int_{\Omega_a} ap(a)da$; Variance $\overline{a^2} = \int_{\Omega_a} (a - \bar{a})^2 p(a)da$;

Standard deviation $\sigma_a = \sqrt{\overline{a^2}}$.

For digital signals, similar histogram moments are considered.

For digital signals $\{a_k\}$, order statistics are introduced as elements of the variational row:

$\{a[n]\}: a[n] \le a[n+1]; n = 0,..., N-1$

Specific order statistics: Minimum $a[0]$; Maximum $a[N-1]$, Median $a\left[\frac{N-1}{2}\right]$ (N -odd number)

Robust statistical estimates

of data bias: Median $a\left[\frac{N-1}{2}\right]$; Trimmed mean: $\overline{a^{tr}} = \frac{1}{n_2 - n_1 + 1}\sum_{n_1>0}^{n_2<N} a[n]$;

of data spread: $QSPR_q = a\left[\frac{N-1}{2} + q\right] - a\left[\frac{N-1}{2} - q\right]$

Signal autocorrelation functions as statistical characterization of signal behavior in its coordinates (time or space):

$$R_a(x_1, x_2) = \frac{1}{X}\int_X a(x + x_1)a(x + x_2)dx ;$$

If $R_a(x_1, x_2) = R_a(x_1 - x_2)$, signals are called "stationary" (spatially homogeneous) signals.

For "stationary" signals, their power spectra are considered as Fourier Transform of the correlation function:

$$PS_a = \int R_a(x)\exp(i2\pi fx)dx$$

Local statistical characteristics of signals: local histograms, local mean, local standard deviation, local minimum, maximum, median and other order statistics, local spectra

Measuring statistical characteristics of signals:

Measuring distribution histograms and moments.

Evaluating autocorrelation functions and spectra by DFT. Boundary effects.

Statistical models of image random interferences

For signal s , statistical models of random interferences:

(i) additive noise: $r = s + n$; where random noise n has zero mean,

(ii) multiplicative noise $r = ns$;

(iii) composite noise: $r = n_m s + n_a$;

(iiii) impulse noise: $r = (1-e)s + en;\ .\ e = \begin{cases} 1, \text{with a certain probabity } P_e \\ 0, \text{otherwise} \end{cases}$

Typical examples of image noise: “white” sensor’s noise; narrow-band (“moire”) noise; speckle noise; quantization noise; image compression noise. Visibility of image noise and noise statistical parameters.

Questions for self-testing

1. Explain differerence between deterministic and statistical treatments of signals.
2. Describe basic statistical characteristics of signals and their meaning
3. How image histograms, histogram moments, image correlation functions and power spectra are computed?
4. What are image local statistical parameters?
5. Describe basic statistical models of image random interferences
6. Describe how visibility of noise interferences depend on noise statistical model and noise statistical parameters

Homework: Using Matlab functions, demonstrate computing image local mean value and standard deviation in window of different size. Explain obtained results

Examples of random interferences in imaging systems

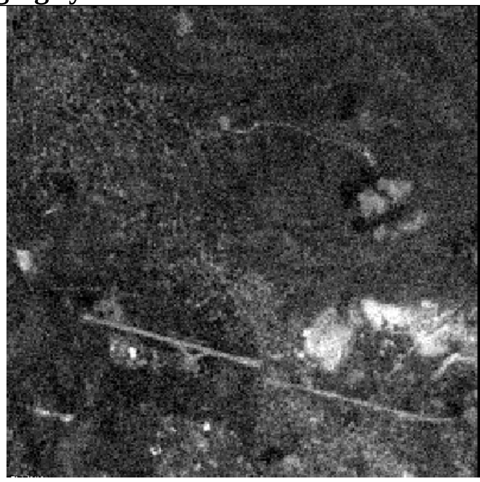
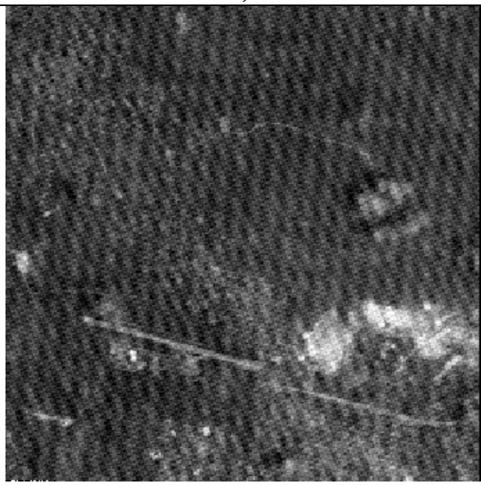
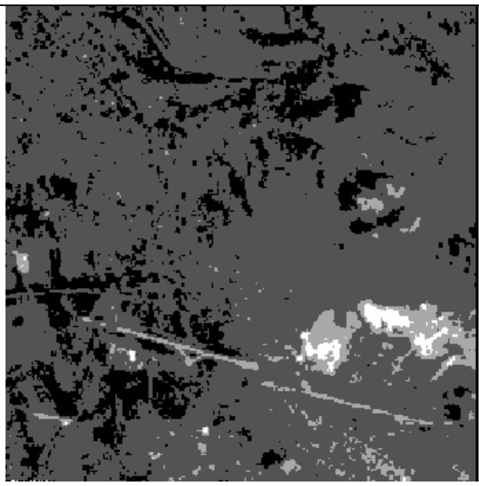
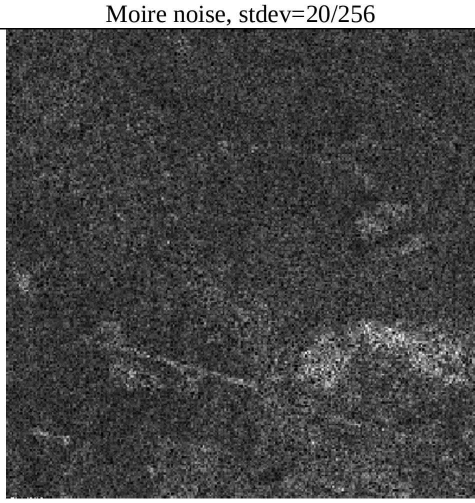
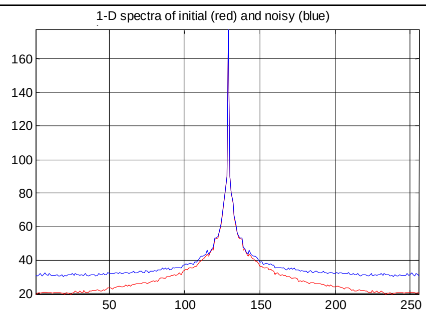
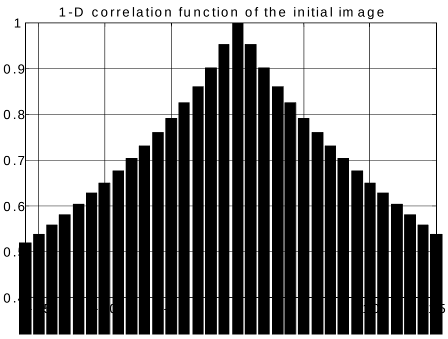
 Noise free image	 Additive noise, stdev=20/256
 Impulse noise, Pe=0.06, stdev=20/256	 Moire noise, stdev=20/256
 Quantization noise, Q=4, stdev=21/256	 "Speckle" noise (local mean value and standard deviation of noise are equal to signal values)

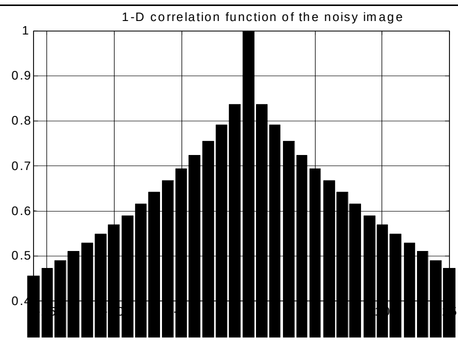
Image noise models and noise statistical characteristics

Diagnostics of additive Gaussian noise

Initial and additively noised image (STDnoise=20)



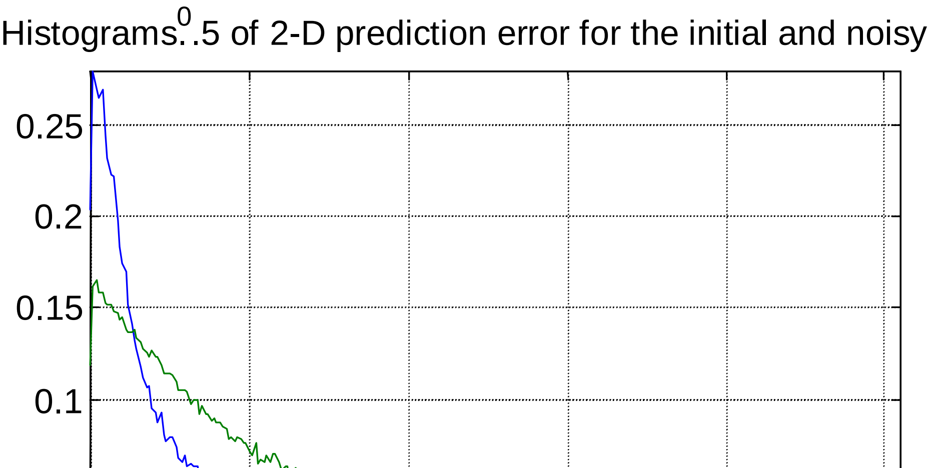




Impulse noise

Noisy image, Per=0.30066

Histograms of 2-D prediction error for the initial and noisy imag



Narrow-band (Moiré) noise