L. Yaroslavsky. *Course* 0510.7211 "Digital Image Processing: Applications"

Lecture 9. Image quantification. Object localization and image registration.

Problem formulation: Given signal $s(x,\rho)$ determine ρ. Statistical formulation. MAP estimation

Additive white Gaussian noise (AWGN) model: $r(x)=s(x,\rho)+n(x)$

$$p(\rho/r(x))=\frac{p(r(x)/\rho)p(\rho)}{p(r(x))}\propto p_n(n(x)=r(x)-s(x,\rho))p(\rho)=$$

$$=\exp\left(-\frac{1}{2\sigma_n^2}\sum_{k=0}|r_k-s_k(\rho)|^2+\ln(p(\rho))\right)\propto\exp\left(-\frac{1}{2\sigma_n^2}\sum_{k=0}^{N-1}|s_k(\rho)|^2+\frac{1}{\sigma_n^2}\sum_{k=0}^{N-1}r_k s_k(\rho)+\ln(p(\rho))\right)$$

MAP-estimate: $\rho_{MAP}^{opt}=\arg\max_{\rho}\left\{\exp\left[-\frac{1}{2\sigma_n^2}\sum_{k=0}^{N-1}|s_k(\rho)|^2+\frac{1}{\sigma_n^2}\sum_{k=0}^{N-1}r_k s_k(\rho)+\ln(p(\rho))\right]\right\}$

ML-estimate: $\rho_{ML}^{opt}=\arg\max_{\rho}\left\{\exp\left[-\frac{1}{2\sigma_n^2}\sum_{k=0}^{N-1}|s_k(\rho)|^2+\frac{1}{\sigma_n^2}\sum_{k=0}^{N-1}r_k s_k(\rho)\right]\right\}$

Multi component images: $r_m(x)=s_m(x,\rho)+n_m(x)$; $m=1,...,M$

MAP-estimate: $\rho_{MAP}^{opt}=\arg\max_{\rho}\left\{\exp\left[-\sum_{m=1}^{M}\frac{1}{2\sigma_m^2}\sum_{k=0}^{N-1}|s_k^{(m)}(\rho)|^2+\sum_{m=1}^{M}\frac{1}{\sigma_m^2}\sum_{k=0}^{N-1}r_n^{(m)}s_k^{(m)}(\rho)+\ln(p(\rho))\right]\right\}$

Estimation of signal position (target location). AWGN-model:

$$(\hat{x}_0,\hat{y}_0)=\arg\max_{(x_0,y_0)}\left\{\frac{1}{\sigma_n^2}\sum_{k=0}^{N-1}r_k s_k(x_0,y_0)+\ln(p(x_0,y_0))\right\}$$

$$(\hat{x}_0,\hat{y}_0)=\arg\max_{(x_0,y_0)}\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}r(x,y)s(x-x_0,y-y_0)+N_0\ln(p(x_0,y_0))\right\}$$

Correlation (Matched filtering in frequency domain): $H_{opt}(f_x,f_y)=\alpha^{\bullet}(f_x,f_y)$.

Non white noise: Signal whitening. Optimal filter: $H_{opt}(f_x,f_y)=\alpha^{\bullet}(f_x,f_y)/N_n(f_x,f_y)$;

Optimality of the filter: computing a posteriori parameter probability distribution for AGN model and providing maximum to (signal peak)-to-(noise standard deviation) ratio.

Accuracy and reliability of parameter estimation.

Normal and anomalous localization errors. Variance of normal error and probability of anomalous errors. Trade-off between estimation accuracy and reliability.

Target location for multiple foreign nonoverlapping object model: Reliability of localization (recognition) in the presence of white Gaussian noise

Application examples. Image recognition and similarity measures. Recognition of objects known to the accuracy of some unknown parameters (scale, rotation, etc.). Trade off between the recognition invariance to unknown parameters and reliability of recognition. Correlational accumulation as a method for signal restoration and superresolution. Optimality of correlational accumulation and iterative correlational accumulation. Signal blur in correlational accumulation. Two noise components in the accumulated signal: normal and abnormal. Signal restoration limits of correlational accumulation. Correlational accumulation and superresolution

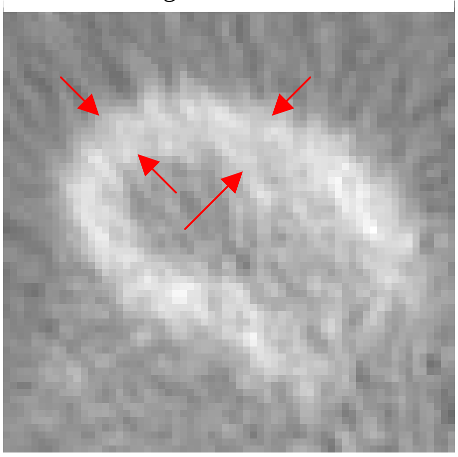
Image segmentation as parameter estimation. Object tracking in image sequences. Fitting 3-D geometrical models to volumetric data.

Problems for self-testing:

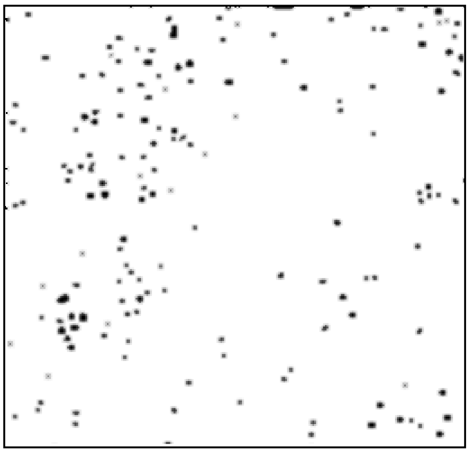
1. Outline statistical approach to image parameter estimation. Derive MAP estimation for the model of additive observation noise. Substantiate correlation and matched filtering methods for shift parameter estimation
2. Describe two types of errors in parameter estimation. Derive formulas for the variance of normal errors and probability of anomalous errors. Explain the threshold in the localization reliability.
3. Derive optimal filter for target location in non white noise. Prove that the optimal filter provides maximum to the ratio of signal peak to noise standard deviation.
4. Outline optimal strategy for target location in clutter of non-overlapping foreign objects

Parameter estimation as an image processing task: Examples

Measuring heart wall thickness

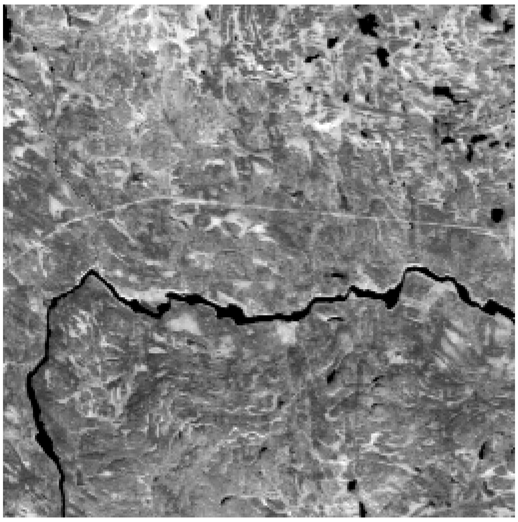


Counting particles and measuring sizes

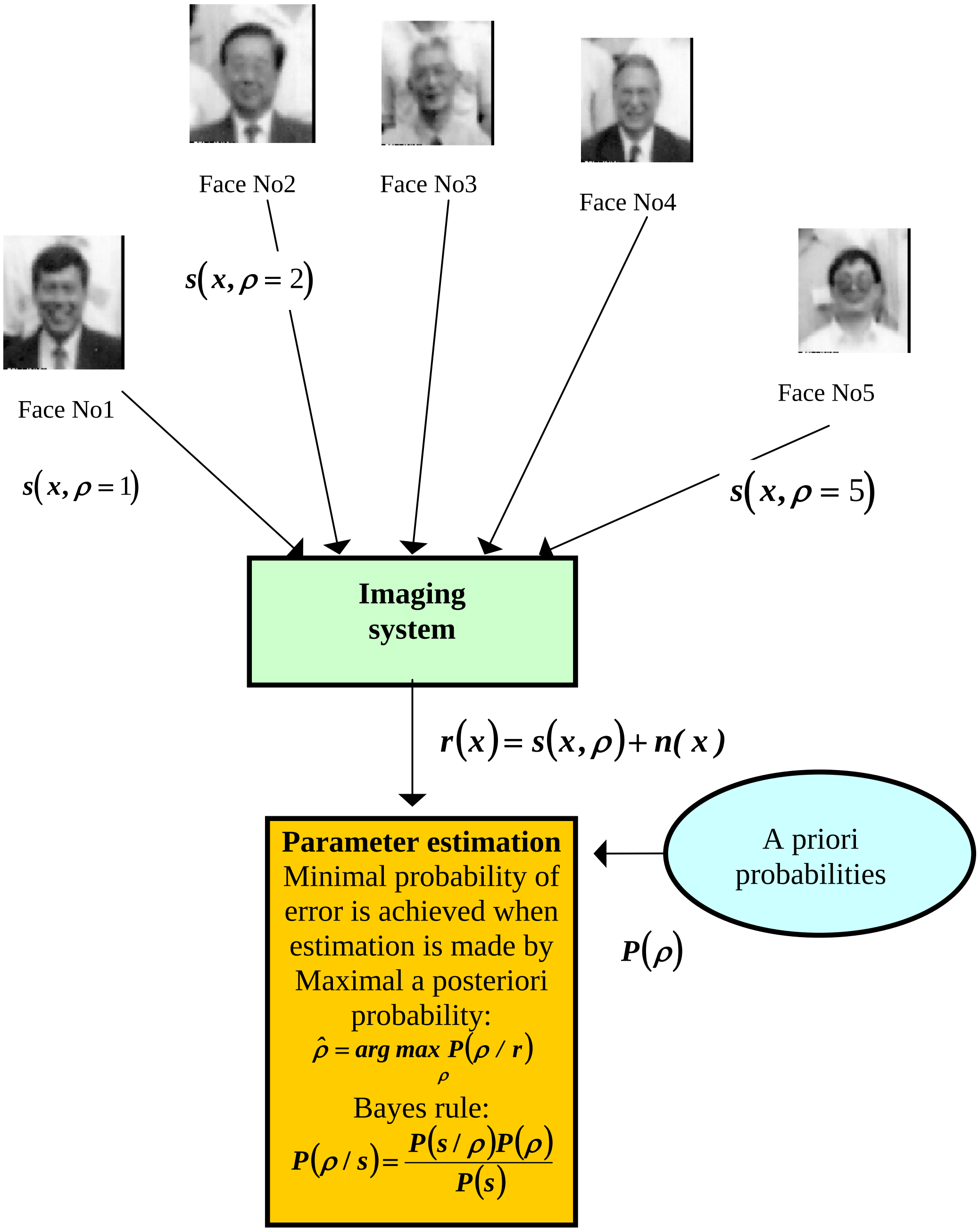


Face detection and recognition

Object detection



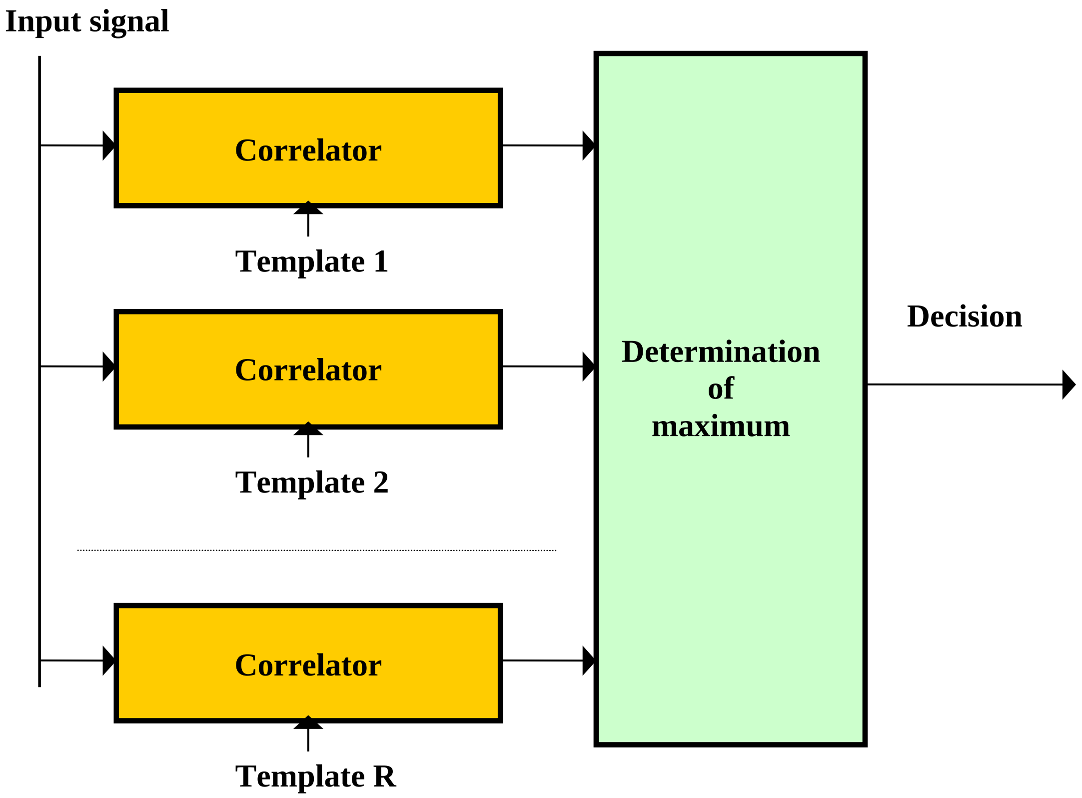
Navigation



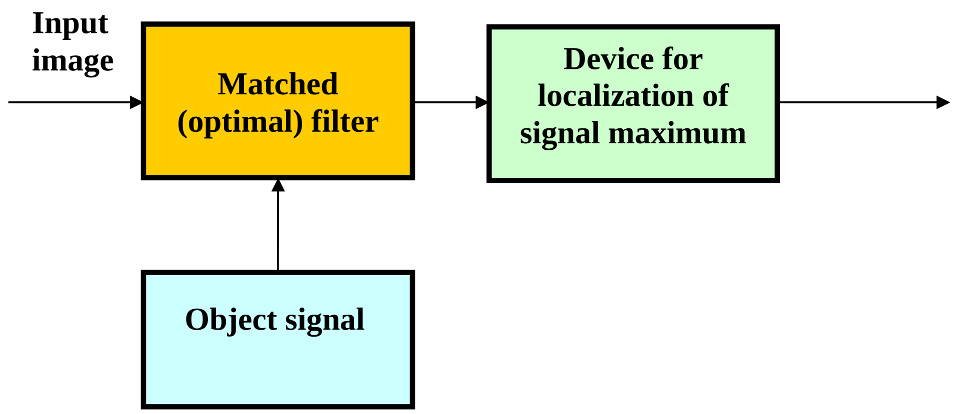
Statistical formulation of the image parameter estimation problem

Bayes' rule is actually a theorem, due to the Reverend Thomas Bayes of England, 1702-1761. This discovery was published posthumously in 1763. Supposedly Bayes did not publish his discovery because he thought it was hubris for humans to investigate the will of God. (see http://www.cs.ucsd.edu/users/elkan/250A/mar12.html)

Bayes, a noncomformist minister, developed his theory as a formal means of arguing for the existence of God. The role of symptoms in this argument is taken by the occurrence of miracles and other manifestations of God's good works, and the two hypotheses are affirmation and denial of God. (P. Szolovits, Uncertainty and decisions in Medical Informatics, Methods of Information in Medicine, 34:11-121, 1995)



Optimal “recognition” device



Optimal localization device

OPTIMAL LOCALIZATION IN COLOR (MULTI COMPONENT) PICTURES

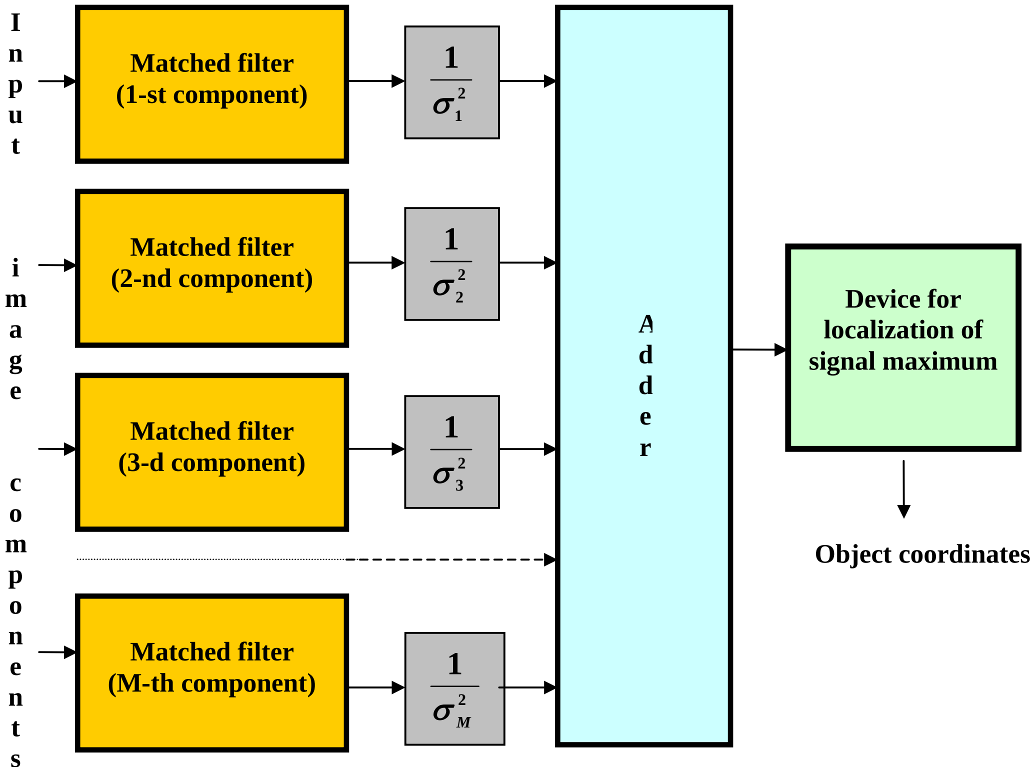
Signal model:

$$b_{k,m} = a_{k,m}(x_0, y_0) + n_{k,m}\,,\;\; m = 1,2,\ldots,M\,;\; k = 0,1,\ldots,N-1$$

The optimal MAP- and ML-estimations:

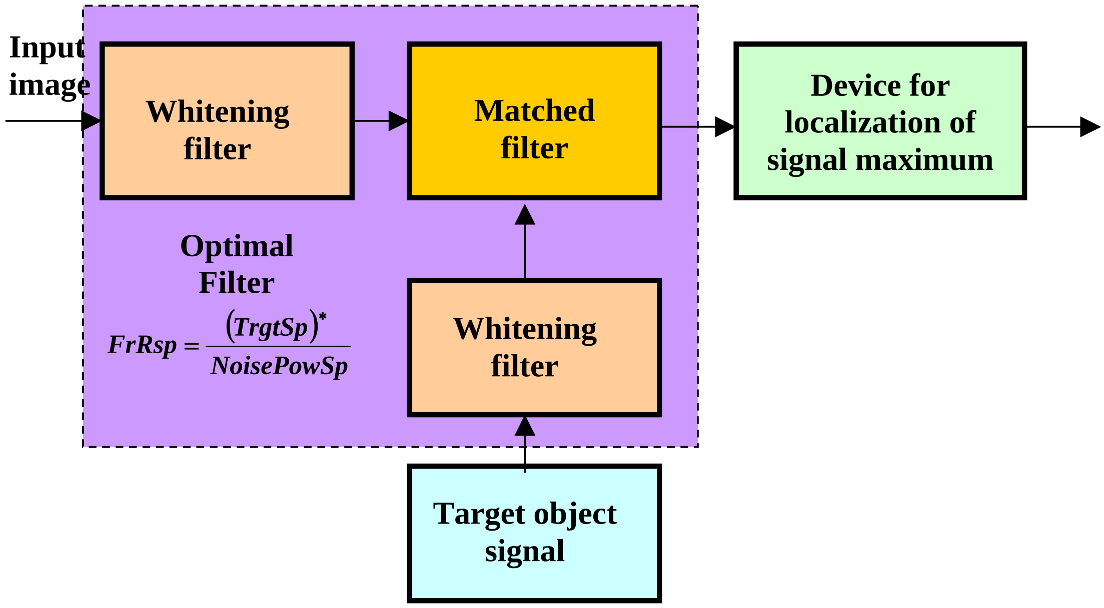
$$\{(\hat{x}_0, \hat{y}_0)\} = \underset{(x_0, y_0)}{\arg\max} \left\{ \sum_{m=1}^{M} \sum_{k=0}^{N-1} b_{k,m} a_{k,m}(x_0, y_0) - \sum_{m=1}^{M} \sigma_m^2 \ln P(x_0, y_0) \right\}$$

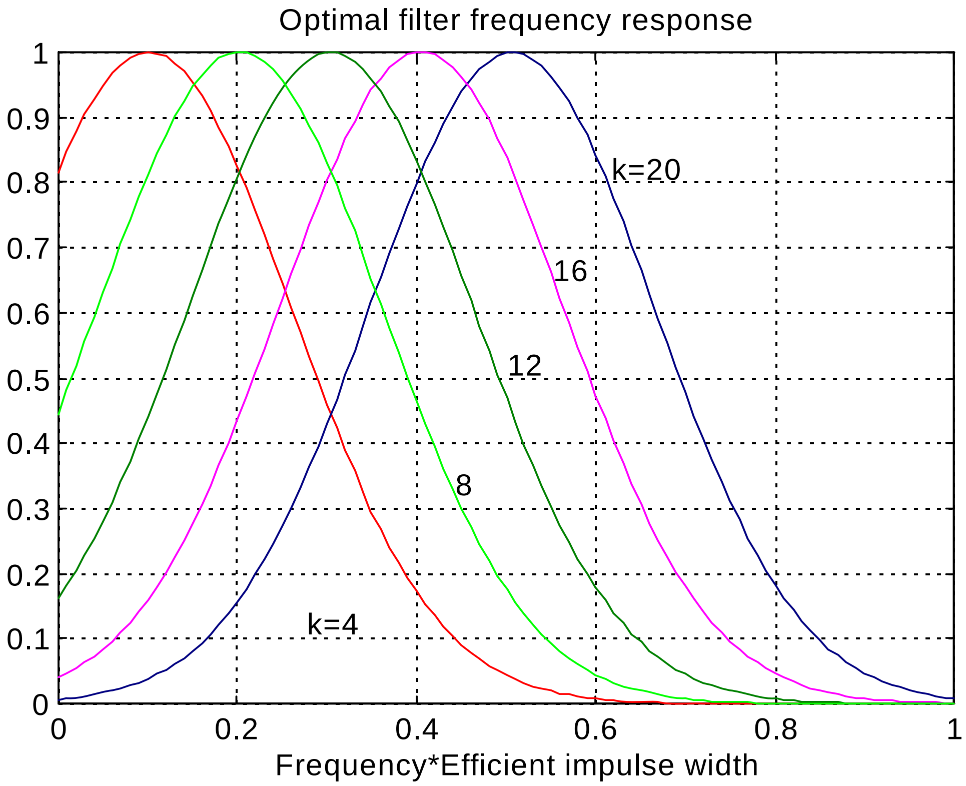
$$\{(\hat{x}_0, \hat{y}_0)\} = \underset{(x_0, y_0)}{\arg\max} \left\{ \sum_{m=1}^{M} \frac{1}{\sigma_m^2} \sum_{k=0}^{N-1} b_{k,m} a_{k,m}(x_0, y_0) \right\}$$



Schematic-diagram of the optimal device for localizing objects in multicomponent images with additive white Gaussian noise in each channel with no inter-channel correlations

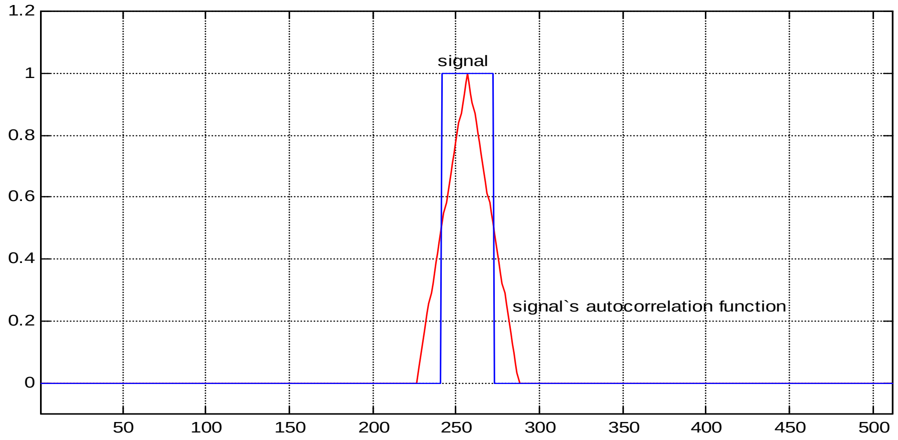
“OPTIMAL” FILTER FOR NON “WHITE” NOISE

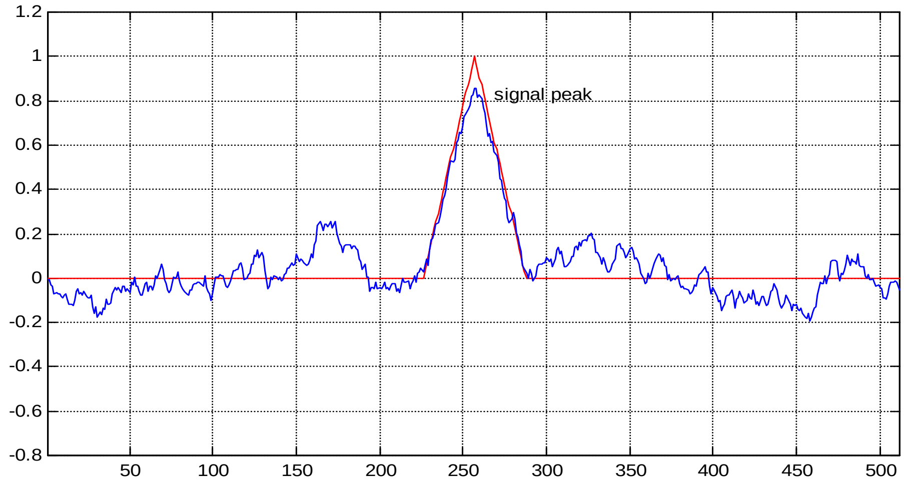




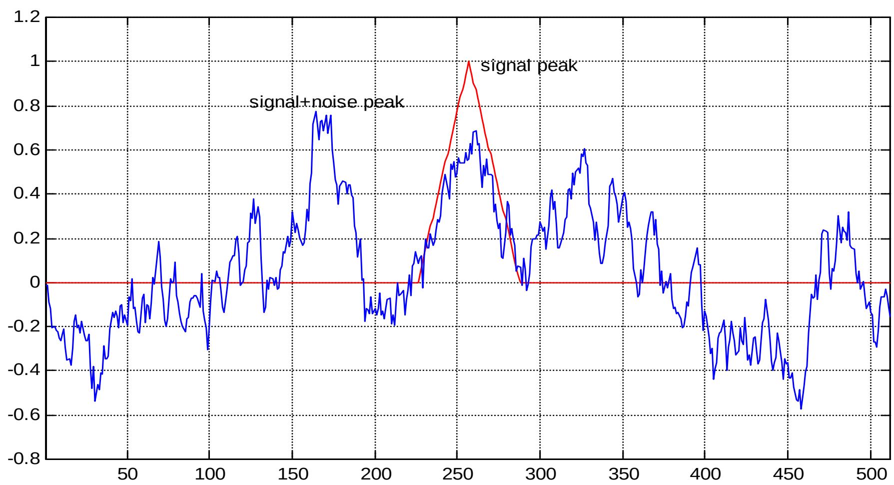
Frequency response of the filter optimal for Gaussian-shaped impulse $a(x) = \exp\left(-\pi x^2 / 2S_a^2\right)$ and non-white noise with exponentially decaying power $N_n(f) = \exp\left(-f^2 / F_n^2\right)$ spectrum; $k = S_a F_n$

TWO TYPES OF THE LOCALIZATION ERRORS





Correlator’s output: the case of “normal” error



Correlator’s output: the case of anomalous error

VARIANCE OF THE LOCALIZATION ERRORS

For one-dimensional signals:

$Corr_{sn}(x) = conv[(s(x-x_0)+n(x))h(x)]$; $h(x)$ - impulse response of the filter

$$\frac{dCorr}{dx}|_{x=\hat{x}_0} = \frac{dCorr_{sh}}{dx}|_{x=\hat{x}_0} + \frac{dCorr_{nh}}{dx}|_{x=\hat{x}_0} = 0; \quad \hat{x}_0 = x_0 + \varepsilon; \quad \frac{dCorr_{sh}(x)}{dx}|_{x=x_0} = 0;$$

$$\varepsilon \frac{d^2Corr_{sh}}{dx^2}|_{x=x_0} + \frac{dCorr_{nh}}{dx}|_{x=\hat{x}_0} = 0; \left\langle \varepsilon^2 \right\rangle \left| \frac{d^2Corr_{sh}}{dx^2}|_{x=x_0} \right|^2 = \left\langle \left| \frac{dCorr_{nh}}{dx} \right|^2 \right\rangle$$

$$\sigma_\varepsilon^2 = \left\langle \varepsilon^2 \right\rangle = \frac{N_0}{4\pi^2 \bar{f}_2^2 E_a} \frac{\int_{-\infty}^{\infty} f^2 |\alpha(f)|^2 df \int_{-\infty}^{\infty} f^2 |H(f)|^2 df}{\left(\int_{-\infty}^{\infty} f^2 \alpha(f) H(f) df \right)^2} \geq \frac{N_0}{4\pi^2 \bar{f}_2^2 E_a} = \frac{N_0}{4\pi^2 E_{aa}}, \quad \text{when}$$

$H(f) = \alpha^*(f)$, where $\alpha(f) = \int_{-\infty}^{\infty} s(x) exp(i2\pi fx) dx$; $H(f) = \int_{-\infty}^{\infty} h(x) exp(i2\pi fx) dx$; $E_a = \int_{-\infty}^{\infty} |\alpha(f)|^2 df$

$$\bar{f}_x^2 = \frac{\int_{-\infty}^{\infty} f^2 |\alpha(f)|^2 df}{\int_{-\infty}^{\infty} |\alpha(f)|^2 df}; \quad E_{aa} = 4\pi^2 \int_{-\infty}^{\infty} f^2 |\alpha(f)|^2 df = \int_{-\infty}^{\infty} \left| \frac{da(x)}{dx} \right|^2 dx$$

- energy of signal's derivative

For 2-D signals:

$$\sigma_x^2 = \frac{1}{4\pi^2} \frac{\bar{f}_y^2}{\bar{f}_x^2 \bar{f}_y^2 - \left(\bar{f}_{xy}^2\right)^2} \frac{N_0}{E_a};$$

$$\sigma_y^2 = \frac{1}{4\pi^2} \frac{\bar{f}_x^2}{\bar{f}_x^2 \bar{f}_y^2 - \left(\bar{f}_{xy}^2\right)^2} \frac{N_0}{E_a};$$

$$\sigma_{xy}^2 = \frac{1}{4\pi^2} \frac{\bar{f}_{xy}^2}{\bar{f}_x^2 \bar{f}_y^2 - \left(\bar{f}_{xy}^2\right)^2} \frac{N_0}{E_a}.$$

with

$$E_a = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\alpha(f_x, f_y)|^2 df_x df_y$$

$$\bar{f}_x^2 = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_x^2 |\alpha(f_x, f_y)|^2 df_x df_y}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\alpha(f_x, f_y)|^2 df_x df_y}$$

$$\bar{f}_y^2 = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_y^2 |\alpha(f_x, f_y)|^2 df_x df_y}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\alpha(f_x, f_y)|^2 df_x df_y}$$

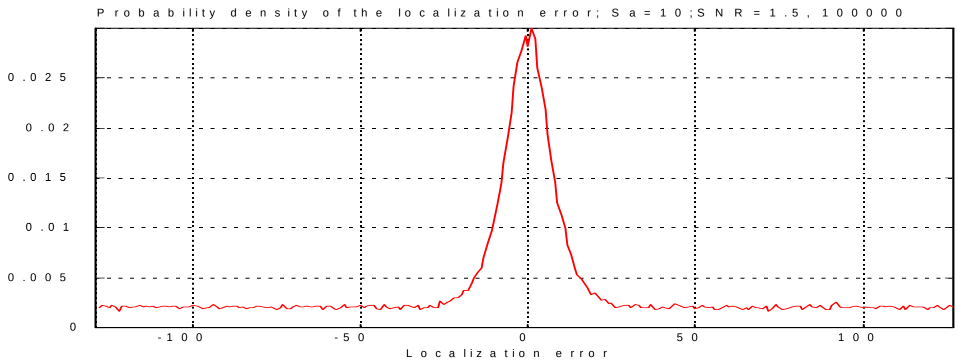
$$\bar{f}_{xy}^2 = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_x f_y |\alpha(f_x, f_y)|^2 df_x df_y}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\alpha(f_x, f_y)|^2 df_x df_y}$$

LOCALIZATION RELIABILITY

Probability of anomalous (false detection) errors:

$$P_{ae} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} exp\left(-\frac{n^2}{2}\right) \left\{1 - \left[\Phi\left(\sqrt{\frac{E_a}{N_0}} + n\right)\right]^{Q-1}\right\} dn$$

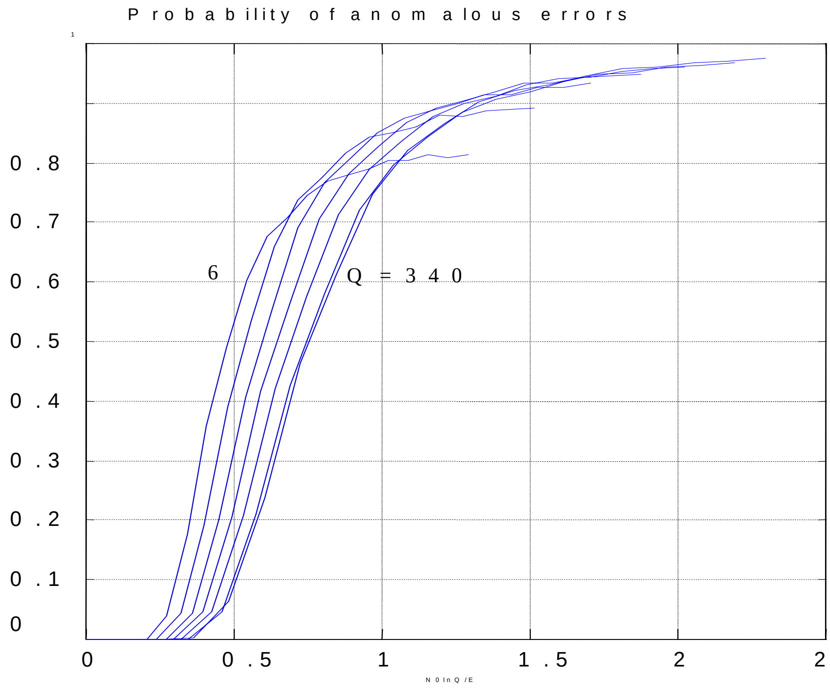
where $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} exp\left(-\frac{n^2}{2}\right) dn$, *Q=O(Search area/Target size).*



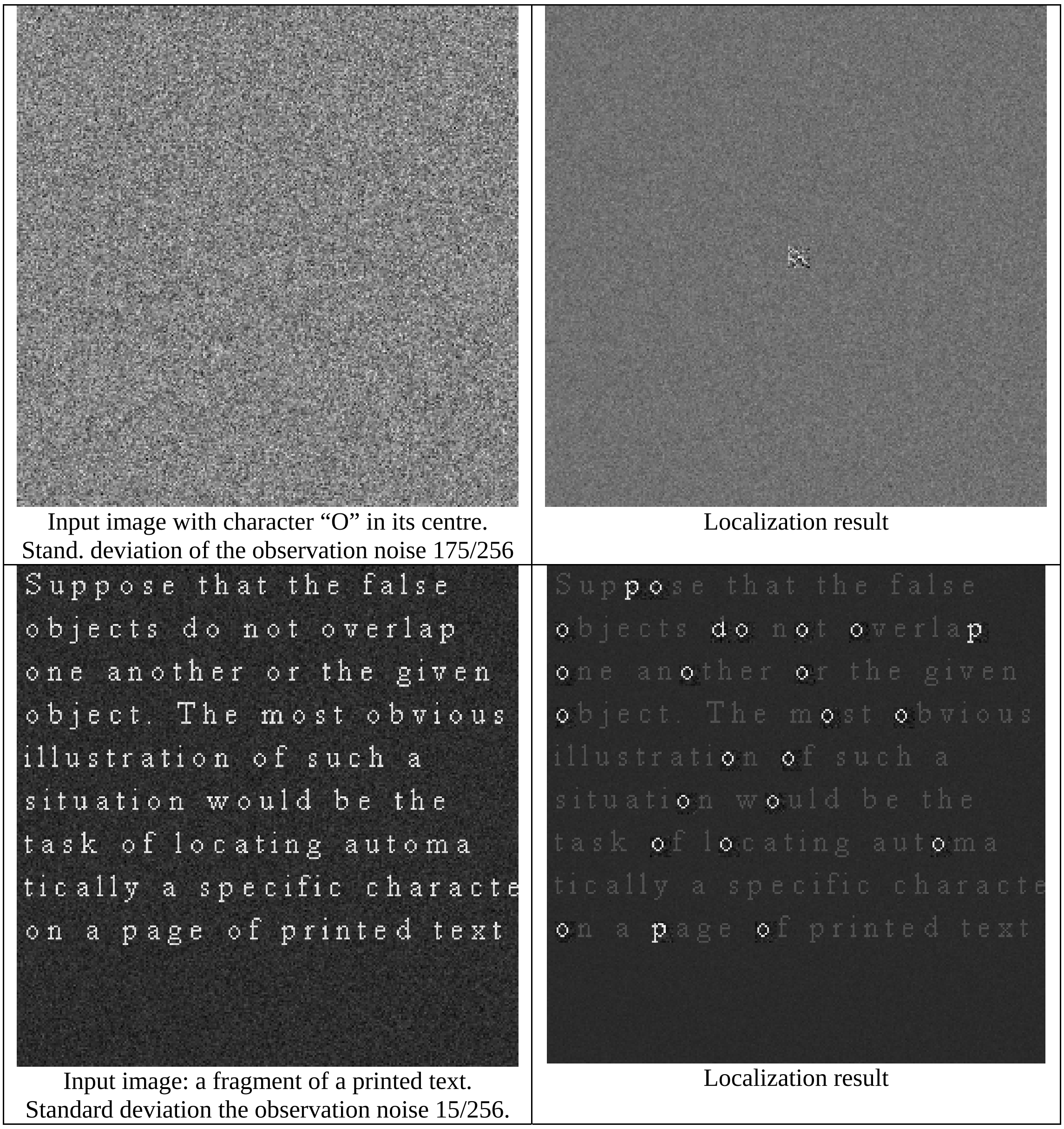
Experimental distribution density of the localization error for Gaussian-shaped impulse ($\sigma_a = 10$) and $\sqrt{E_a / N_0} = 1.5$ (100000 realizations)

The localization reliability threshold:

$$\lim_{Q \to \infty} P_{ae} = \begin{cases} 1, if\ E_a / 2N_0 \le \ln Q \\ 0, if\ E_a / 2N_0 > \ln Q \end{cases}$$



Probability of anomalous errors as a function of normalised noise-to-signal ratio $\sqrt{N_0 \ln Q / E_a}$ for localization of rectangle impulses of 2;5;11;21;41;81;161 samples within an interval of 1024 samples (10000 realizations). The theoretical threshold value of the normalised noise-to-signal ratio is $\sqrt{2}/2 \approx 0.707$



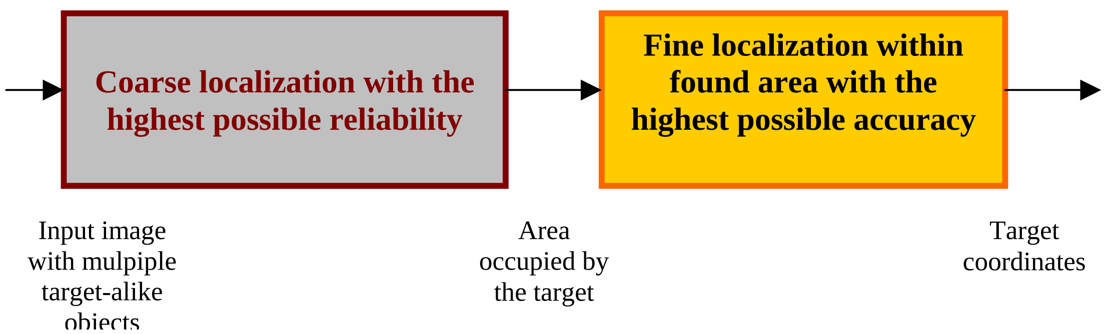
Input image with character "O" in its centre. Stand. deviation of the observation noise 175/256

Localization result

Input image: a fragment of a printed text. Standard deviation the observation noise 15/256.

Localization result

Localization of a target on uniform background and in the presence of multiple non overlapping target-alike objects.



Localization strategy for target observed with multiple target-alike objects