Properties of 1-D DFT

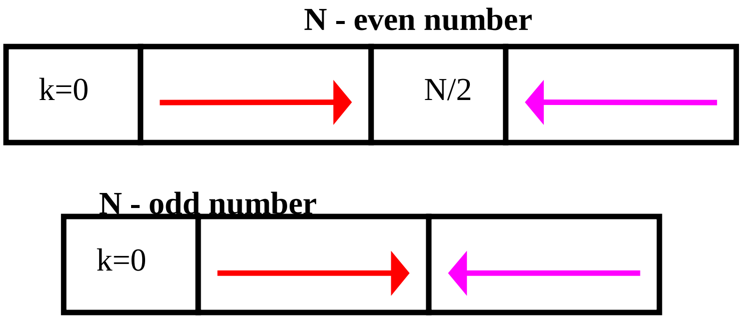
$a_{(k)\bmod N} = \frac{1}{\sqrt{N}}\sum_{r=0}^{N-1}\alpha_r \exp\left(-i2\pi\frac{kr}{N}\right)$	$\alpha_r = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} a_k \exp\left(i2\pi\frac{kr}{N}\right)$
$a_k = a_{(k)\bmod N}$	$\alpha_r = \alpha_{(r)\bmod N}$
$\pm a_{N-k}$	$\pm \alpha_{N-r}$
$a_k = \pm a_{N-k}$	$\alpha_k = \pm\alpha_{N-r}$
$(a_k)^*$	$(\alpha_{N-r})^*$
$a_k = \pm(a_k)^*$	$\alpha_r = \pm(\alpha_{N-r})^*$
$a_k = (a_k)^* = a_{k-n}$	$\alpha_r = \pm(\alpha_{N-r})^* = \alpha_{N-r}$
$\alpha_0 = \frac{1}{\sqrt{N}}\sum_{r=0}^{N-1}\alpha_r$	$\alpha_0 = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} a_k$
For N – *even number*	
$\alpha_{N/2} = \frac{1}{\sqrt{N}}\sum_{r=0}^{N-1}(-1)^r \alpha_r$;	$\alpha_{N/2} = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}(-1)^r a_k$
$\frac{1}{\sqrt{N}}\sum_{k=0}^{(N/2)-1} a_{2k} = (\alpha_0 + \alpha_{N/2})/2$	
$\frac{1}{\sqrt{N}}\sum_{k=0}^{(N/2)-1} a_{2k+1} = (\alpha_0 - \alpha_{N/2})/2$	
Shift theorem	
$a_{(k+k_0)\bmod N}$	$\alpha_r \exp\left(-i2\pi\frac{k_0 r}{N}\right)$
$a_k \exp\left(i2\pi\frac{kr_0}{N}\right)$	$\alpha_{(r+r_0)\bmod N}$
$a_k = 1$	$\sqrt{N}\exp\left(i\pi\frac{N-1}{N}r\right)\frac{\sin(\pi r)}{\sin(\pi r/N)} = \sqrt{N}\delta_r = \sqrt{N}\,0^r$
$\delta_{k-k_0} = 0^{k-k_0}$	$\frac{1}{\sqrt{N}}\exp\left(i2\pi\frac{k_0 r}{N}\right)$
$\frac{1}{\sqrt{N}}\exp\left(i2\pi\frac{kr_0}{N}\right)$	$\delta_{r-r_0} = 0^{r-r_0}$
$\frac{1}{\sqrt{N}}\cos\left(2\pi\frac{kr_0}{N}\right)$	$(\delta_{r-r_0} + \delta_{r+r_0})/2$
$\frac{1}{\sqrt{N}}\sin\left(2\pi\frac{kr_0}{N}\right)$	$(\delta_{r-r_0} - \delta_{r+r_0})/2i$
$(a_k + a_{k-1})/2$	$\alpha_r \exp\left(i\pi\frac{r}{N}\right)\cos\left(\pi\frac{r}{N}\right)$
$a_k - a_{k-1}$	$2i\alpha_r \exp\left(i\pi\frac{r}{N}\right)\sin\left(\pi\frac{r}{N}\right)$

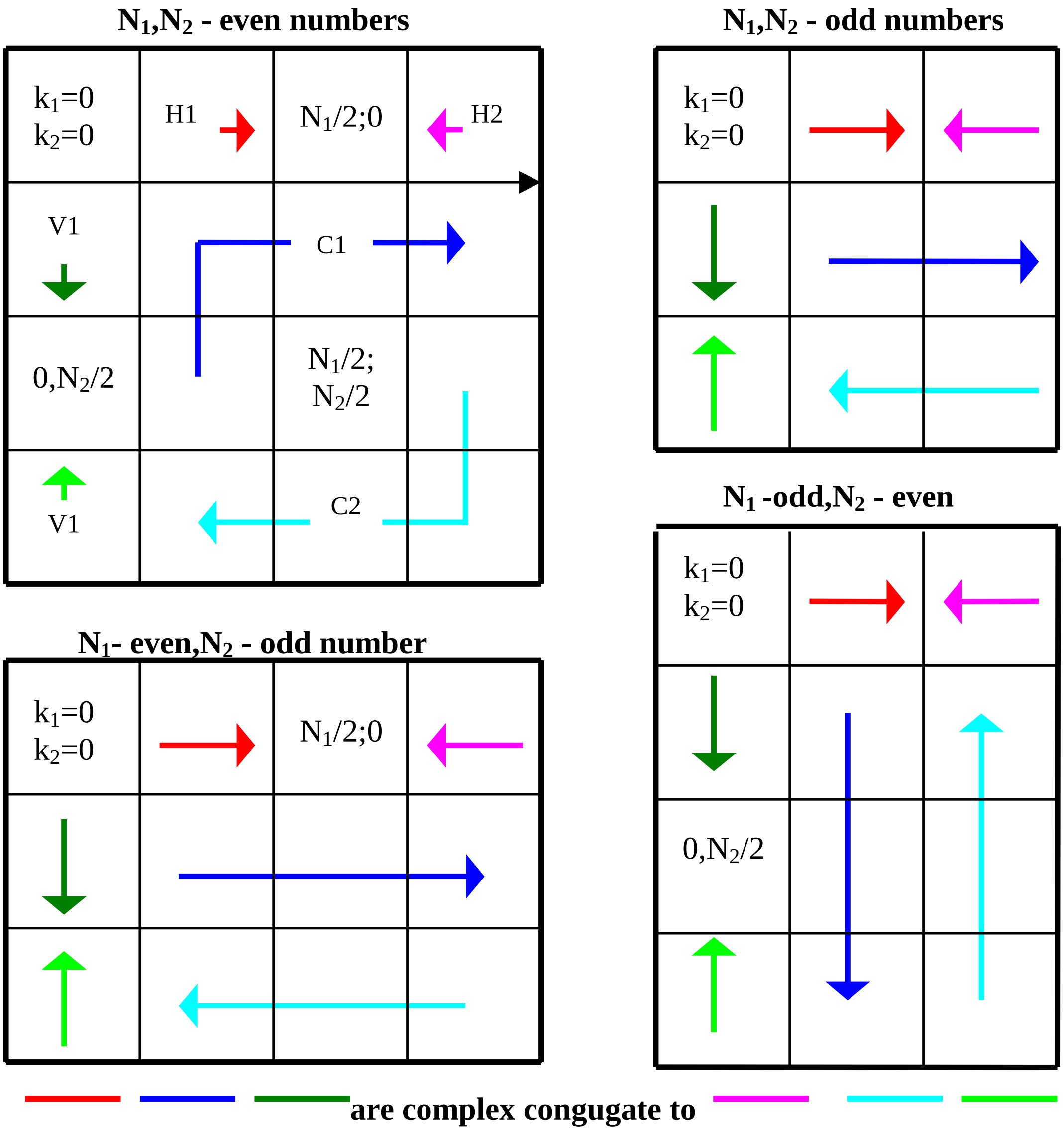
Properties of 1-D DFT

Convolution theorem. For $\{a_k\} \xleftrightarrow{DFT} \{\alpha_r\}, \{b_k\} \xleftrightarrow{DFT} \{\beta_r\}$	
$c_n = \frac{1}{\sqrt{N}} \sum a_{(k)\bmod N}\, b_{(n-k)\bmod N}$	$\gamma_r = \alpha_r \beta_r$
Parceval's relationship: $\sum_{k=0}^{N-1} a_k b_k^* = \sum_{r=0}^{N-1} \alpha_r \beta_r^*$,	
Sampling theorem	
$\tilde{a}_{Lk_2+k_1} = a_{k_2}\delta_{k_1}$; $k_1 = 0,...,L-1$; $k_2 = 0,...,N-1$	$\tilde{\alpha}_{r=Nr_1+r_2} = \frac{1}{\sqrt{N}}\alpha_{r_2} = \frac{1}{\sqrt{N}}\alpha_{(r)\bmod N}$
N – even number	
$\tilde{\tilde{a}}_k = \frac{1}{\sqrt{L}} \sum_{k_2=0}^{N-1} a_{k_2} \frac{\sin\left[\pi \frac{N-1}{LN}(k - Lk_2)\right]}{N \sin\left[\pi \frac{(k-Lk_2)}{LN}\right]}$	$\tilde{\alpha}_{r=Nr_1+r_2} = \frac{1}{\sqrt{N}}\alpha_{(r)\bmod N}\left[1 - rect\frac{r - N/2}{(L-1)N}\right]$
$\tilde{\tilde{a}}_k = \frac{1}{\sqrt{L}} \sum_{k_2=0}^{N-1} a_{k_2} \frac{\sin\left[\pi \frac{N+1}{LN}(k - Lk_2)\right]}{N \sin\left[\pi \frac{(k-Lk_2)}{LN}\right]}$	$\tilde{\alpha}_{r=Nr_1+r_2} = \frac{1}{\sqrt{N}}\alpha_{(r)\bmod N}\left[1 - rect\frac{r - N/2 - 1}{(L-1)N - 1}\right]$
N – odd number	
$\tilde{\tilde{a}}_k = \frac{1}{\sqrt{L}} \sum_{k_2=0}^{N-1} a_{k_2} \frac{\sin\left[\pi \frac{(k - Lk_2)}{LN}\right]}{N \sin\left[\pi \frac{(k-Lk_2)}{LN}\right]}$	$\tilde{\alpha}_{r=Nr_1+r_2} = \frac{1}{\sqrt{N}}\alpha_{(r)\bmod N}\left[1 - rect\frac{r - (N+1)/2}{(L-1)N}\right]$

Properties of 2-D DFT

$a_{k,l} = \frac{1}{\sqrt{N_1 N_2}} \sum_{r=0}^{N_1-1}\sum_{s=0}^{N_2-1} \alpha_{r,s} \exp\left[-i2\pi\left(\frac{kr}{N_1}+\frac{ls}{N_2}\right)\right]$	$\alpha_{r,s} = \frac{1}{\sqrt{N_1 N_2}} \sum_{k=0}^{N_1-1}\sum_{l=0}^{N_2-1} a_{k,l} \exp\left[i2\pi\left(\frac{kr}{N_1}+\frac{ls}{N_2}\right)\right] = \frac{1}{\sqrt{N_1 N_2}} \sum_{k=0}^{N_1-1} \exp\left(i2\pi\frac{kr}{N_1}\right)\sum_{l=0}^{N_2-1} a_{k,l} \exp\left(i2\pi\frac{ls}{N_2}\right)$
$a_{k,l} = a_{(k)\bmod N_1,(l)\bmod N_2}$	$\alpha_{r,s} = a_{(r)\bmod N_1,(s)\bmod N_2}$
$\pm a_{N_1-k,N_2-l}$	$\pm\alpha_{N_1-r,N_2-s}$
$a_{k,l} = \pm a_{N_1-k,N_2-l}$	$\alpha_{r,s} = \pm\alpha_{N_1-r,N_2-s}$
$(a_{k,l})^*$	$(\alpha_{N_1-r,N_2-s})^*$
$a_{k,l} = \pm(a_{k,l})^*$	$\alpha_{r,s} = \pm(\alpha_{N_1-r,N_2-s})^*$
$a_{k,l} = (a_{k,l})^* = a_{N_1-k,N_2-l}$	$\alpha_{r,s} = (\alpha_{N_1-r,N_2-s})^* = \alpha_{N_1-r,N_2-s}$
$\pm a_{N_1-k,l}$	$\pm\alpha_{N_1-r,s}$
$\pm a_{k,N_2-l}$	$\pm\alpha_{r,N_2-s}$
$\frac{a_{0,0}}{\sqrt{N_1 N_2}} = \frac{1}{N_1 N_2}\sum_{r=0}^{N_1-1}\sum_{s=0}^{N_2-1}\alpha_{r,s}$	$\frac{\alpha_{0,0}}{\sqrt{N_1 N_2}} = \frac{1}{N_1 N_2}\sum_{k=0}^{N_1-1}\sum_{l=0}^{N_2-1}a_{k,l}$
N_1, N_2 – even numbers	
$a_{N_1/2,N_2/2} = \frac{1}{\sqrt{N_1 N_2}}\sum_{r=0}^{N_1-1}\sum_{s=0}^{N_2-1}(-1)^{r+s}\alpha_{r,s}$	$\alpha_{N_1/2,N_2/2} = \frac{1}{\sqrt{N_1 N_2}}\sum_{k=0}^{N_1-1}\sum_{k=0}^{N_2-1}(-1)^{k+l}a_{k,l}$
Shift theorem	
$a_{(k+k_0)\bmod N_1,(l+l_0)\bmod N_2}$	$\alpha_{r,s}\exp\left[-i2\pi\left(\frac{k_0 r}{N_1}+\frac{l_0 s}{N_2}\right)\right]$
$a_{k,l}\exp\left[-i2\pi\left(\frac{kr_0}{N_1}+\frac{ls_0}{N_2}\right)\right]$	$\alpha_{(r+r_0)\bmod N_1,(s+s_0)\bmod N_2}$
Convolution theorem	
$c_{n,m} = \frac{1}{\sqrt{N_1, N_2}}\sum_{k=0}^{N_1-1}\sum_{l=0}^{N_2-1} a_{(k)\bmod N_1,(l)\bmod N_2} \times b_{(n-k)\bmod N_1,(m-l)\bmod N_2}$	$\gamma_{r,s} = \alpha_{r,s}\beta_{r,s}$





Types of the DFT spectra symmetry for one and two-dimensional real-valued signals