

## Properties of 1-D DFT

$a_{(k)\bmod N} = \frac{1}{\sqrt{N}} \sum_{r=0}^{N-1} \alpha_r \exp\left(-i2\pi \frac{kr}{N}\right)$	$\alpha_r = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} a_k \exp\left(i2\pi \frac{kr}{N}\right)$
$a_k = a_{(k)\bmod N}$	$\alpha_r = \alpha_{(r)\bmod N}$
$\pm a_{N-k}$	$\pm \alpha_{N-r}$
$a_k = \pm a_{N-k}$	$\alpha_k = \pm \alpha_{N-r}$
$(a_k)^*$	$(\alpha_{N-r})^*$
$a_k = \pm (a_k)^*$	$\alpha_r = \pm (\alpha_{N-r})^*$
$a_k = (a_k)^* = a_{k-n}$	$\alpha_r = \pm (\alpha_{N-r})^* = \alpha_{N-r}$
$\alpha_0 = \frac{1}{\sqrt{N}} \sum_{r=0}^{N-1} \alpha_r$	$\alpha_0 = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} a_k$
For $N - \text{even number}$	
$\alpha_{N/2} = \frac{1}{\sqrt{N}} \sum_{r=0}^{N-1} (-1)^r \alpha_r ;$	$\alpha_{N/2} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (-1)^k a_k$
$\frac{1}{\sqrt{N}} \sum_{k=0}^{(N/2)-1} a_{2k} = (\alpha_0 + \alpha_{N/2})/2$	
$\frac{1}{\sqrt{N}} \sum_{k=0}^{(N/2)-1} a_{2k+1} = (\alpha_0 - \alpha_{N/2})/2$	
<b>Shift theorem</b>	
$a_{(k+k_0)\bmod N}$	$\alpha_r \exp\left(-i2\pi \frac{k_0 r}{N}\right)$
$a_k \exp\left(i2\pi \frac{kr_0}{N}\right)$	$\alpha_{(r+r_0)\bmod N}$
$a_k = 1$	$\sqrt{N} \exp\left(i\pi \frac{N-1}{N} r\right) \frac{\sin(\pi r)}{\sin(\pi r/N)} = \sqrt{N} \delta_r = \sqrt{N} \mathbf{0}^r$
$\delta_{k-k_0} = \mathbf{0}^{k-k_0}$	$\frac{1}{\sqrt{N}} \exp\left(i2\pi \frac{k_0 r}{N}\right)$
$\frac{1}{\sqrt{N}} \exp\left(i2\pi \frac{kr_0}{N}\right)$	$\delta_{r-r_0} = \mathbf{0}^{r-r_0}$
$\frac{1}{\sqrt{N}} \cos\left(2\pi \frac{kr_0}{N}\right)$	$(\delta_{r-r_0} + \delta_{r+r_0})/2$
$\frac{1}{\sqrt{N}} \sin\left(2\pi \frac{kr_0}{N}\right)$	$(\delta_{r-r_0} - \delta_{r+r_0})/2i$
$(a_k + a_{k-1})/2$	$\alpha_r \exp\left(i\pi \frac{r}{N}\right) \cos\left(\pi \frac{r}{N}\right)$
$a_k - a_{k-1}$	$2i \alpha_r \exp\left(i\pi \frac{r}{N}\right) \sin\left(\pi \frac{r}{N}\right)$

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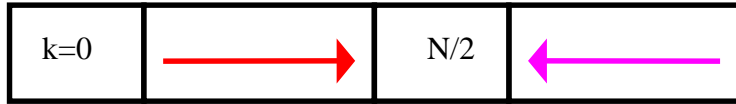
<b>Convolution theorem.</b> For $\{a_k\} \xleftrightarrow{DFT} \{\alpha_r\}, \{b_k\} \xleftrightarrow{DFT} \{\beta_r\}$	
$c_n = \frac{1}{\sqrt{N}} \sum a_{(k) \bmod N} b_{(n-k) \bmod N}$	$\gamma_r = \alpha_r \beta_r$
<b>Parseval's relationship:</b> $\sum_{k=0}^{N-1} a_k b_k^* = \sum_{r=0}^{N-1} \alpha_r \beta_r^*$ ,	
<b>Sampling theorem</b>	
$\tilde{a}_{Lk_2+k_1} = a_{k_2} \delta_{k_1};$ $k_1 = 0, \dots, L-1; k_2 = 0, \dots, N-1$	$\tilde{\alpha}_{r=Nr_1+r_2} = \frac{1}{\sqrt{N}} \alpha_{r_2} = \frac{1}{\sqrt{N}} \alpha_{(r) \bmod N}$
$N - \text{ even number}$	
$\tilde{\tilde{a}}_k = \frac{1}{\sqrt{L}} \sum_{k_2=0}^{N-1} a_{k_2} \frac{\sin \left[ \pi \frac{N-1}{LN} (k - Lk_2) \right]}{N \sin \left[ \pi \frac{(k - Lk_2)}{LN} \right]}$	$\tilde{\tilde{\alpha}}_{r=Nr_1+r_2} = \frac{1}{\sqrt{N}} \alpha_{(r) \bmod N} \left[ 1 - \text{rect} \frac{r - N/2}{(L-1)N} \right]$
$\tilde{\tilde{a}}_k = \frac{1}{\sqrt{L}} \sum_{k_2=0}^{N-1} a_{k_2} \frac{\sin \left[ \pi \frac{N+1}{LN} (k - Lk_2) \right]}{N \sin \left[ \pi \frac{(k - Lk_2)}{LN} \right]}$	$\tilde{\tilde{\alpha}}_{r=Nr_1+r_2} = \frac{1}{\sqrt{N}} \alpha_{(r) \bmod N} \left[ 1 - \text{rect} \frac{r - N/2 - 1}{(L-1)N - 1} \right]$
$N - \text{ odd number}$	
$\tilde{\tilde{a}}_k = \frac{1}{\sqrt{L}} \sum_{k_2=0}^{N-1} a_{k_2} \frac{\sin \left[ \pi \frac{(k - Lk_2)}{LN} \right]}{N \sin \left[ \pi \frac{(k - Lk_2)}{LN} \right]}$	$\tilde{\tilde{\alpha}}_{r=Nr_1+r_2} = \frac{1}{\sqrt{N}} \alpha_{(r) \bmod N} \left[ 1 - \text{rect} \frac{r - (N+1)/2}{(L-1)N} \right]$

## Properties of 2-D DFT

$a_{k,l} = \frac{1}{\sqrt{N_1 N_2}} \sum_{r=0}^{N_1-1} \sum_{s=0}^{N_2-1} \alpha_{r,s} \exp \left[ -i2\pi \left( \frac{kr}{N_1} + \frac{ls}{N_2} \right) \right]$	$\alpha_{r,s} = \frac{1}{\sqrt{N_1 N_2}} \sum_{k=0}^{N_1-1} \sum_{l=0}^{N_2-1} a_{k,l} \exp \left[ i2\pi \left( \frac{kr}{N_1} + \frac{ls}{N_2} \right) \right] =$ $\frac{1}{\sqrt{N_1 N_2}} \sum_{k=0}^{N_1-1} \exp \left( i2\pi \frac{kr}{N_1} \right) \sum_{l=0}^{N_2-1} a_{k,l} \exp \left( i2\pi \frac{ls}{N_2} \right)$
$a_{k,l} = a_{(k) \bmod N_1, (l) \bmod N_2}$	$\alpha_{r,s} = a_{(r) \bmod N_1, (s) \bmod N_2}$
$\pm a_{N_1-k, N_2-l}$	$\pm \alpha_{N_1-r, N_2-s}$
$a_{k,l} = \pm a_{N_1-k, N_2-l}$	$\alpha_{r,s} = \pm \alpha_{N_1-r, N_2-s}$
$(a_{k,l})^*$	$(\alpha_{N_1-r, N_2-s})^*$
$a_{k,l} = \pm (a_{k,l})^*$	$\alpha_{r,s} = \pm (\alpha_{N_1-r, N_2-s})^*$
$a_{k,l} = (a_{k,l})^* = a_{N_1-k, N_2-l}$	$\alpha_{r,s} = (\alpha_{N_1-r, N_2-s})^* = \alpha_{N_1-r, N_2-s}$
$\pm a_{N_1-k, l}$	$\pm \alpha_{N_1-r, s}$
$\pm a_{k, N_2-l}$	$\pm \alpha_{r, N_2-s}$
$\frac{a_{0,0}}{\sqrt{N_1 N_2}} = \frac{1}{N_1 N_2} \sum_{r=0}^{N_1-1} \sum_{s=0}^{N_2-1} \alpha_{r,s}$	$\frac{\alpha_{0,0}}{\sqrt{N_1 N_2}} = \frac{1}{N_1 N_2} \sum_{k=0}^{N_1-1} \sum_{l=0}^{N_2-1} a_{k,l}$
$N_1, N_2 - \text{even numbers}$	
$a_{N_1/2, N_2/2} = \frac{1}{\sqrt{N_1 N_2}} \sum_{r=0}^{N_1-1} \sum_{s=0}^{N_2-1} (-1)^{r+s} \alpha_{r,s}$	$\alpha_{N_1/2, N_2/2} = \frac{1}{\sqrt{N_1 N_2}} \sum_{k=0}^{N_1-1} \sum_{l=0}^{N_2-1} (-1)^{k+l} a_{k,l}$
<b>Shift theorem</b>	
$a_{(k+k_0) \bmod N_1, (l+l_0) \bmod N_2}$	$\alpha_{r,s} \exp \left[ -i2\pi \left( \frac{k_0 r}{N_1} + \frac{l_0 s}{N_2} \right) \right]$
$a_{k,l} \exp \left[ -i2\pi \left( \frac{kr_0}{N_1} + \frac{ls_0}{N_2} \right) \right]$	$\alpha_{(r+r_0) \bmod N_1, (s+s_0) \bmod N_2}$
<b>Convolution theorem</b>	
$c_{n,m} = \frac{1}{\sqrt{N_1, N_2}} \sum_{k=0}^{N_1-1} \sum_{l=0}^{N_2-1} a_{(k) \bmod N_1, (l) \bmod N_2} \times$ $b_{(n-k) \bmod N_1, (m-l) \bmod N_2}$	$\gamma_{r,s} = \alpha_{r,s} \beta_{r,s}$



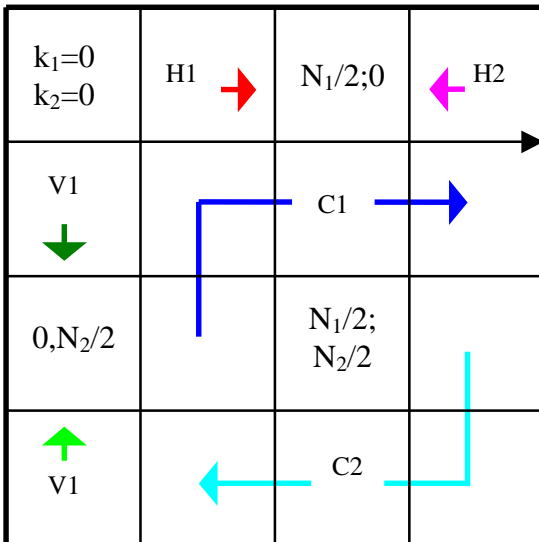
**N - even number**



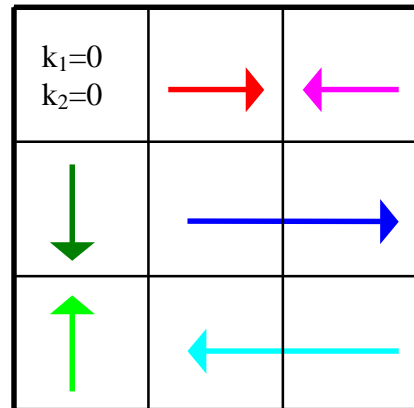
**N - odd number**



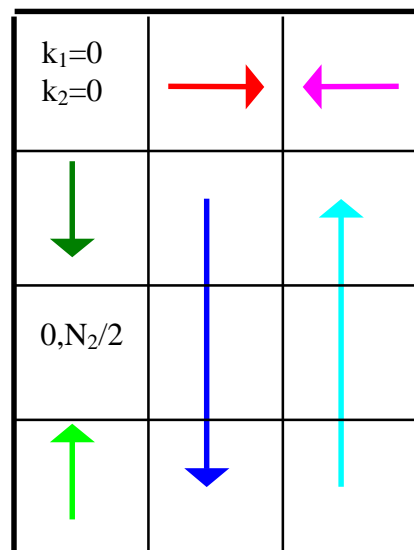
**$N_1, N_2$  - even numbers**



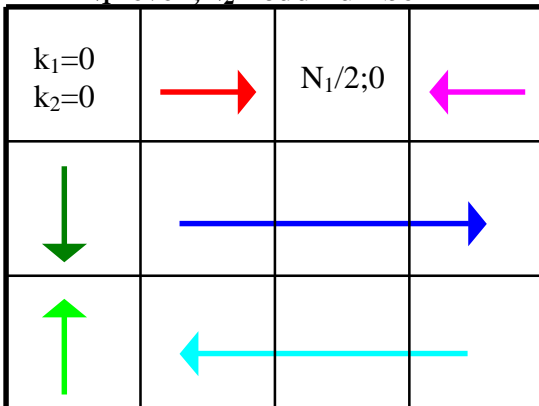
**$N_1, N_2$  - odd numbers**



**$N_1$  - odd,  $N_2$  - even**



**$N_1$  - even,  $N_2$  - odd number**



are complex conjugate to

**Types of the DFT spectra symmetry for one and two-dimensional real-valued signals**