L. Yaroslavsky

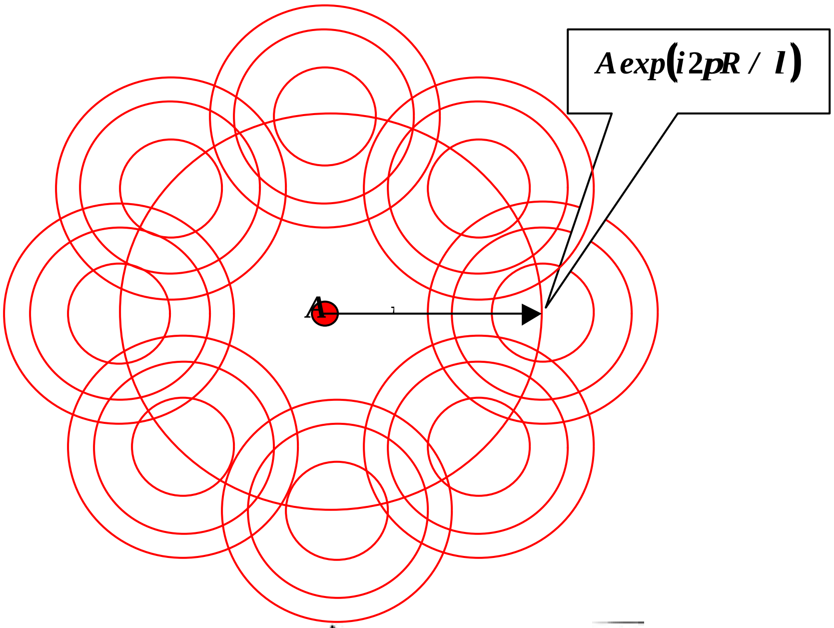
FROM PHOTOGRAPHY TO *.GRAPHIES: UNCONVENTIONAL IMAGING TECHNIQUES

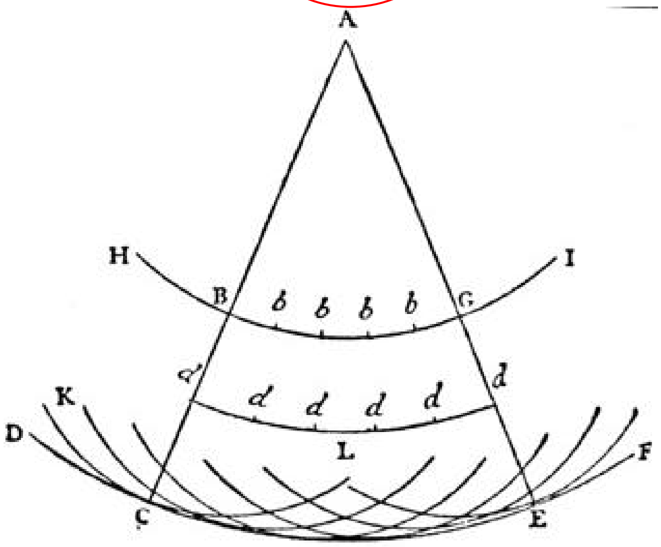
A short course at Tampere University of Technology,
Tampere, Finland, Sept. 3 – Sept. 14, 2001

Lecture 3.
PRINCIPLES OF FOURIER OPTICS

Lecture 3 **PRINCIPLES OF FOURIER OPTICS**

<u>**Huygens' principle :**</u> In light propagation, every point through which light passes can be regarded as a source of a spherical wavefront:





Wave front modulation as a result of its reflection or transmission:

Reflection (transmission) factor of an object is defined as a ratio of outcoming and incoming wave front complex amplitudes:

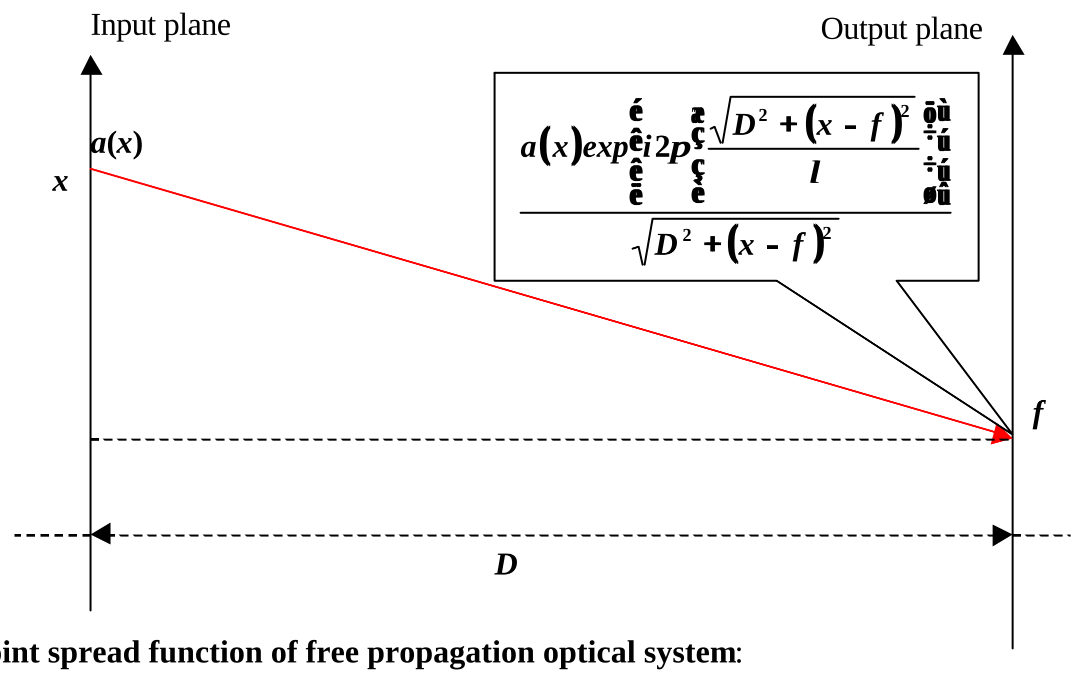
$$Obj(x,y) = \frac{A_{out}\exp(i\varphi_{out})}{A_{in}\exp(i\varphi_{in})}$$

such that, if object's reflection/transmission factor is given as $A_{obj}\exp(i\varphi_{obj})$,

$$A_{out}\exp(i\varphi_{out}) = A_{obj}\exp(i\varphi_{obj})A_{in}\exp(i\varphi_{in})$$

The Fresnel-Kirchhof theory

Point spread function of optical systems with free propagation



Point spread function of free propagation optical system:

$$PSF(x,f)=\frac{\exp\left[i2\pi\left(\frac{\sqrt{D^2+(x-f)^2}}{\lambda}\right)\right]}{\sqrt{D^2+(x-f)^2}}$$

Kirchhof equation:

$$a(f)=\int_x a(x)\frac{\exp\left[i2\pi\left(\frac{\sqrt{D^2+(x-f)^2}}{\lambda}\right)\right]}{\sqrt{D^2+(x-f)^2}}dx$$

For $D >> max|x-f|$ ("near zone" propagation), *Fresnel approximation*:

$$a(f)=\int_x a(x)\frac{\exp\left[i2\pi\left(\frac{\sqrt{D^2+(x-f)^2}}{\lambda}\right)\right]}{\sqrt{D^2+(x-f)^2}}dx \cong C\int_x a(x)\exp\left[i\pi\frac{(x-f)^2}{\lambda D}\right]dx$$

with C as an irrelevant constant.

If $\exp(i\pi x^2/D^2)\cong 1$ and $\exp(i\pi f^2/D^2)\cong 1$, ("far zone" propagation) *Fraunhofer approximation*:

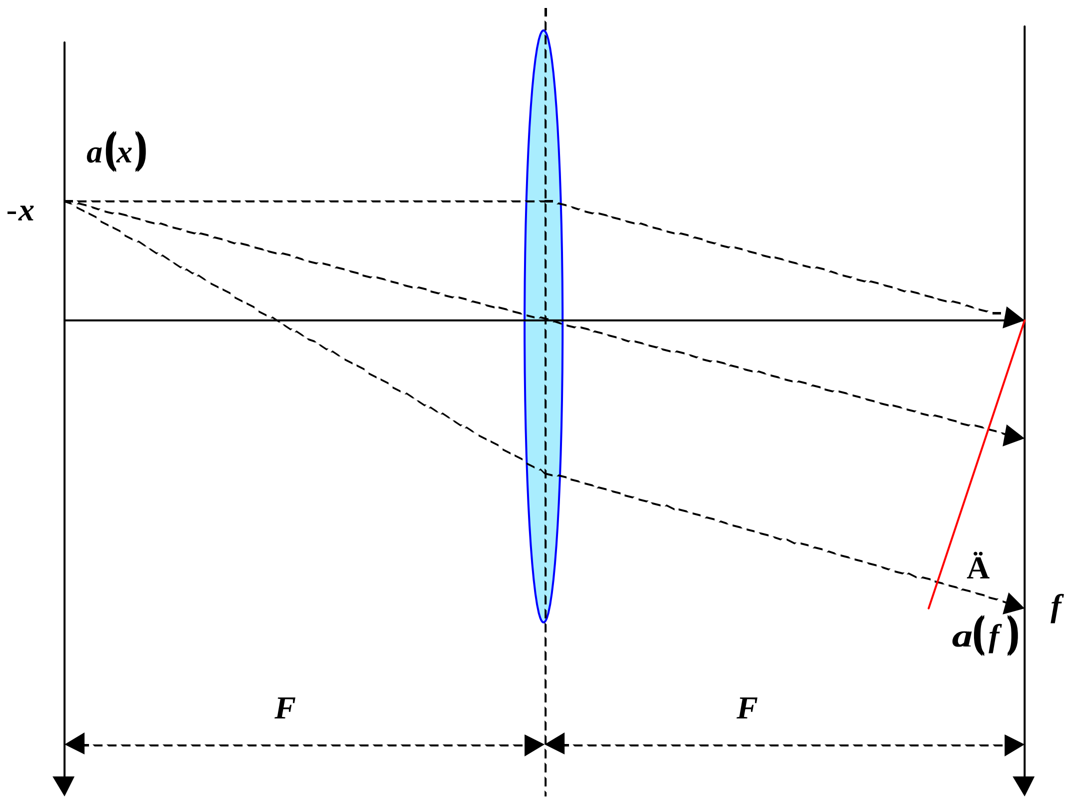
$$a(f)\cong C\int_x a(x)\exp\left[-i2\pi\frac{xf}{\lambda D}\right]dx$$

Lens a Fourier Transformer.

Two reciprocal main properties of a lens (a definition of a lens) are:

- lens focuses parallel beam of light;
- light from a point source in the lens focal plane is converted by the lens into a parallel beam propagating along the line passing through the point source and center of the lens.

One can use this property to determine point spread function of the optical system with a lens and input image plane and output image plane in front and rear focal planes of the lens.



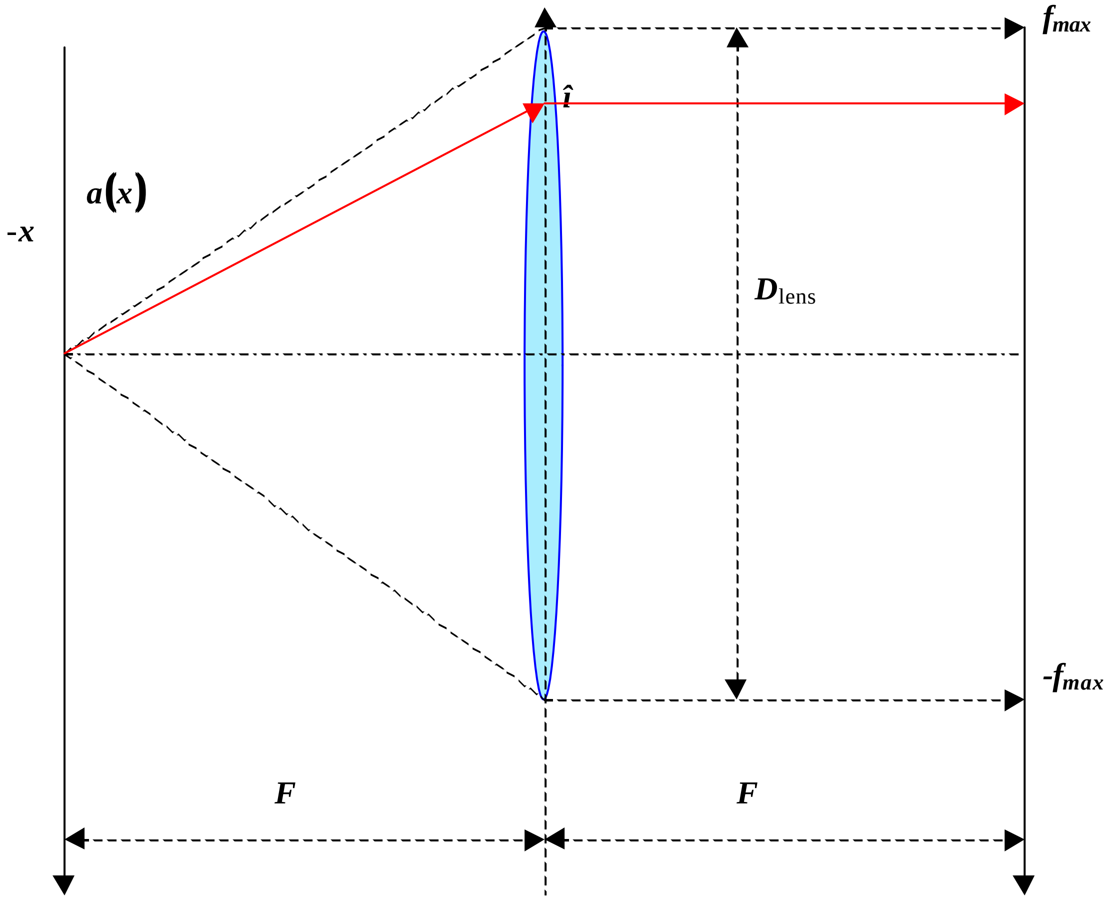
For $x \ll F$ ("paraxial" approximation),

$$\Delta = f\left(\frac{-x}{F}\right)$$

$$PSF(x,f) = exp(i2\pi\Delta/\lambda) = exp\left(-i2\pi\frac{fx}{\lambda F}\right);$$

$$a(f) = \int_x a(x)exp\left(-i2\pi\frac{fx}{\lambda F}\right)dx$$

Lens as a low-pass spatial filter:



$$a(f) = \left\{ \int_x a(x) \exp\left(-i2\pi \frac{fx}{\lambda F} \right) dx \right\} rect\left(\frac{f + D_{lens}/2}{D_{lens}} \right)$$

Lens as a "chirp"-modulator:

Spherical wave at point ***î*** in lens plain has complex amplitude

$$A = A_0 \exp\left(i2\pi \frac{\sqrt{F^2 + x^2}}{\lambda} \right).$$

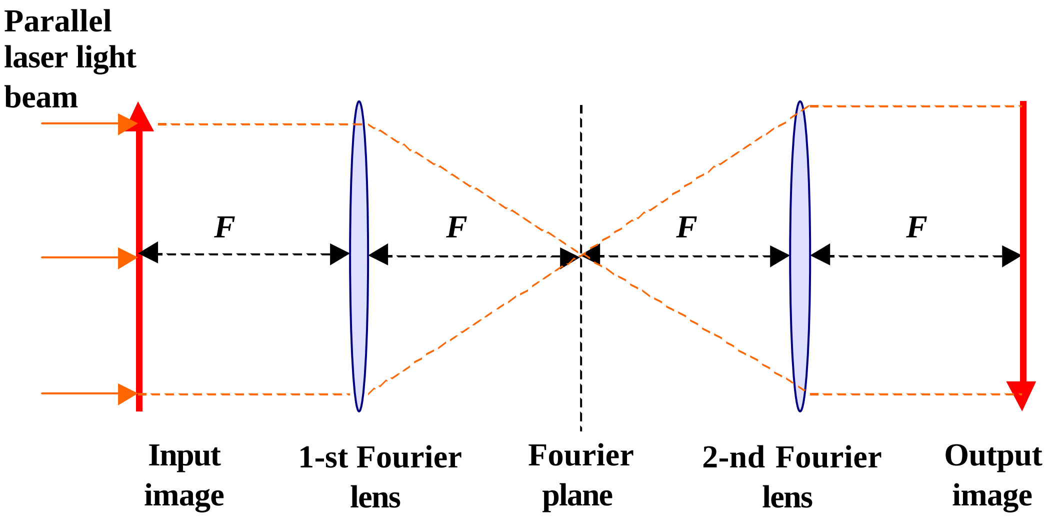
In "paraxial" approximation, when size of the lens is much less than its focal distance, wave amplitude is approximately

$$A \cong A_0 \exp\left(i\pi \frac{x^2}{\lambda F} \right).$$

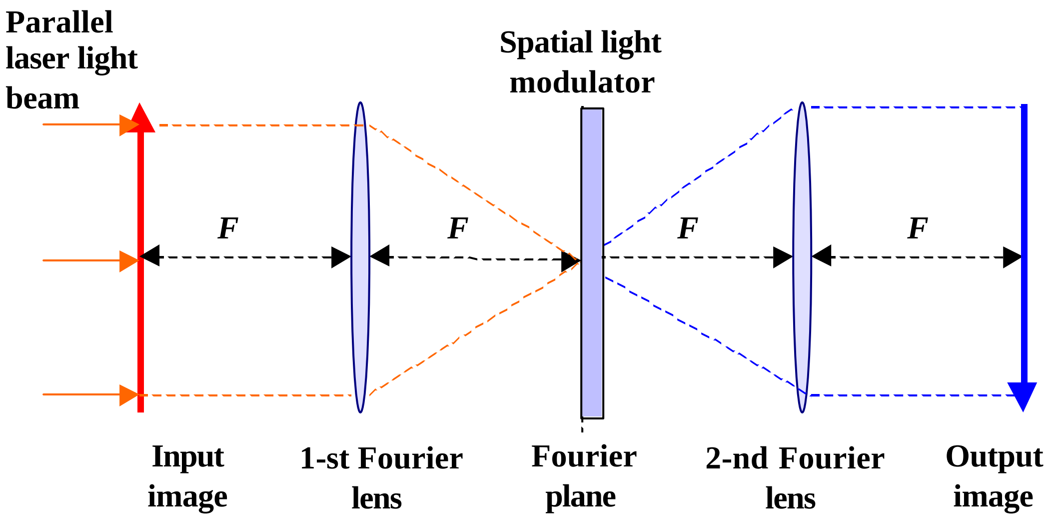
Since this spherical wave is converted by the lens into a plain wave, complex transparency of the lens is then

$$\exp\left(-i\pi \frac{x^2}{\lambda F} \right)$$

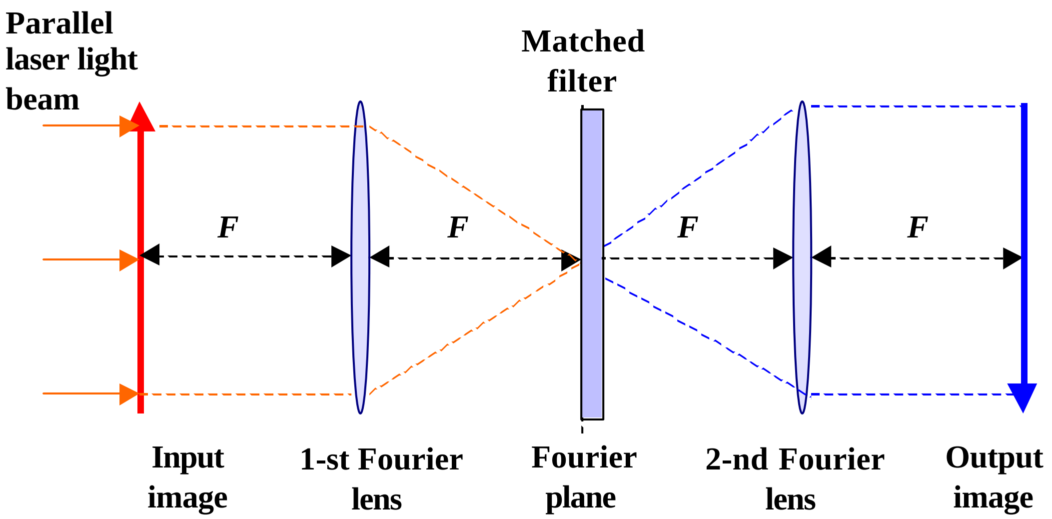
4-F imaging system



4-F coherent spatial filtering system



4-F coherent optical correlator



Nonlinear optical correlators

Optimal adaptive correlator (*L. Yaroslavsky, M. Eden, Fundamentals of Digital Optics, Birkhauser, Boston, 1996*):

$$H_{opt}(f) = \frac{a^*(f)}{AV_{x_0}\left|a_{bg}(f)\right|^2}$$

where $a^*(f)$ is complex conjugate spectrum of the target object; $AV_{x_0}\left|a_{bg}(f)\right|^2$ is power spectrum of the image background component averaged over unknown co-ordinate x_0 of the target

Evaluating power spectrum of the image background component: zero order approximation

$$a_{bg}(x) \approx b(x);\; AV_{x_0}\left|a_{bg}(f)\right|^2 = \left|b_{img}(f)\right|^2,$$

where $\left|b_{img}(f)\right|^2$ is power spectrum of the image $b(x)$.

Optimal adaptive correlator can be represented as a composition of a "whitening" filter $WF(f)$ and a modified matched filter $MMF(f)$:

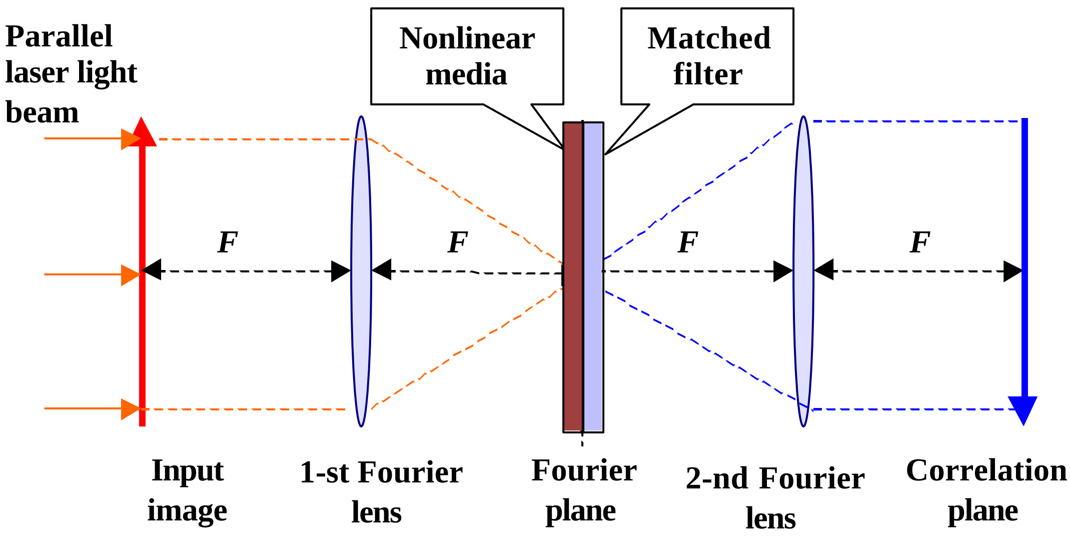
$$H_{opt}(f) = WF(f) \cdot MMF(f) = \frac{1}{\left[AV_{x_0}\left|a_{bg}(f)\right|^2\right]^{1/2}} \frac{a^*(f)}{\left[AV_{x_0}\left|a_{bg}(f)\right|^2\right]^{1/2}}$$

Optimal adaptive correlator can be implemented in coherent optical image processing systems with a matched filter and a nonlinear optical media placed in the system's Fourier plane. Matched filter can be fabricated optically as a Fourier hologram of the target object or generated by computer as a digital (computer generated) hologram

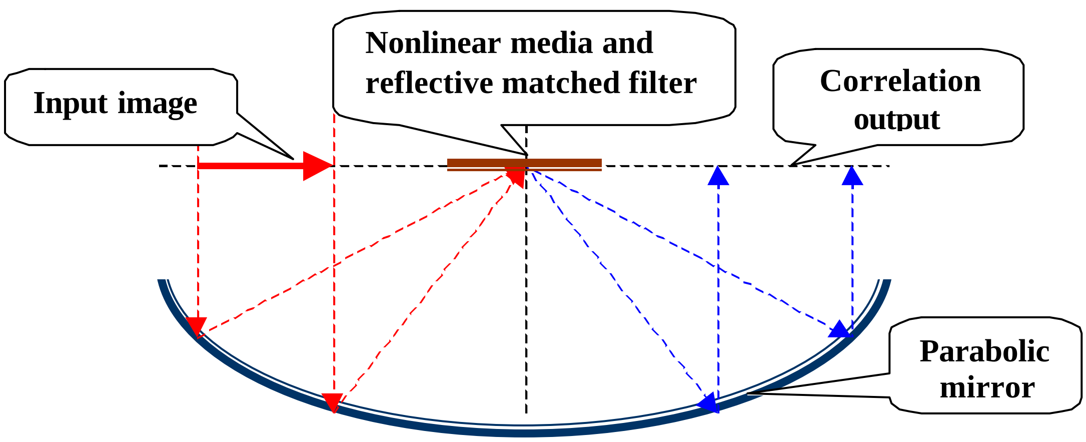
Optical correlators with (-*k*)th law nonlinearity

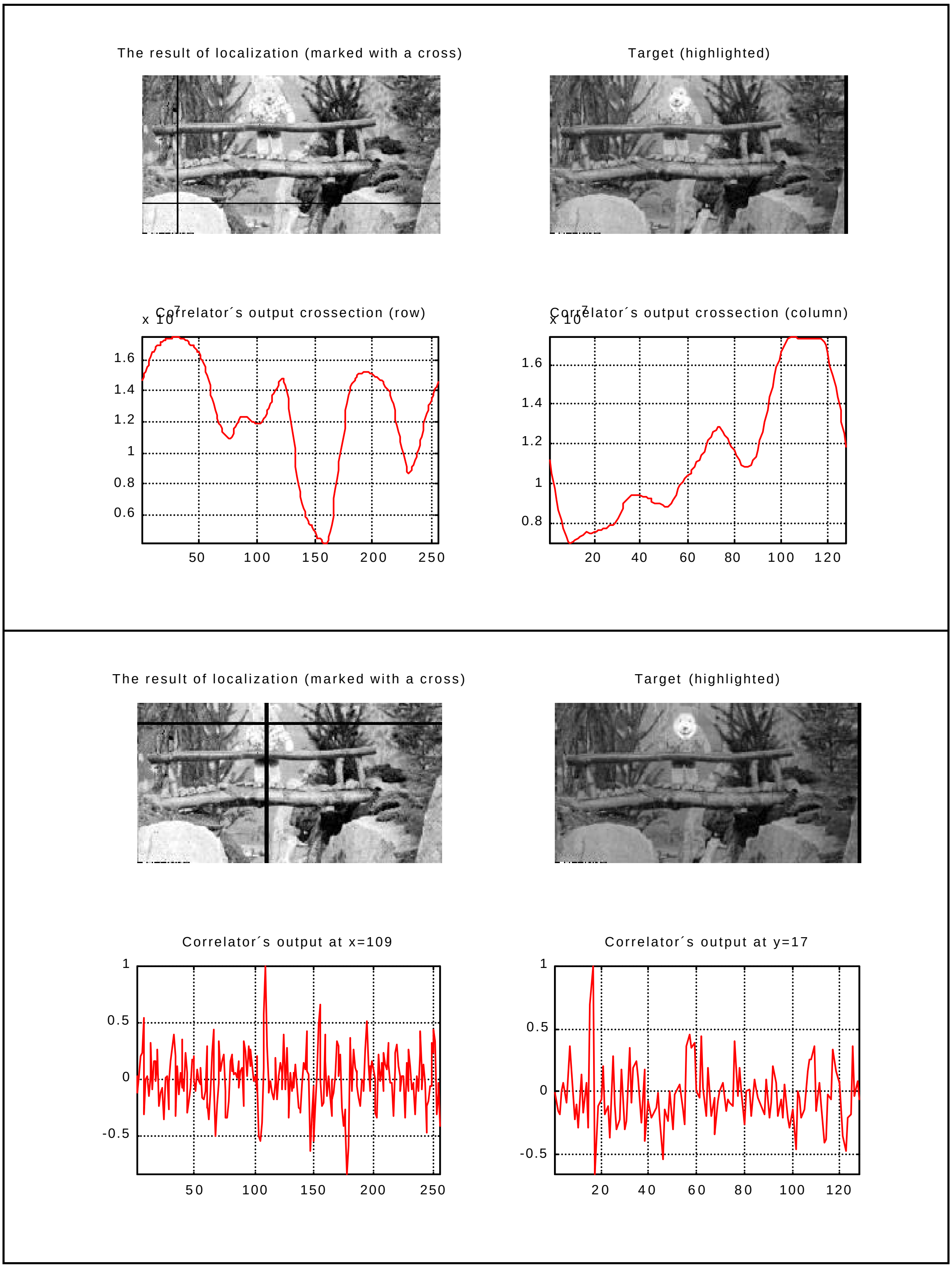
(L.P. Yaroslavsky, Optical correlators with (-k)th law nonlinearity: optimal and suboptimal solutions, Applied Optics, v. 34, No. 20 (10 July, 1995), pp. 3924-3932)

4-F nonlinear optical correlator



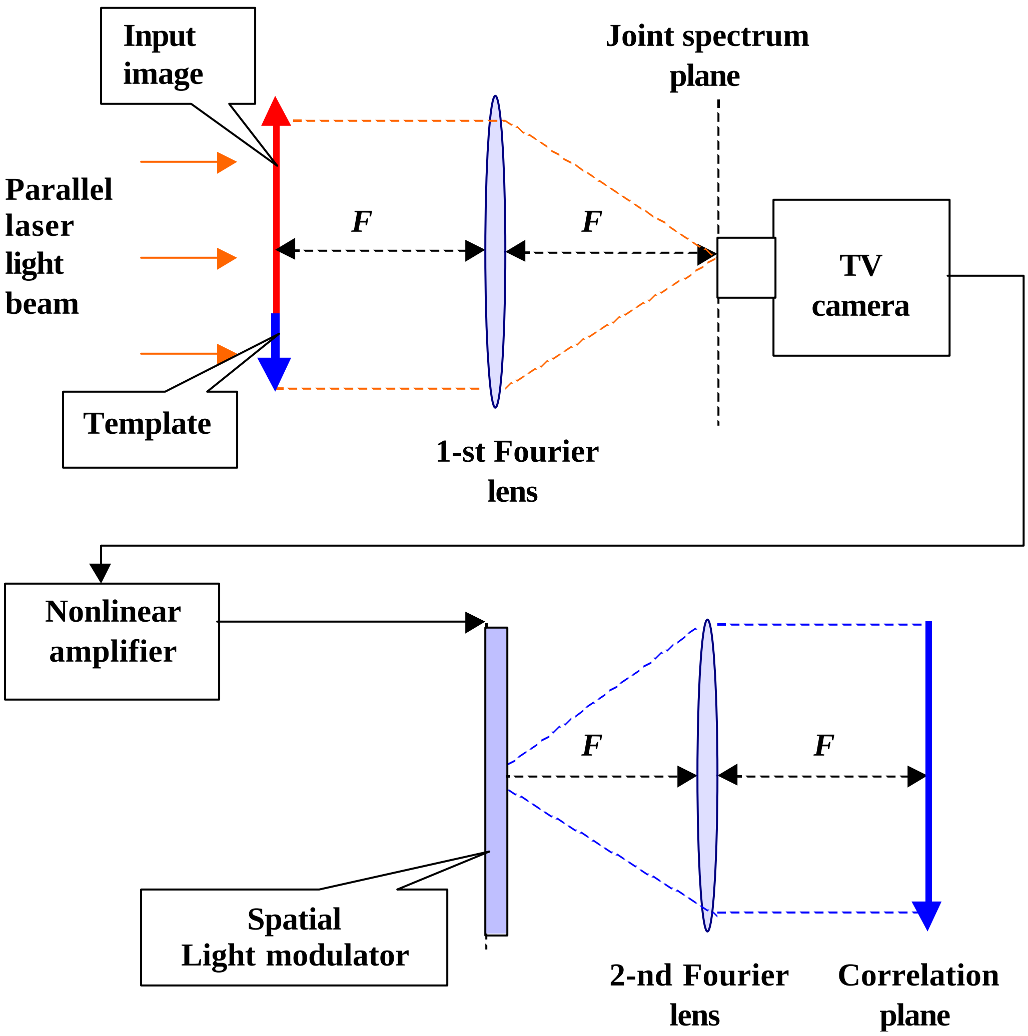
Nonlinear optical correlator with a parabolic mirror:





Comparison of the matched (upper) and optimal adaptive filter (bottom) correlators for localization of corresponding points in stereoscopic images

Electro-optical Joint Transform Nonlinear Correlator



Logarithmic amplifier:

$$OUT = \ln\left(\left|ImgSp + TrgtSp\right|^2\right) =$$

$$\ln\left(\left|ImgSp\right|^2 + \left|TrgtSp\right|^2 + ImgSp^* \cdot TrgtSp + ImgSp \cdot TrgtSp^*\right) =$$

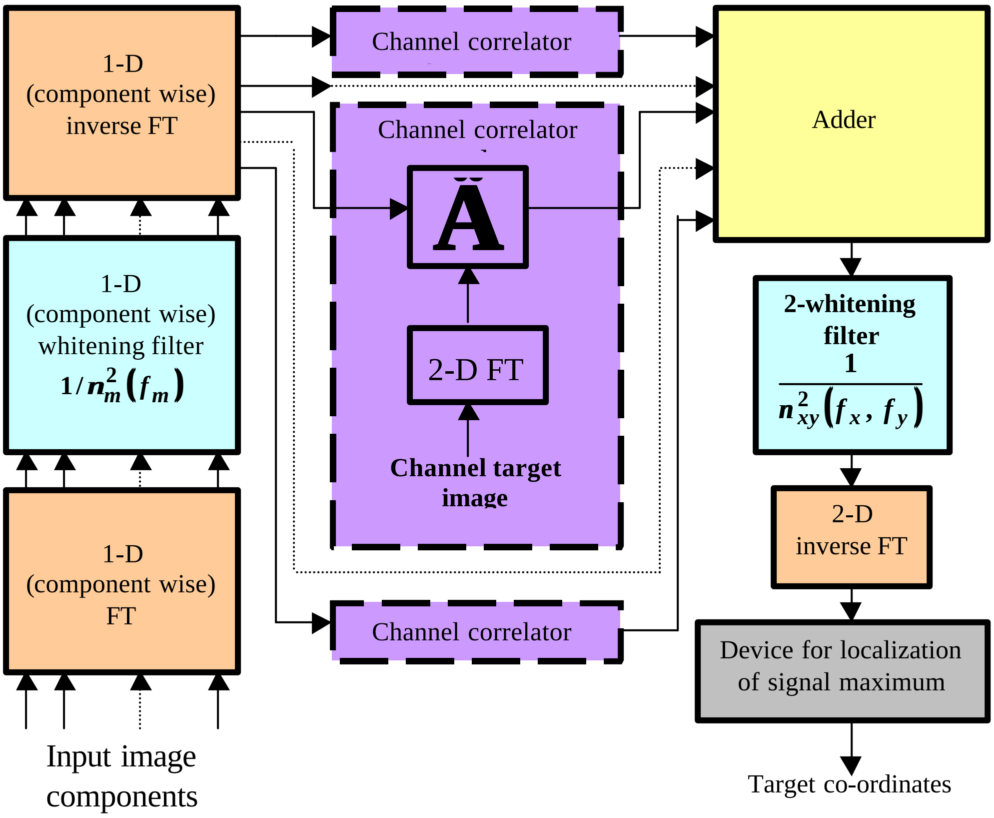
$$\ln\left(\left|ImgSp\right|^2\left[1 + \frac{\left|TrgtSp\right|^2}{\left|ImgSp\right|^2} + \frac{ImgSp^* \cdot TrgtSp}{\left|ImgSp\right|^2} + \frac{ImgSp \cdot TrgtSp^*}{\left|ImgSp\right|^2}\right]\right) \cong$$

$$\ln\left(\left|ImgSp\right|^2\right) + \frac{\left|TrgtSp\right|^2}{\left|ImgSp\right|^2} + \frac{ImgSp^* \cdot TrgtSp}{\left|ImgSp\right|^2} + \frac{ImgSp \cdot TrgtSp^*}{\left|ImgSp\right|^2}$$

Useful output

Optical adaptive correlators for color images

L. Yaroslavsky, Optimal target location in color and multi component images, Asian Journal of Physics , Vol 8, No 3 (1999) 100-113



Multi channel correlator with separated component wise and spatial wise whitening.

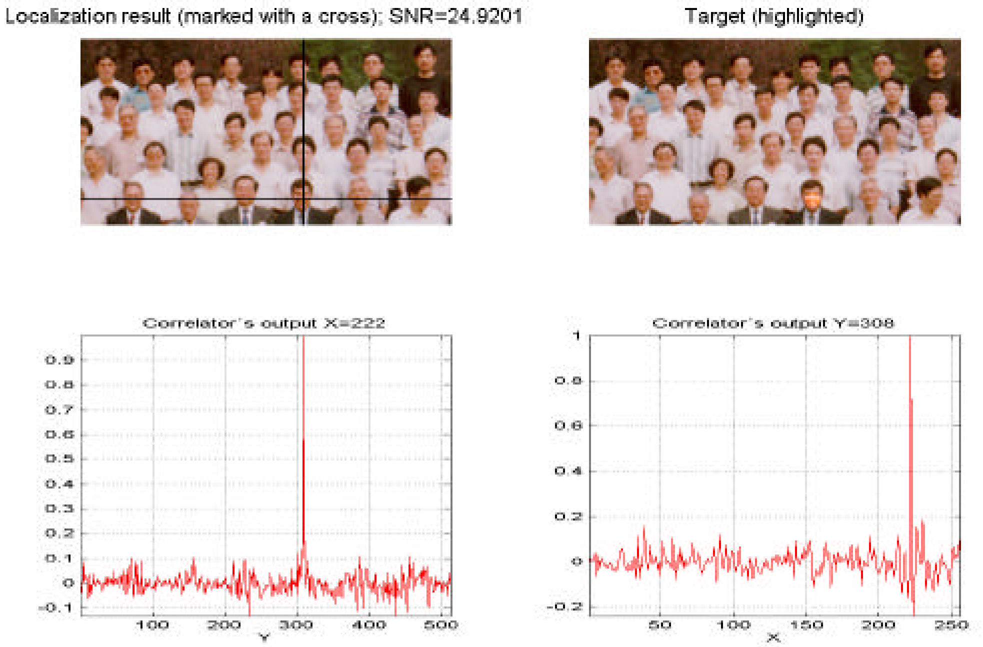


Fig. 8 a. Localization of a target in color image: separable optimal correlator (images are printed in black/white)

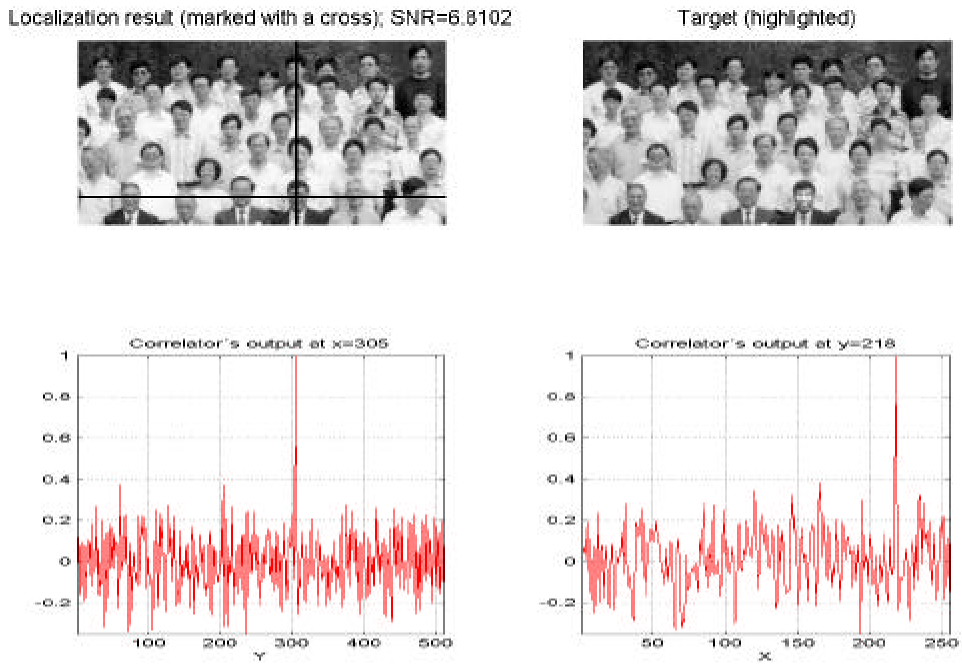


Fig. 8 b. Localization of a target in single component image

Test questions

1. Formulate Huygens's principle of light wave propagation.
2. Derive the equation of wave propagation in free space. Explain "near" zone and "far" zone approximations.
3. Prove that lens is a Fourier Transformer.
4. Prove that lense is a chirp-modulator.
5. Describe the structure and capabilities of coherent optical systems for image processing.
6. Describe coherent optical correlators.
7. What are adaptive nonlinear optical correlators?