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Not all Nearest Neighbors are Equal in Bounded Distance Decoding

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Abstract — **Bounded distance soft decoders guarantee correct decoding at least up to half the minimum Euclidean distance of a code, $\frac{1}{2}d_o$. Under practical working conditions, performance degradation - as compared to optimal decoding - is dominated by the behavior of the decoder outside the (bounded distance) hyper-spheres whose centers are the codewords. We carefully investigate this issue and reveal an interesting phenomenon: there are three different types of nearest neighbors classified according to their affect on the decision region. Simulation results are presented to demonstrate this phenomena.**

I. Introduction

For a given codeword, the geometrical interpretation of bounded distance decoding is simply a hyper-sphere of radius $\frac{1}{2}d_o$, whose center is the codeword. Clearly, the spheres around the codewords may touch but never intersect; it does not mean, however, that bounded distance algorithms necessarily fail to decode outside these spheres. In fact, in practical working conditions (rather than asymptotically), there is a lot to gain by correctly decoding beyond the spheres. Thus, the performance of two bounded distance decoders may greatly vary. Clearly, the decision regions of a bounded distance algorithm[1] are most important as they determine the performance of the algorithm. While the decision regions of optimal decoding algorithms, known as *Voronoi regions*, have been extensively studied (see for instance [1, 2, 3], and the references therein), little is known about the decision regions of suboptimal algorithms.

In [7] we present bounded distance decoding algorithms for a family of linear *q-ary* block codes, with various performance/complexity tradeoffs. The complexity of the proposed algorithms is low, both in terms of real-number operations and in terms of algebraic operations. Moreover, unlike some of the most eminent suboptimal algorithms [4, 5] and their successors, e.g. [6], our algorithms involve no algebraic decoding.

II. Classification of Nearest Neighbors, Decision Regions and Performance Analysis

The most common analytical way to estimate the probability of error of a bounded distance algorithm is to identify and count the overall number of nearest neighbors and then employ some variation of the *union bound.* The overall number of nearest neighbors, known as the *effective error coefficient*, is given by $N_{o,eff} = N_o + N_{BDD}$, where N_o denotes the number of **codewords** at distance d_o, and N_{BDD} denotes the number of **non-codeword** points at distance d_o that are generated in the decoding process. Note that it is inherently assumed that all nearest neighbors affect the decision region in a similar manner. We investigate the decision regions of several bounded distance algorithms [7], and reveal some interesting phenomena:

- **1.** in addition to the $N_{o,eff}$ nearest neighbors, there is another cause for errors at Euclidean distance $\frac{1}{2}d_o$. In this case, rather than a hyper-plane, one segment from the border of the decision region coincides with a fragment of the bounded distance hyper-sphere itself. We associate this phenomenon with a new type of nearest neighbors and refer to them as *pseudo nearest neighbors.*
- **2.** a nearest neighbor generated in the decoding process affects the decision region (and thus the error probability) less than a nearest neighbor that is a codeword.
- **3.** distinct algorithms may have identical nearest neighbors, yet the same neighbor (that is not a codeword) can affect the decision region of each algorithm differently.

Consequently, the number of nearest neighbors of a bounded distance algorithm (as used with the union bound) can be a very misleading measure for performance. In fact, as suggested by the second phenomenon, it is possible for a bounded distance algorithm, which considerably increases the number of nearest neighbors, to perform very close to the optimum even for high noise levels. We explicitly identify and enumerate the different types of neighbors, and discuss their affect on the decision regions and consequently on the probability of error.

Additionaly, we evaluate the performance of the decoders of [7] by computer simulations for several interesting yet simple codes. The simulation results support our analytical derivations. A comparison by simulations to GMD decoding [4, 6] is also given. Finally, we present some interesting images that vividly exhibit the different phenomena described above.

[1] To be precise, the decision regions of a *decoding algorithm* $\mathcal{D}$, are actually the decision regions of the *code* C, obtained when decoded by $\mathcal{D}$. The shape of the decision regions, however, are typical to the decoding algorithm.

References

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