Bounded-Distance Soft Decision Decoding of Binary Product Codes

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Abstract — **A two-step sub-optimal algorithm for decoding binary product codes is discussed. This algorithm realizes at least half the minimum Euclidean distance of the code. The fundamental geometric properties associated with the algorithm are investigated, and bounds on the number of nearest neighbors are derived. This investigation also results with an improved algorithm which achieves the minimum effective error coefficient, the number of minimum-weight codewords in the product code.**

I. Introduction

A product code $\mathcal{C}_p = \mathcal{C}_r \times \mathcal{C}_c$ contains all the matrices whose columns are codewords in the code $\mathcal{C}_c$ and the rows are codewords in $\mathcal{C}_r$. The parameters of the product code are given by $[n_p, k_p, d_p] = [n_r n_c,\ k_r k_c,\ d_r d_c]$, where n denotes the length, k, the dimension, and d, the minimum Hamming distance of the corresponding code.

Product (iterated) codes were introduced by Elias in 1954 [3], and studied by many researchers until the late 70's. Several hard decision decoding techniques were proposed at that time for decoding a product code up to its guaranteed error-correction capability. Reddy and Robinson [6] gave a general decoder for any product code, with good correction capabilities for simultaneous burst and random errors. Yu and Costello [8] proposed a generalized minimum distance decoder for Q-ary output channels. In 1993 product codes gained renewed attention with the soft decision decoder of Lodge *et al.* [5], and the birth of turbo (iterative) decoding. While Lodge *et al.* [5] used the *a posteriori* probability as the reliability-measure for each bit, others, e.g. [7], employed suboptimal reliability measures that are less computationally involved.

II. Decoding

The proposed decoding technique [2] is not an iterative one, nor does it require explicit reliability-measure calculations for each bit. Rather, it is a suboptimal soft decision decoding scheme, more in the line of the aforementioned work [6], [8], operating as follows. Each of the component codes is soft-decision decoded separately, rows (columns) and then columns (rows), while passing a simple, hard-limited, reliability measure from the rows (columns) to the columns (rows). The result of the columns (rows) decoders is taken as the output. Generally speaking, while turbo decoding reduces the probability of bit error, the proposed technique is aimed at reducing the probability of codeword error.

III. Analysis and conclusions

We prove [2] that if the decoders of the component codes realize half the minimum Euclidean distance of these codes, then the complete decoding scheme realizes half the minimum Euclidean distance of the product code. Such a scheme is known as bounded distance (BD) decoding. An analysis of the decision region associated with this decoding scheme is given, revealing the following phenomena: **i)** regardless of the specific choice of a BD decoder used for decoding the component codes, the complete decoding scheme is always better than strictly BD decoding; **ii)** The algorithm contains *pseudo nearest neighbors* [1]. Based on the above analysis, an upper bound on the number of conventional nearest neighbors, i.e. the effective error coefficient, is derived. Furthermore, it is shown that the minimum effective error coefficient is achievable, as in the case of optimal decoding, by using a slightly modified decoding scheme.

The proposed decoding algorithms may be attractive for practical implementation due to their low decoding complexities. Decoding involves an order of $n_r + n_c$ applications of a component code decoder. For comparison, a single iteration of a block turbo-decoding scheme requires an order of $O(n_r n_c)$ such applications. Also, due to their geometrical properties, the algorithms can be employed as stopping-criteria (within the framework of block turbo-decoding) for terminating the iterative process. Since these algorithms aim at reducing the probability of word error, they are good candidates for the decoding of coset product codes [4].

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