Iterative Decoding of Product Block Codes based on the Least Mean Square Error Criterion

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Abstract — **A soft-input soft-output decoder based on the least mean square error (LMSE) criterion is presented and employed for iterative decoding of product codes. The *residual estimation error* is derived and used for analyzing the convergence of the iterative process as well as a stopping criterion. A relation between the LMSE and the MAP decoders is also established. Simulation results are presented for several codes revealing near-optimal performance.**

I. Introduction

Turbo codes, as introduced in the celebrated paper of Berrou et al [1], exhibit excellent performance, just a few tenths of decibels from the channel capacity. The heart of a turbo decoder is the so-called soft-input soft-output (SISO) decoder. Product codes were introduced in 1954 by P. Elias [2]. A (two-dimensional) product code can be defined as the set of all the arrays whose rows are codewords of one code and whose columns are codewords of (possibly) another code. Owing to their structure, product codes can be 'turbo decoded' by iteratively operating on the rows and the columns using some kind of a SISO MAP decoder [4]. In this work, a different type of SISO algorithm is proposed for iterative decoding of product codes [3]. This decoder is based on the Least Mean Squared Error criterion. The LMSE estimator minimizes the mean square distance between the estimated word and all the code words. Unlike other criteria (*e.g.* Maximum Likelihood or Maximum *a posteriori*), averaging is inherent to the LMSE estimation process. The primary motivation for considering the LMSE estimator as a SISO decoder is due to the fact that estimation theory, being equipped with an arsenal of analysis tools, may be used for studying the convergence mechanism of the iterative decoder.

II. Iterative decoding using LMSE estimation

Let $\underline{c}$ be a codeword in a block code of length n and let $\underline{c}$ be transmitted over an additive white Gaussian noise channel. The vector $\underline{r} = \underline{c} + \underline{\mu}$ is observed at the output of the channel, where $\underline{\mu}$ is a Gaussian noise vector. LMSE decoding amounts to estimating the vector $\hat{\underline{v}}$ that minimizes the mean square error $\frac{1}{n}E\{(\underline{v}-\underline{c})(\underline{v}-\underline{c})^T\}$. $\hat{\underline{v}}$ is given by $\hat{\underline{v}} = E\{\underline{c}|\underline{r}\}$. We take the elements of $\hat{\underline{v}}$, $\hat{v}_1, \hat{v}_2, ..., \hat{v}_n$, as the soft-outputs of the LMSE decoder, while the signs of these elements are taken as the decoded (hard) outputs. Associated with $\hat{\underline{v}}$ is an error estimation referred herein as the *residual estimation error* (REE):

$$e^2 = \frac{1}{n}E_{\underline{r}}\{E\{\underline{c}\cdot\underline{c}^T|\underline{r}\} - E\{\underline{c}|\underline{r}\}\cdot E\{\underline{c}|\underline{r}\}^T\} =$$

$$1 - \frac{1}{n}E_{\underline{r}}\{\hat{\underline{v}}\cdot\hat{\underline{v}}^T\} \quad (1)$$

The LMSE-based iterative decoder is similar to a conventional iterative decoder, except that the MAP algorithm used for decoding the component codes is replaced with the above LMSE decoder. The REE may be computed after each iteration and employed as a criterion for terminating the iterative decoding process.

III. The LMSE and the MAP decoders

Both the MAP and the LMSE SISO decoders yield excellent performance when employed for iterative decoding. Let $x_k = P(\underline{r}|c_k = 1)/P(\underline{r}|c_k = 0)$ denote the likelihood ratio associated with the bit in position k. For MAP decoding, the soft-output associated with that bit is given by $m_k = log(x_k)$. It is shown that the output of the LMSE decoder is given by $v_k = (x_k - 1)/(x_k + 1)$, and so the following relation between MAP and LMSE decoding is established

$$m_k = log\frac{(1+v_k)}{(1-v_k)}.$$

This relation suggests that both algorithms may exhibit similar behavior, and therefore the understanding of one can reflect on the other. Convergence analysis of the LMSE-based iterative decoder has been performed assuming that the row and column decoders can be treated independently, and while considering full, rather than extrinsic, information exchange. When the iterative process converges, the REE (1) typically approaches a constant value. It is shown that the REE decreases after each iteration if the soft-input is in the vicinity of a codeword. Explicit expression is given for the guaranteed radius of convergence about the codewords.

IV. Simulation results

Comprehensive computer simulations of the proposed LMSE approach and a conventional turbo decoder have been performed for several product codes. For comparison, non-iterative maximum-likelihood (ML) decoding has been simulated, or otherwise upper bounded, depending on the complexity of the product code. The results show that the performance of the iterative LMSE and MAP decoders are comparable. Furthermore, for the selected codes, the iterative LMSE approach is close to realizing the ML performance.

References

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