Quantifying Operational Synergies in a Merger/Acquisition

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ABSTRACT

Merger and acquisition activity has increased sharply in the last decade. While the shareholders of the target firms appear to reap economic benefits from mergers, those of the bidder firms often do not fare well. In such an environment, it seems useful to have models that can help senior managers of bidder firms make informed decisions about the amount of premium, over target's share prices prevailing prior to merger announcement, that can be justified on the basis of *operational synergies*. The goal of this article is to capture important parameters from the production side that have a bearing on the valuation of target's shares. We show that the production characteristics of both the bidder and the target matter in a significant way. For example, if the bidder and target operate in independent markets, bidder has flexible production facilities, but target's production facilities are inflexible, then an increase in the bidder's demand can make the target less attractive, and lower the value of operational synergy.

Keywords: Mergers and Acquisitions,
Production and Finance
Newsvendor Problem, Capacity Management.

1 Introduction

Merger and acquisition (M&A) activity has increased significantly since the early 1990's. The worldwide dollar value of M&A[1] in 1998 was 50% higher than in 1997, and more than twice as much as in 1996 (Anonymous, 1999a). More specifically, there were approximately 26,200 mergers worldwide in 1998, involving companies worth nearly 2.4 trillion dollars. This is up from 11,300 mergers worth approximately half a trillion dollars in 1990 (Anonymous, 1999b) In addition to top-managers' ego and hubris, which are often cited as reasons for increased M&A (Anonymous, 1999b) several economic arguments can be made in favor of M&A. These include increased efficiency, operating synergy, diversification, financial synergy, strategic realignment, and the fact that target firms with low q-ratios[2] provide an economical way of gaining market entry for the bidder firm (see, for example, Hirshleifer (1993, 1995), and pages 75-79 in Weston et al. (1998) for a detailed description of these reasons).

M&A is widely believed to be a *net value increasing activity.* This is based on empirical evidence that the value of the combined firm is typically larger than the sum of the values of each individual firm. A number of articles have studied and measured the benefit of M&A and found that while the shareholders of both the target and the bidder firms experience positive returns, the gains of the bidder's shareholders are not significantly different from zero. Some examples include Jarrell, Brickley and Netter (1988) who studied 663 successful tender offers from 1962 to 1985, and Bradley, Desai and Kim (1988) who covered 236 successful tender offer contests from 1963 through 1984. Later work of Schwert (1996), and Howe and Wo (1994) observed similar patterns as well as increasing returns to target firm's shareholders in recent years (see also Hirshleifer, 1995). Similarly, Jensen and Ruback (1983) find that the overall excess return to bidder firms is positive (albeit small).

Of the many regulatory, economic and organizational factors that determine whether or not a merger is an economic success, two of the most important are proper valuation of the target firm, and reconciling organizational differences between the target and acquiring firms after the merger takes place (Anonymous, 1999b). This article is concerned with the former. More specifically, it

[1] Our use of the term M&A relates to all mergers and tender offers, whether friendly or hostile, contested or single bidder. Furthermore, we also use the term mergers to refer to all M&A activities.

[2] The q-ratio is the ratio of the market value of a firm's securities to the replacement cost of its assets. When q-ratio is low, acquisition may be a cheaper means of adding capacity and gaining entry to new markets, as opposed to building new capacity from scratch.

develops simple models that a senior manager of a firm contemplating a bid[3] can use to quantify the effect of production and demand characteristics of the target and her own firm. Such characteristics include production capacity, manufacturing flexibility, and demand correlation and volatility. Thus, our point of view is that of a bidder firm and the focus is on valuation, rather than organizational, regulatory or technical (such as compatibility of information systems) issues. Furthermore, we are interested not in valuation for legal purposes, such as may be necessary in case of a legal challenge to the tender offer, but rather valuation for the purpose of helping would-be bidders gain a better understanding of key drivers of *operational synergy*. This, in turn, will help them determine how much premium (over prevailing market price prior to tender announcement) can be justified on the basis of improved production efficiency. Note that our goal is consistent with observations made by Hirshleifer (1993) who listed scale economies and complementarities as major value-improving motives for M&A's.

The corporate finance literature presents several approaches to valuation of target firms. As explained in Chapter 9 of Weston et al. (1998), these methods fall broadly into two categories: the comparable companies or transactions approach, and the discounted cash flow (DCF) approach. In addition, since the synergistic benefits that we wish to quantify arise as a result of the ability of the merged entity to exploit operational flexibility, recent models that use *real options* to evaluate benefits of flexibility in capital budgeting and resource allocation decisions are also relevant. We are not aware of a direct application of this approach to M&A (see Trigeorgis, 1999, and Dixit and Pindyck, 1994, for a review of literature on the real options approach), although our method can be viewed as a simple real option model in which we find the value of the option to use excess capacity of one firm to fill the other firm's excess demand.

The DCF approach could be performed either using spreadsheets (involving numerical estimation of demand, growth over time, tax considerations, etc.) or the formulae approach (based on an assumed pattern of growth, tax rates, discount rates, and other time-sensitive factors). Under the comparable companies or transactions approach, key relationships (e.g., ratio of market value to book value, price to earnings ratio, and the ratio of market value to sales revenue) are estimated from a group of similar companies or a group of similar transactions using historical data. Usually more than one ratio is estimated and market value of the target firm is taken to be the average

[3]In the remainder of this article a "firm contemplating a bid" is simply called the "bidder" for sake of convenience. It should be noted that "bidder" in our terminology may thus not actually bid, if bidding is not worth its while.

value that results upon applying these ratios.

To the best of our knowledge, DCF based valuation methods calculate a target firm's market value independent of who is acquiring it. Thus, by default, they do not account for operating synergies, or lack thereof. Theoretically, it is possible to include synergistic benefits by affecting cash flows in a spreadsheet based DCF model. However, this can prove to be tedious and inappropriate for developing managerial insights. The goal of this article is to bridge this gap by providing simple valuation models, similar in spirit to the DCF formula approach, that explicitly account for operating synergies. These models expose important relationships between the value of the target firm and factors such as the amount of excess capacity of the target and acquiring firms, volatility of target and acquiring firm's demand, the degree of demand correlation, and the degree of flexibility of the production capacities of the target and acquiring firms.

As noted in Trigeorgis (1999), pp. 18-21, it is often difficult to evaluate complex real options in practice since real life examples involve multiple interdependent options. Numerical techniques are possible, but require either the specification of underlying stochastic processes, or approximating the underlying differential equations. In contrast, our model, which is a simple real option model, offers analytical insights even though the value perceived by the bidder is a complex function of both the need and the ability of each entity to produce the other's product line.

We use the newsvendor model framework (see Porteus, 1990 for a review) to quantify operating synergies. This framework is applied repeatedly while no inventory is maintained from one period to the next. Revenues realized are derived from the sale of what is produced in each period, i.e., the firm's production either equals total capacity or total demand, whichever is smaller. This formulation is easy to analyze and consistent with our goal of obtaining managerial insights. Also, such a framework should indeed reflect a firm's modus operandi when averaged over a long period of time. In other words, if each unit sold earns a net positive contribution, the expected profit maximizing manufacturer should on average produce either at its peak capacity (when demand exceeds this capacity), or what is demanded (if capacity exceeds demand). We do, however, discount to the present all future cash flows derived from the repeated newsvendor setting.

Although motivated by the need to quantify synergistic benefits from M&A activity, our framework and mathematical model are in fact more general. They apply to all situations in which managers need to evaluate investment in new product lines and capacity, irrespective of whether it arises through M&A or developmental activity. Similarly, after suitable modifications, our model could be used to capture synergies that arise for reasons other than operational flexibility. Essentially, our approach is applicable where ever synergies depend on the state-dependent availability of resources of one entity and the state-dependent need for these resources by another entity. Finally, the labels bidder and target are somewhat arbitrary in the broader context of our model and either firm could be considered a bidder. The terminology is used largely to identify the two firms by different labels, and to be consistent with terminology used in the M&A literature.

The main analytical results obtained in this article are summarized in an easy-to-read tabular format in section 4. It shows that the size of bidder's capacity, demand volume, and volatility affect its valuation of the target in a significant and non-monotonic way. Changing any one of these three characteristics can cause either an increase or a decrease in target's value depending on the values of other parameters. Since popular valuation methods do not explicitly model bidder firm's operational characteristics, our model can be used to make senior managers aware of the importance of these issues.

2 Market Value of the Target Before Tender Announcement

Consider a target firm that makes items belonging to a single product family. The product has perpetual instantaneous demand denoted by the random variable X which takes values in the interval $[0, \infty)$. The firm possesses production capacity (assets) Q with a market value of $C(Q)$. Since its product is an unstorable service or a perishable commodity, the firm maintains no inventory from one period to the next. Thus, the firm can sell products at a rate which is the smaller of its production rate and realized demand rate. It earns a fixed contribution margin p per unit sold. Denote demand's distribution by F, and let $\bar{F} = 1 - F$. The discount rate (or cost of capital) is assumed fixed and it is denoted by r. In this article we use the terms *increasing* and *decreasing* to mean non-decreasing and non-increasing respectively. Stronger relationships are recognized as strict.

A bidder firm that wishes to purchase this facility and product line pays an amount $P(Q)$ up front, i.e., prior to demand realization. It will experience either underage or overage capacity penalties once the true demand is known. We assume that the underage cost is simply the foregone revenue whereas the overage cost is counted in what the acquiring firm pays for the target firm.

Incremental production costs are ignored, as the focus is on valuation of production capacity. Clearly, under equilibrium conditions, $P(Q)$ is the expected NPV of all future cash flows[4], which can then be written as:

$$P(Q) = E\left[\int_{t=0}^{\infty} p \cdot \min\{X, Q\}\, e^{-rt} dt\right]. \tag{1}$$

Equation 1 can be simplified to obtain the following equivalent version (proved in Appendix A):

$$P(Q) = \frac{p}{r}\int_{0}^{Q} \bar{F}(x)dx. \tag{2}$$

Thus, a bidder who views the opportunity to purchase the target firm as an independent investment opportunity, without regard to synergies with its own product lines and assets (capacity size, manufacturing flexibility, demand, etc.), will pay up to the amount $P(Q)$ for the firm's assets. If an independent valuation is performed ignoring any future tender offer announcements, $P(Q)$ will also be the market value of the target firm. Put differently, under the model assumptions proposed here, an independent valuation is equal to the target's market value prior to tender announcement, i.e., $C(Q) = P(Q)$. Henceforth, we shall only retain the notation $C(Q)$ to denote both the expected NPV of the target's cash flows as well as its market value prior to tender announcement. Any increase in target's price, which comes from the supply and demand tension, and the fact that the acquiring firm has a limited time window within which to acquire a controlling interest in target's shares, must be justifiable on the basis of operating and other synergies that the merger is expected to bring. Before attempting to develop a model for that purpose, we first investigate certain properties of $C(Q)$. In order to facilitate that discussion and to provide a common basis of comparison to readers with different backgrounds, we begin with a brief review of some equivalent notions of stochastic orders relations used in operations research and economics literature.

If A and B are random variables and $E[f(A)] \leq E[f(B)]$ for all non–decreasing functions f, then $A \leq_{st} B$, i.e., A is smaller than B in the usual stochastic order (see, for example, Shaked and Shanthikumar (1994), pp. 3-4.). The usual stochastic order can be shown to be equivalent to the intuitive condition that $P(A \leq x) \geq P(B \leq x)$ for all x; that is, A is more likely to be small than B. It is customary in the economics literature to call it *first-order stochastic dominance* (FSD) (see Fishburn and Vickson (1978) for details).

[4] It is possible for this firm to manufacture another compatible product, say via subcontracting its excess capacity, and thereby affect its cash flows. However, such arrangements involve fixed transaction costs and are the exception rather than the norm. In this article, we do not consider such possibilities.

Similarly, if $E[f(A)] \leq E[f(B)]$ for all convex functions f, then $A \leq_{\text{cx}} B$, i.e., A is stochastically smaller in *convex order*. Note that functions f_1 and f_2 defined by $f_1(a) = a$, and $f_2(a) = -a$ are both convex functions of a. Thus, $E[f_1(A)] \leq E[f_1(B)] \Rightarrow E[A] \leq E[B]$, and $E[f_2(A)] \leq E[f_2(B)] \Rightarrow -E[A] \leq -E[B]$. That is, if $A \leq_{\text{cx}} B$, then $E(A) = E(B)$. Similarly, since $f_3(a) = a^2$ is also a convex function of a, it follows that $A \leq_{\text{cx}} B \Rightarrow \text{Var}(A) \leq \text{Var}(B)$. Thus, convex order is a type of variability order. A *concave order* is the exact opposite of the convex order described above, i.e., if $A \leq_{\text{cx}} B$, then this is equivalent to $A \geq_{\text{cv}} B$ (see Shaked and Shanthikumar (1994), pp. 55-56, for details and other examples of variability orders).

The *second-order stochastic dominance* (SSD) is a common notion of variability ordering used in economics literature. If X is smaller than Y in SSD order, then it means that X is both smaller and more variable than Y. Formally, $X \leq_{\text{SSD}} Y \Rightarrow E(X) \leq E(Y)$ and $\text{Var}(X) \geq \text{Var}(Y)$. A necessary and sufficient condition for $X \leq_{\text{SSD}} Y$ to hold is that $\int_{-\infty}^{a} F_X(x)dx \geq \int_{-\infty}^{a} F_Y(y)dy$, for all a (see, for example, Fishburn and Vickson, 1978). SSD is equivalent to the an *increasing concave order* (Shaked and Shanthikumar (1994), pp. 85, Theorem 3.A.1), and it is related to the convex order as explained next. An increasing concave order, $A \leq_{\text{icv}} B$, is obtained if $E[f(A)] \leq E[f(B)]$ for all increasing concave functions f. If the random variables being compared also have the same means, then the increasing concave order and the SSD order are both equivalent to the concave order, and the exact opposite of the convex order. Since the set of all increasing functions includes the set of all increasing concave functions, it is easy to see that FSD implies SSD, but that the opposite is not necessarily true. Similarly, since the set of all concave functions includes the set of increasing concave functions, a concave order implies SSD, but the opposite is not necessarily true. In short, SSD is the weakest of the various orders discussed above and that explains our interest in it.

Proposition 1 *The target firm's market value, prior to tender announcement (or, equivalently, in an independent valuation), is*

1. *proportional to p/r,*
2. *increasing in Q, but at a decreasing rate,*
3. *increases when X is larger in the sense of first-degree stochastic dominance,*
4. *increases when X is larger in the sense of second-degree stochastic dominance.*

A proof of these claims is presented in Appendix A. We see that these observations are consistent with intuition, that is, larger contribution margin, lower capital costs, larger demand, and lower demand variability (with the same or higher mean demand) increase value.

In many M&A's, the market price of the target firm's shares starts to rise as soon as an impending tender becomes public knowledge. The bidder then pays a substantial premium over the market price that prevailed prior to the tender offer announcement. A natural question that arises in this situation is how much additional investment can be justified by the bidder firm on the basis of operational synergies that might accrue from the merger. That issue is studied in the remaining sections.

3 A Model for Quantifying Operational Synergies

In order to differentiate the bidder's assets and demand pattern from those of the target, we shall use index 1 to denote the bidder and index 2 to denote the target. Thus, after a successful merger, the buyer "inherits" the demand for product 2 with distribution F_2 and acquires an additional capacity Q_2. Let $C(Q_1, Q_2)$ denote the maximum amount that a bidder with existing capacity Q_1 should be willing to pay for the target after accounting for operational synergies, whereas $C_2(Q_2)$, as obtained in equation 1 of the previous section, denotes the target's market value prior to the tender announcement, and $\Delta(Q_1, Q_2) = C(Q_1, Q_2) - C(Q_2)$ is the net benefit from operational synergies.

Suppose that 1 unit of type 1 capacity can be used to make $\nu_2 \geq 0$ units of product type 2. A similar parameter for facility 2 is denoted by ν_1. Clearly, when $\nu_1 = \nu_2 = 0$, there is no production flexibility. The per unit contribution margin from making a unit of product i on either facility is p_i. Any processing inefficiencies that might arise from using facility 2 for making product line 1, or vice versa, are accounted for in the parameter ν_i. In this setting, the merged entity needs to decide how to allocate capacity at the beginning of each planning period and immediately after observing demands for the two product types. Its decision problem can be formulated as a linear program, as shown in Appendix B. Due to the simplicity of its construction, the decision problem can be resolved relatively easily without actually having to solve a LP. Furthermore, if we assume that each facility is ideally suited to making its own product type, which translates to the conditions that $p_1 \geq p_2\nu_2$ and $p_2 \geq p_1\nu_1$, then each facility should make its own product type first and only allocate excess capacity, if any, to the other product type (see Appendix B for details). Other cases under which p_i's do not satisfy the above ordering are possible, though less likely, and they are discussed briefly in the next paragraph. Finally, ignoring switchover costs, the expected NPV of incremental future cash in-flows can be written as shown below.

$$C(Q_1,Q_2) = \begin{cases} C_2(Q_2) + E[\int_{t=0}^{\infty} p_2 \min\left(\max(0, X_2 - Q_2), \nu_2 \max(0, Q_1 - X_1)\right) e^{-rt} dt] \\ +E[\int_{t=0}^{\infty} p_1 \min\left(\max(0, X_1 - Q_1), \nu_1 \max(0, Q_2 - X_2)\right) e^{-rt} dt]. \end{cases} \tag{3}$$

This relationship is similar to expected profit functions derived in studies dealing with substitutable products (see, for example, Bassok, Anupindi, and Akella, 1999 and references therein), and in the evaluation of investments in flexible capacities (see, for example, Gupta, Gerchak and Buzacott, 1992, and Pennings and Natter, 2001). Furthermore, it simplifies as follows:

$$C(Q_1,Q_2) = \begin{cases} C_2(Q_2) + (p_2/r)\{\int_{x_1=0}^{Q_1} \int_{x_2=Q_2}^{\nu_2(Q_1-x_1)+Q_2} (x_2 - Q_2) g(x_1,x_2) dx_2 dx_1 + \\ \int_{x_1=0}^{Q_1} \int_{x_2=\nu_2(Q_1-x_1)+Q_2}^{\infty} \nu_2(Q_1 - x_1) g(x_1,x_2) dx_2 dx_1\} + \\ (p_1/r)\{\int_{x_2=0}^{Q_2} \int_{x_1=Q_1}^{\nu_1(Q_2-x_2)+Q_1} (x_1 - Q_1) g(x_1,x_2) dx_1 dx_2 + \\ \int_{x_2=0}^{Q_2} \int_{x_1=\nu_1(Q_2-x_2)+Q_1}^{\infty} \nu_1(Q_2 - x_2) g(x_1,x_2) dx_1 dx_2\} \end{cases} \tag{4}$$

In the above equation $g(\cdot,\cdot)$ denotes the PDF of the joint distribution of demands for the two product lines. From 4, it follows that $\Delta(Q_1,Q_2) = 0$ when $\nu_1 = \nu_2 = 0$, i.e., when there is no manufacturing flexibility. In this situation, there are no operational synergies. However, $\Delta(Q_1,Q_2) > 0$ so long as at least one of the ν_i's is strictly positive.

Three other cases concerning the relative magnitudes of p_i's are possible. If $p_1/p_2 \leq \min[1/\nu_1, \nu_2]$, then we will preferentially use facility 1 to make any excess demand for item 2, i.e., any demand that exceeds Q_2. Item 1 will be produced only when all demand for item 2 is satisfied and there is leftover capacity. Similarly, when $p_1/p_2 \geq \max[1/\nu_1, \nu_2]$, we will preferentially use facility 2 to make any excess demand for item 1, i.e., any demand that exceeds Q_1. Item 2 will be produced only when all demand for item 1 is satisfied and there is leftover capacity. In the third instance, $p_1/p_2 \in (1/\nu_1, \nu_2)$. Here it would be better to produce item 1 on facility 2 and item 2 on facility 1. This case is not realistic. We expect at least one of the two facilities to be more economical at making its own product lines. Also, this case is mathematically equivalent to the first case with capacity labels switched. In each of the two remaining cases, it is possible to write the objective function in a manner similar to equation 4. Details are omitted since case 1 is considered to be the most relevant from a practical standpoint. Thus, all subsequent analysis applies only to case 1 in which each facility is preferentially used to manufacture the product line that it was designed to make.

In the following two subsections we study how demand volume and volatility, and the size and flexibility of the two firms' production capacities affect the magnitude of C and Δ when $\nu_i > 0$ for at least one i. In particular, we show that although C is increasing in Q_2 and X_2, Δ is not monotone in these parameters. We also derive sufficient conditions under which C and Δ are monotonically increasing or decreasing in different characteristics of the target and acquiring firms.

The Effect of Production Capacity and Flexibility

Proposition 2 *Upon examining the sign of the derivatives of $C(Q_1, Q_2)$ and $\Delta(Q_1, Q_2)$ with respect to Q_1, Q_2, ν_1, and ν_2, we discover the following:*

1. *Changes in Q_2 affect C and Δ in a different fashion. Specifically, the maximum amount (C) that the bidder should be willing to pay for the target is increasing, at a decreasing rate, in the target firm's production capacity. However, the value of operational synergy (Δ) is not monotone (see item 2 for details).*

2. *Changes in Q_1 affect C and Δ in the same manner. Both C and Δ are not monotone in the size of the bidder firm's production capacity. In fact, we can identify the following special cases relating to how Q_2 affects Δ, and how Q_1 affect C and Δ:*

   (a) *Δ is decreasing in Q_2 if $\nu_1 = 0$, i.e., if the target's production capacity is inflexible. Analogously, C and Δ are decreasing in Q_1 if $\nu_2 = 0$, i.e., if the bidder's production capacity is inflexible.*

   (b) *Δ is decreasing in Q_2, and C and Δ are increasing in Q_1, if $X_1 \leq Q_1$ with probability 1, i.e., if the bidder has sufficient capacity relative to its demand, irrespective of target's demand level.*

   (c) *Δ is increasing in Q_2, and C and Δ are decreasing in Q_1, if $X_2 \leq Q_2$ with probability 1, i.e., if the target has ample capacity relative to its demand, irrespective of bidder's demand level.*

(d) Δ *is virtually unaffected by* Q_2*, and* C *and* Δ *are virtually unaffected by* Q_1*, when* $X_i < Q_i$ *with probability 1, both for* $i = 1$ *and* $i = 2$*.*

3. *The maximum that the bidder should be willing to pay for the target firm, and the net benefit from operational synergy, are increasing in both* ν_1 *and* ν_2*, albeit at a decreasing rate. A bidder with more flexible production facilities will find the same target more attractive, as compared to another bidder with less flexible facilities.*

A proof of the claims made above can be found in Appendix C. This appendix also obtains insights into the special case of independent demands. Note that the relationships mentioned above are free of any distributional assumptions concerning the demand for the two firms' product lines. It is also easy to confirm that both C and Δ decrease when cost of capital (r) increases.

Whereas the effect of Q_2 on C (item 1) and the effect of production flexibilities (item 3 in proposition 2) can be understood on an intuitive level, it is harder to relate to the observations made in item 2. We offer the following explanations for the four cases described in item 2 viz. a viz. the effect of Q_1 on C and Δ (a similar explanation can be constructed to explain the effect of Q_2 on Δ). When $\nu_2 = 0$, the bidder's excess capacity cannot be used to make excess demand for target's products. On the other hand, if $P(X_2 \leq Q_2) = 1$, then the target never has demand in excess of its capacity. In both these situations, larger production capacity of the bidder makes it less likely that its demand will exceed its capacity. Thus, it has less need for target's capacity, and target's value decreases. Similarly, when $P(X_1 \leq Q_1) = 1$, larger capacity at the bidder makes it more likely that it will have excess capacity to manufacture target's product when the latter has demand in excess of capacity. Thus, the value of acquiring target's demand is greater. Finally if $P(X_i \leq Q_i) = 1$, for each i, then neither entity's demand exceeds its demand and no synergistic benefits can be derived from having flexible capacities.

The Effect of Demand Characteristics

Next, we shall investigate the effect of the size of demand and its variability on the valuation of the target by the bidder firm. In some instances, we are able to derive meaningful results only when the two firms face independent demands. Numerical analysis is possible for specific demand distributions, e.g., bivariate normal distribution, which is discussed in the next section.

Denote the marginal PDF of product i demand by f_i, i.e., $f_i(x_i) = \int_{x_j} g(x_1, x_2)dx_j$, $i \neq j$.

We use the notation $g_{X_j}(x_j \mid x_i)$ for the conditional PDF of X_j, $j \neq i$, given that $X_i = x_i$. The underlying random variable is denoted by $X_{j|x_i}$, and its CDF by $G_{X_j}(\cdot \mid x_i)$, where $j \neq i$. Since our primary concern here are demand characteristics, we set $C_0 = rC$ and $\Delta_0 = r\Delta$, and then study the effect of changing X_i on C_0 instead of C, and on Δ_0 instead of Δ. Note that equation 4 simplifies as follows:

$$C_0(Q_1, Q_2) = \begin{cases} p_2 \int_0^{Q_2} [1 - F_2(x_2)] dx_2 + (p_1 \nu_1) \int_0^{Q_2} F_2(x_2) dx_2 \\ -p_1 \int_0^{Q_2} \int_{Q_1}^{Q_1 + \nu_1(Q_2 - x_2)} G_{X_1}(x_1 \mid_{x_2}) f_2(x_2) dx_1 dx_2 \\ +(p_2 \nu_2) \int_0^{Q_1} F_1(x_1) dx_1 - p_2 \int_0^{Q_1} \int_{Q_2}^{Q_2 + \nu_2(Q_1 - x_1)} G_{X_2}(x_2 \mid x_1) f_1(x_1) dx_2 dx_1. \end{cases} \tag{5}$$

We say that the target's conditional demand $X'_{2|x_1}$ is stochastically larger than $X_{2|x_1}$ for a given x_1, i.e., $X'_{2|x_1} \geq_{\text{st}} X_{2|x_1}$, iff $G'_{X'_2}(y \mid x_1) \leq G_{X_2}(y \mid x_1)$ for all y. It is easy to see that when the above ordering holds for all x_1, it implies that $X'_{2|x_1}$ dominates $X_{2|x_1}$ in the sense of first-degree stochastic dominance. Put differently, if $X'_{2|x_1} \geq_{\text{st}} X_{2|x_1}$ for all x_1, then $X'_2 \geq_{\text{st}} X_2$ (see, for example, Shaked and Shanthikumar, 1994, page 6). In the following treatment, stochastically increasing conditional demand of target's product implies that the ordering holds for all x_1.

A related concept, which is also used in the sequel, is that of *positive dependence through stochastic ordering*, or *PDS* in short. A random variable X_1 is PDS on another random variable X_2, if $X_{1|x_2}$ is stochastically increasing in x_2, for each x_2. Thus, if X_i is positively dependent on X_j, $i \neq j$, it means that $\left(\frac{\partial G_{X_i}(y|x_j)}{\partial x_j}\right) < 0$, for all y. We shall denote this property as $PDS(X_i \mid X_j)$. Positive dependence implies positive correlation, but the opposite is not necessarily true. However, positive correlation does imply positive dependence for several common distributions. For example, it is found to hold for multivariate normal distribution with positive pairwise correlations (see Block, Savits, and Shaked, 1985, for details). It should also be noted that positive dependence cannot hold when the random variables are negatively correlated.

Proposition 3 *When the bidder's demand is PDS on the target's demand, the target firm's value* (C) *is increasing in stochastically increasing* <u>*conditional*</u> *demand for its products. If the bidder and target demands have a bivariate normal distribution, then positive dependence is implied by positive correlation.*

We discuss the effect of the target's demand on the value of operational synergy (Δ) in Proposition 4 and present a formal proof of Proposition 3 in Appendix D. In what follows, we explain the result in Proposition 3 and its implications through informal arguments. If $X_{2|_{x_1}}$ increases for each x_1, and this in turn makes larger X_1 more likely, then larger $X_{2|_{x_1}}$ has the effect of increasing both the target's and the bidder's demands, which increases target's value, as expected. It needs to be stressed that having positive dependence is a sufficient, but not a necessary condition for value to be increasing. In general, the nature of dependence will also change as the conditional demand distribution of X_2 is varied. Our result holds so long as the dependence remains positive. For most practical situations, PDS might actually represent a strong condition. For example, the value is always increasing when demands are independent. However, what is equally interesting is the fact that it is possible to find examples (see Example 1, section 4) with negatively correlated demands where having larger conditional demand for the target could *lower* its overall value to the bidder.

Let us consider a specific example of positive and negative dependence of demand. Suppose a manufacturer of carbonated soft drinks (bidder) wishes to acquire a company that makes frozen fruit juice (target). One could argue that in this case their demands are positively dependent. Demand is affected by factors like the intensity and length of the summer season. On the other hand, if the same carbonated soft drink manufacturer wishes to acquire another carbonated soft drink manufacturer with a competing brand name, then the bidder and target are likely to have negatively correlated demands. In this situation, our model predicts the following. The higher the demand for frozen fruit juice, all other things being equal, the carbonated drink manufacturer should be willing to pay more for the frozen juice canning operations. However, the higher the demand for competing soft drink brand, its value to the bidder may actually be lower. Numerical examples illustrating such cases are presented in the next section.

Proposition 4 *The effect on Δ of increasing X_2, and the effect on C and Δ of increasing X_1 are, in general, not monotone. However, if the bidder firm's demand is positively dependent (PDS) on target's demand, and the target's capacity is inflexible ($\nu_1 = 0$), then Δ is increasing in stochastically increasing $X_{2|_{x_1}}$. Similarly, if target firm's demand is PDS on bidder's demand, and the bidder's capacity is inflexible ($\nu_2 = 0$), then C and Δ are increasing in stochastically increasing $X_{1|_{x_2}}$. In addition, the following sufficient conditions can be identified in the situation where target and bidder have independent demands.*

1. *Δ is increasing in X_2, and C and Δ are decreasing in X_1, if either $\nu_1 = 0$ (target has inflexible capacity) or $X_1 \leq Q_1$ with probability 1 (bidder has ample capacity to meet its demand), or*

both.

2. *Δ is decreasing in X_2, and C and Δ are increasing in X_1, if either $\nu_2 = 0$, (bidder has inflexible capacity) or $X_2 \leq Q_2$ with probability 1 (target has ample capacity to meet its demand), or both.*

A proof of proposition 4 is presented in Appendix E. Informally, the central observation in proposition 4 can be explained as follows by taking as an example how X_1 affects C and Δ. When $X_1 \mid_{x_2}$ becomes larger for each x_2, it is more likely that the bidder's demand will exceed its capacity. In those situations, the merged entity can utilize any excess capacity of the target to reap economic benefits. The value of the target, and of operational synergy, goes up monotonically because the bidder's capacity has no other use ($\nu_2 = 0$). If that is not the case, a higher bidder's demand can reduce the availability of bidder's capacity to produce any excess demand for the target's product line, which may lead to a non-monotonic effect. The requirement of PDS ensures that the target's demand does not decrease as a result of $X_1 \mid_{x_2}$ becoming larger, and thus there is no negative effect on target's valuation. As before, PDS is a sufficient condition that can be replaced with the positive correlation condition for many common distributions.

We noted on page 7 that SSD is weaker than both FSD and concave orders. Therefore, in what follows we identify sufficient conditions under which SSD-ordered demand results in an ordering of the target's value (C) and of the value of operational synergy (Δ). Since FSD $\Rightarrow$ SSD, our goal is to find conditions that are strictly weaker than those identified in Propositions 3 and 4. We assume in this section of the article that the target and bidder firms have independently demanded product lines. When demands are correlated, it is possible to study the effect of variability for specific joint demand distributions via numerical examples, as shown in section 4. Distribution-free analysis of the most general model does not yield meaningful results (other than what are already reported in Propositions 3 and 4) in this case.

Assuming independent demands, i.e., $g(x_1, x_2) = f_1(x_1)f_2(x_2)$, equation 5 can be simplified as follows:

$$C_0(Q_1, Q_2) = \begin{cases} p_2 Q_2 - (p_2 - p_1\nu_1)\int_0^{Q_2} F_2(x_2)dx_2 + (p_2\nu_2)\int_0^{Q_1} \bar{F}_2(Q_2 + \nu_2(Q_1 - x_1))F_1(x_1)dx_1 \\ -(p_1\nu_1)\int_0^{Q_2} F_1(Q_1 + \nu_1(Q_2 - x_2))F_2(x_2)dx_2. \end{cases} \tag{6}$$

Upon applying the necessary and sufficient condition for SSD ordering presented on page 7 to equation 6, we see that the first term is neutral in SSD-larger X_2, the second term is increasing, and the third and the fourth term are not, in general, monotone. However, the third term vanishes if $\nu_2 = 0$, and the fourth term is monotone increasing if $F_1(Q_1) = 1$. Thus, when the bidder has inflexible but ample capacity to meet its demand, increasing demand for target's product lines (in the SSD sense) increases the value of the target. This result is presented formally in a proposition below.

PROPOSITION 5 *The maximum value of the target firm (C) increases as its demand becomes larger and less volatile, if the bidder has inflexible but ample capacity to meet its own demand.*

The effect of having a larger and more variable target's demand on Δ, and a similar effect of larger and more variable bidder's demand on C and Δ are by and large not monotone. This can be seen by first writing C_0 in the following equivalent form:

$$C_0(Q_1, Q_2) = rC_2(Q_2) + \Delta_0(Q_1, Q_2), \quad \text{where,} \tag{7}$$

$$\Delta_0(Q_1, Q_2) = \begin{cases} (p_2\nu_2) \int_0^{Q_1} \bar{F}_2(Q_2 + \nu_2(Q_1 - x_1))F_1(x_1)dx_1 \\ \\ +(p_1\nu_1) \int_0^{Q_2} \bar{F}_1(Q_1 + \nu_1(Q_2 - x_2))]F_2(x_2)dx_2. \end{cases} \tag{8}$$

Consider, for instance, what happens to C_0 and Δ_0 when we make X_1 larger in the SSD sense. For this purpose, we can focus simply on Δ_0 since $C_2(Q_2)$ is independent of X_1. Both terms on the right hand side of 8 are, in general, not monotone. However, the first term is decreasing if $\bar{F}_2(Q_2) = 1$, and under the same condition the integrand of the second term is zero. Thus, if the target firm is severely capacity constrained, larger and more volatile demand for bidder's product line reduces the value of operational synergy. An analogous result is also obtained when we vary X_2 and study its effect on Δ_0.

PROPOSITION 6 *The effect of larger and more variable X_2 on Δ, and of larger and more variable X_1 on C and Δ, is not monotone. However, when the bidder's (target's) demand always exceeds its capacity, then Δ (C and Δ) is decreasing in larger and more variable demand for target's (bidder's) product line.*

In some situations, the senior managers might be interested in obtaining a better understanding of the magnitude of change (as opposed to simply the direction of change) in valuation of the target when its or bidder's demand becomes more volatile. In such case, a useful analytical tool is the mean-preserving transformation (see Gerchak and Mossman, 1992 for details).

4 Insights and Numerical Examples

We begin by summarizing the how the various characteristics of the target and acquiring firms affect the value of operational synergy (Δ) in Table 1. The reader should note that the effect of bidder's characteristics on C is identical their effect on Δ (shown in Table 1). However, changes in target's characteristics (Q_2 and X_2) affect C in a different fashion. Specifically, C is increasing in Q_2. It is also increasing in FSD-larger $X_{2|x_1}$ under PDS($X_1 \mid X_2$), and in SSD-larger X_2 when X_1 and X_2 are independent and the bidder has ample capacity to meet its demand. In addition, it is well known from previous studies dealing with risk pooling and diversification that greater negative correlation is value enhancing (see, for example, Samuelson 1967 for an application in finance, and Eppen 1979 for an application in inventory literature). Such benefits also apply here and greater negative correlation between the demands of the target and the bidder firms increases the value of the target firm when all other parameters are kept fixed.

Insert Table 1 about here

In order to illustrate the effect of various production capacities and demand characteristics of the target and the bidder firms, we present three numerical examples in this section. In these examples we assume that the joint demand for the product lines of the two firms has a bivariate normal distribution. The joint density function can be written as follows:

$$g(x_1, x_2) = \left(\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \right) e^{-q/2}, \tag{9}$$

where $\sigma_i > 0$, $\mid \rho \mid < 1$ is the correlation coefficient, $q = [1/(1-\rho^2)]\{[(x_1-\mu_1)/\sigma_1]^2 + [(x_2 - \mu_2)/\sigma_2]^2 - 2\rho[(x_1-\mu_1)/\sigma_1][(x_2-\mu_2)/\sigma_2]\}$, and $-\infty < x_1, x_2 < \infty$. Although, theoretically, X_i's can take negative values, we only consider examples in which the probability of this happening is virtually zero, i.e., $\sigma_i/\mu_i < \frac{1}{3}$, for each $i = 1,\ 2$. For all practical purposes, this is a reasonable and often used model of product demand distribution (see, for example, Silver, Pyke and Peterson, 1998). Another reason for using bivariate Normal demand distribution is that in this case, it is relatively easy to ascertain PDS. Positive dependence is implied by positive correlation. Also, for the bivariate normal (BN) distribution:

$$\begin{aligned} f_i(x_i) &= \textstyle\int_{x_j=0}^{\infty} g(x_1, x_2)dx_j, \qquad j \neq i, \\ &= \left(\tfrac{1}{\sqrt{2\pi}\sigma_i}\right) e^{-\frac{1}{2}(\frac{x_i-\mu_i}{\sigma_i})^2}, \end{aligned} \tag{10}$$

and the conditional PDF of X_1, given that $X_2 = x_2$ can be written as:

$$
\begin{aligned}
g_{X_1}(y\,|_{x_2}) &= \left(\frac{1}{\sqrt{2\pi(1-\rho^2)}\sigma_1}\right) e^{-\frac{1}{2}\left(\frac{y-\mu_1(x_2)}{\sigma_1\sqrt{1-\rho^2}}\right)^2}, \\
g_{X_2}(y \mid x_1) &= \left(\frac{1}{\sqrt{2\pi(1-\rho^2)}\sigma_2}\right) e^{-\frac{1}{2}\left(\frac{y-\mu_2(x_1)}{\sigma_2\sqrt{1-\rho^2}}\right)^2}.
\end{aligned}
\tag{11}
$$

The terms $\mu_1(x_2)$ and $\mu_2(x_1)$ represent the conditional means $E[X_1\,|_{x_2}]$ and $E[X_2 \mid x_1]$. These are given as follows:

$$
\begin{aligned}
\mu_1(x_2) &= \mu_1 + \rho(\sigma_1/\sigma_2)(x_2 - \mu_2), \\
\mu_2(x_1) &= \mu_2 + \rho(\sigma_2/\sigma_1)(x_1 - \mu_1).
\end{aligned}
\tag{12}
$$

We also solved the same examples with bivariate lognormal (BLN) demand distribution. Although the numerical values differ, the key insights and patterns observed in our examples remain unchanged. We illustrate this in example 1 by showing results for both BN and BLN distributions. A brief description of the BLN distribution, and its various parameters, is included in Appendix F for sake of completeness. Since it is more common to use normal distribution to model product demands, we show the remaining examples only with the BN distribution.

Example 1 Let $\Phi(y)$ denote the CDF of a unit normal distribution, i.e., a distribution with mean of 0 and standard deviation of 1. Since $F_2(y) = \Phi(\frac{y-\mu_2}{\sigma_2})$, it follows that a stochastically increasing sequence of distributions is obtained if we increase μ_2, while keeping other parameters of target's demand constant.

Our first example deals with a situation in which the demand for target's products is stochastically increasing, and the bidder and the target have negatively correlated demands. Specifically, the various parameters values are as follows:

$$
\begin{aligned}
&p_1 = 100,\ p_2 = 10,\ Q_1 = Q_2 = 3,\ \nu_1 = \nu_2 = 0.5, \\
&\rho = -0.5,\ \mu_1 = 10,\ \text{and}\ \sigma_1 = \sigma_2 = 1.
\end{aligned}
$$

The mean demand for the target is varied from 4 units to 6.25 units in steps of size 0.25. The results are shown in Figure 1. As predicted immediately following Proposition 3, for such cases the value of the target is decreasing even as its demand grows stochastically larger.

Insert Figure 1 about here

Next, we also solved the same example with bivariate lognormal (BLN) demand distribution (see details in Appendix F). To allow quick comparison, the underlying distribution of X_i's (where

$X_i = \ln(Z_i)$) were assumed to have the same parameter values as above. Figure 2 shows how C varies as a function of $E(Z_2) = e^{\mu_2 + 0.5\sigma_2^2}$. Note that the pattern observed in Figure 1 is preserved.

Insert Figure 2 about here

Example 2 In this example, we study how the size of bidder and target firms' capacities affect the value of the target and the value of operational synergies. In the first example, we vary bidder's capacity in a situation where the target has sufficient capacity to meet its peak demand. In this case the target's value and the value of operational synergy are decreasing in Q_1 (see Figure 3 and Table 1). Relevant data for Figure 3 is as follows:

$$p_1 = p_2 = 10,\ Q_2 = 6,\ \nu_1 = \nu_2 = 0.5,$$
$$\rho = 0.5,\ \mu_1 = 10,\ \mu_2 = 4,\ \text{and } \sigma_1 = \sigma_2 = 0.5.$$

The bidder's capacity is varied from $Q_1 = 5$ to 7.25 in steps of 0.25.

In the second part of this example, we observe how C and Δ vary with target's capacity Q_2, when the bidder has ample capacity to meet all its demand. The results are shown in Figure 4. Notice that C, shown by the solid line, increases in Q_2, whereas Δ, shown by the dotted line, decreases. This shows that even as the value of the target increases, the value of synergistic benefits decline. Relevant data for Figure 4 is as follows:

$$p_1 = p_2 = 10,\ Q_1 = 6,\ \nu_1 = \nu_2 = 0.5,$$
$$\rho = 0.5,\ \mu_1 = 3,\ \mu_2 = 5.5,\ \text{and } \sigma_1 = \sigma_2 = 0.5.$$

The target's capacity is varied from $Q_2 = 4.5$ to 6.75 in steps of 0.25.

Insert Figures 3 and 4 about here

Example 3 Finally, we demonstrate the effect of larger and more variable demand for target's product line on C and Δ. For this purpose, we model a situation in which bidder and target's demand are independent, and the bidder has inflexible but ample capacity to meet its demand. Specifically, the following data is considered:

$$p_1 = p_2 = 10,\ Q_1 = 6,\ \nu_1 = 0.5,\ \nu_2 = 0,$$
$$\rho = 0,\ \mu_1 = 4,\ \text{and } \sigma_1 = 0.5.$$

μ_2 and σ_2 are varied in a lock-step fashion such that as the target's demand becomes more variable, its mean becomes smaller. In particular, σ_2 is varied from 0.5 to 1.4, and at the same time, μ_2 is varied from 7.25 to 5 in steps of 0.25. Results of our experiments are shown in Figure 5. Notice that while the value of the target decreases in increasing σ_2, there is no effect on operational synergy. In fact, the value of Δ remains zero as we vary σ_2.

Insert Figure 5 about here

5 Concluding Remarks

Widely used methods for valuing target firms in M&A do not account for bidder firm characteristics. We show in this article that these have a significant, and often non-monotone impact on the target's value and on the value of operational synergy. We also show how size and flexibility of capacity, and larger/less variable demand patterns interact to produce different levels of operational synergy in a typical M&A. In the remainder of this section, we comment on two situations for which our model provides bounds. A detailed analysis of these scenarios remains a challenge for future research.

We assume that the output of the two firms is unstorable/perishable and hence no inventory is carried. For firms that make products that can be stored, inventory offers a means to smooth capacity requirements that fluctuate due to demand uncertainty. Thus, when inventory can be carried, the option value of using excess capacity of one firm to fill the other firm's excess demand will be reduced (see, for example, Hillier, 1999, for a similar result with substitutable products). Put differently, the option to carry inventory is likely to lower the benefit of operational synergy from a possible M&A, making our valuation an upper bound on the value of the target firm. If inventory is reviewed periodically and setup costs are negligible, the optimum inventory control policy is known to be myopic and thus, in principle, the benefits of synergy can be calculated.

The model presented in this article assumes that the bidder will make long term use of the target firm's entire capacity. Many actual M&A's, however, are followed by cost cutting, and other rationalization measures directed at the merged entity. Even outright sale of some of target's (or bidder's) assets/subsidiaries is not uncommon. Assuming that such steps are financially beneficial, our valuation becomes a lower bound on the value of the target firm.

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Appendix

A Proof of Proposition 1

First, we show that $E[\int_{t=0}^{\infty} e^{-rt} p \min\{X, Q\} dt]$ can be simplified to obtain 2 as follows:

$$C(Q) = \int_{t=0}^{\infty} \int_{x=0}^{Q} pxf(x)dx\, e^{-rt}dt + \int_{t=0}^{\infty} \int_{x=Q}^{\infty} pQf(x)dx\, e^{-rt}dt \tag{A.1}$$

Next, integrating A.1 by parts and rearranging terms on its RHS, we obtain:

$$C(Q) = (p/r)(Q[1 - F(Q)] + \int_{0}^{Q} xf(x)dx) = (p/r) \int_{0}^{Q} \bar{F}(x)dx. \tag{A.2}$$

Differentiating A.2 with respect to p/r, and then with respect to Q, we find that these derivatives are positive, and that the second derivative with respect to Q is negative, confirming claims 1 and 2 in proposition 1.

For claim 3, we note that if $X \geq_{\text{st}} Y$ (i.e., X is stochastically larger than Y), then clearly $\min[X, Q] \geq_{\text{st}} \min[Y, Q]$, since $\min[x, Q]$ is an increasing function of x. Therefore, $E[\min\{X, Q\}] \geq E[\min\{Y, Q\}]$ follows from Ross (1996, proposition 9.1.2, pp. 405), and this completes the proof of claim 3.

Writing equation 2 as $C(Q) = (p/r)[Q - \int_0^Q F(x)dx]$, and applying the necessary and sufficient condition on page 7 for SSD-order, we see that as X become SSD-larger, the negative term on the right hand side of 2 becomes smaller, and therefore $C(Q)$ becomes larger. That is $C(Q)$ increases when X becomes less variable with the same or higher mean.

B Developing the One-Period Expected Profit Function for the Merged Entity

At the beginning of each planning period, after demands have realized, the merged entity must decide how to allocate available capacity to the two product lines. Let x_i denote the realized demand in an arbitrary period and y_{ij} denote the amount of product type i produced on facility type j. Then, the decision problem faced by the merged entity can be written as the following linear program:

$$\text{Maximize} \quad p_1(y_{11} + y_{12}) + p_2(y_{21} + y_{22}) \tag{B.1}$$

Subject to:

$$
\begin{aligned}
y_{11} + y_{21}/\nu_2 &\leq Q_1 \\
y_{22} + y_{12}/\nu_1 &\leq Q_2 \\
y_{11} + y_{12} &\leq x_1 \\
y_{22} + y_{12} &\leq x_1 \\
y_{ij} &\geq 0 \text{ for all } i \text{ and } j.
\end{aligned}
\tag{B.2}
$$

The problem described in B.1 - B.2 is relatively simple and can be solved analytically without actually having to solve a linear program. We present a solution below which is obtained if we assume that each facility is ideally suited to making its own product, i.e., $p_1 \geq \nu_2 p_2$ and $p_2 \geq \nu_1 p_1$.

1. If $x_i \leq Q_i$, the optimal solution is not unique and any solution that sets $y_{11}+y_{12} = x_1$ and $y_{22}+y_{21} = x_2$ is optimal. For example, the optimal solution is obtained trivially by setting $y^*_{ii} = x_i$ and $y^*_{ij} = 0$, whenever $i \neq j$.
2. If $x_1 > Q_1$ and $x_2 \leq Q_2$, we need to decide how to allocate capacities more carefully. One unit of type 1 capacity can either produce 1 unit of type 1 product, or ν_2 units of product type 2. The corresponding revenues earned are p_1 and $p_2\nu_2$. Thus, if $p_1 \geq p_2\nu_2$, it is always economical to use type 1 capacity to product type 1 product, provided demand exists. Furthermore since $x_1 > Q_1$, all of type 1 capacity can be allocated to type 1 product, i.e., we have $y^*_{11} = Q_1$, $y^*_{21} = 0$. Similarly, if we consider 1 unit of type 2 capacity, it can either produce 1 unit of product type 2, or ν_1 units of type 1 product. So long as $p_2 \geq p_1\nu_1$, preference will be given to the production of type 2 items, until all type 2 demand is satisfied (recall that $x_2 \leq Q_2$ and therefore this is indeed possible). Putting it together, we get $y^*_{22} = x_2$ and $y_{12} = \min\{\nu_2(Q_2 - x_2), x_1 - Q_1\}$.
3. If $x_1 \leq Q_1$ and $x_2 > Q_2$, then the optimal allocation decisions are: $y^*_{11} = x_1$, $y^*_{21} = \min\{\nu_1(Q_1 - x_1), x_2 - Q_2\}$, $y^*_{22} = Q_2$, and $y^*_{12} = 0$. Arguments similar to those presented above can be used to support this conclusion.
4. Finally, when $x_1 \geq Q_1$, and $x_2 \geq Q_2$, the optimal allocation decisions are trivially as follows: $y^*_{11} = Q_1$, $y^*_{21} = 0$, $y^*_{22} = Q_2$, and $y^*_{12} = 0$. This follows directly from our assumption that each facility is ideally suited for making its own product.

Combining the solution in items 1–4 above and taking expectations results in expression 3.

C Proof of Proposition 2 and Its Specialization to Independent Demands

The claims listed in proposition 2 can each be proved by studying the sign of each partial derivative of C and Δ. The following result is obtained after differentiating equation (4) and simplifying:

$$
\begin{aligned}
\frac{\partial C}{\partial Q_2} &= (\frac{p_2}{r}) \left(\int_0^\infty \int_{Q_2}^\infty g(x_1,x_2)dx_2dx_1 - \int_0^{Q_1} \int_{Q_2}^{Q_2+\nu_2(Q_1-x_1)} g(x_1,x_2)dx_2dx_1 \right) \\
&+ (\frac{p_1}{r}) \left(\nu_1 \int_0^{Q_2} \int_{Q_1+\nu_1(Q_2-x_2)}^\infty g(x_1,x_2)dx_1dx_2 \right) \geq 0.
\end{aligned}
\tag{C.1}
$$

The above is non-negative since $g(x_1, x_2) \geq 0$, for all x_1 and x_2, and the summation limits for the second double integral of the first term lie strictly within the limits for the first double integral. We also see that $\frac{\partial C}{\partial Q_2}$ is strictly positive when $\nu_1 > 0$, i.e., when the target firm has flexible production facilities. The second derivative is obtained as shown below:

$$\begin{aligned}\frac{\partial^2 C}{\partial Q_2^2} &= -\left(\frac{p_2 - p_1\nu_1}{r}\right)\left(\int_{Q_1}^{\infty} g(x_1, Q_2)dx_1\right) - \left(\frac{p_2}{r}\right)\left(\int_0^{Q_1} g(x_1, Q_2 + \nu_2(Q_1 - x_1))dx_1\right) \\ &- \left(\frac{p_1\nu_1^2}{r}\right)\left(\int_0^{Q_2} g(Q_1 + \nu_1(Q_2 - x_2))dx_2\right) \leq 0. \end{aligned} \tag{C.2}$$

C.2 is clearly negative if $p_2 \geq p_1\nu_1$. This is guaranteed by our earlier assumption that each facility is most economical for making its own product lines (recall this happens when $\nu_2 \leq p_1/p_2 \leq 1/\nu_1$). Thus, the maximum value of the target firm increases at a decreasing rate as its production capacity increases, irrespective of bidder's characteristics. The proof of how Q_2 affects Δ is presented after we treat how Q_1 affects and C and Δ.

The effect on C and Δ of changing Q_1 is the same. This is because the term $C_2(Q_2)$ is unaffected by Q_1. The first partial derivative is as follows:

$$\frac{\partial C}{\partial Q_1} = \frac{\partial \Delta}{\partial Q_1} = (\frac{p_2\nu_2}{r})\int_0^{Q_1}\int_{Q_2+\nu_2(Q_1-x_1)}^{\infty} g(x_1, x_2)dx_2dx_1 - (\frac{p_1}{r})\int_0^{Q_2}\int_{Q_1}^{Q_1+\nu_1(Q_2-x_2)} g(x_1, x_2)dx_1dx_2. \tag{C.3}$$

Since C.3 contains both a negative and a positive term, $\frac{\partial C}{\partial Q_1}$ and $\frac{\partial \Delta}{\partial Q_1}$ are neither always positive nor always negative. We note that the first (positive) term disappears when either $\nu_2 = 0$ or $\int_0^{Q_1}\int_{Q_2}^{\infty} g(x_1, x_2)dx_2dx_1 \rightarrow 0$, making C.3 negative for these cases. The double integral goes to zero when for all $y \geq 0$ and $0 \leq x_1 \leq Q_1$, the conditional CDF $G_{X_2}(Q_2 + y \mid x_1) \approx G_{X_2}(Q_2 \mid x_1) \approx 1$. $G_{X_2}(\cdot \mid x_1)$ is the conditional CDF of X_2 given that $X_1 = x_1$. This can be explained as follows. Writing $g(x_1, x_2) = g_{X_2}(x_2 \mid x_1)f_1(x_1)$, where $g_{X_2}(\cdot \mid \cdot)$ is the conditional PDF of X_2 given x_1, and $f_1(\cdot)$ is the marginal PDF of X_1, we can obtain the following identity:

$$\int_0^{Q_1}\int_{Q_2+\nu_2(Q_1-x_1)}^{\infty} g(x_1, x_2)dx_2dx_1 = \int_0^{Q_1} f_1(x_1)[1 - G_{X_2}(Q_2 + \nu_2(Q_1 - x_1) \mid x_1)]dx_1,$$

which approaches zero when $G_{X_2}(Q_2 + y \mid x_1) \approx G_{X_2}(Q_2 \mid x_1) \approx 1$ for all $y \geq 0$ and $0 \leq x_1 \leq Q_1$. In other words, a sufficient condition for C and Δ to be decreasing in Q_1 is that the target's demand should not exceed Q_2, irrespective of the level of bidder's demand. Similarly, when $G_{X_1}(Q_1 + y \mid_{x_2}) \approx G_{X_1}(Q_1 \mid x_1) \approx 1$, for all $y \geq 0$, and all $0 \leq x_2 \leq Q_2$, then the term $\int_0^{Q_2}\int_{Q_1}^{Q_1+\nu_1 Q_2} g(x_1, x_2)dx_1dx_2 \rightarrow 0$. When both the positive and the negative terms disappear, we observe no significant effect of Q_1.

Returning now to how Q_2 impacts Δ, the value of operational synergy, note that

$$\frac{\partial \Delta}{\partial Q_2} = (\frac{p_1\nu_1}{r})\int_0^{Q_2}\int_{Q_1+\nu_1(Q_2-x_2)}^{\infty} g(x_1, x_2)dx_1dx_2 - (\frac{p_2}{r})\int_0^{Q_1}\int_{Q_2}^{Q_2+\nu_2(Q_1-x_1)} g(x_1, x_2)dx_1dx_2 \tag{C.4}$$

has a structure similar to C.3. This means that the conditions under which it is positive or negative are derived in the same fashion as C.3. Together they give rise to the four cases described in proposition 2.

In order to better understand the conditions identified in proposition 2, it is instructive to consider the instance when the bidder and target face independent demands. In that case, equation C.3 simplifies as follows:

$$\frac{\partial C}{\partial Q_1} = \left(\frac{p_2\nu_2}{r}\right)\left(\int_0^{Q_1} [1 - F_2(Q_2 + \nu_2(Q_1 - x_1))]f_1(x_1)dx_1\right)$$

$$- \quad \left(\frac{p_1}{r}\right)\left(\int_0^{Q_2}[F_1(Q_1+\nu_1(Q_2-x_2))-F_1(Q_1)]f_2(x_2)dx_2\right). \qquad \text{(C.5)}$$

The first of the two terms of C.5 vanishes when either $\nu_2 = 0$ or $F_2(Q_2) \approx 1$, and the second term vanishes when $F_1(Q_1) \approx 1$. In other words, maximum value of the target firm is decreasing in Q_1 when either the bidder's facility is inflexible or the target has sufficient capacity to meet its peak demand. On the other hand, the maximum value is increasing when bidder's capacity is flexible and sufficient to meet the peak demand for its own product line.

Next, we compute the first and second derivative of C and Δ with respect to ν_1 and ν_2. These are:

$$\frac{\partial C}{\partial \nu_1} = \frac{\partial \Delta}{\partial \nu_1} = \left(\frac{p_1}{r}\right)\left(\int_0^{Q_2}\int_{Q_1+\nu_1(Q_2-x_2)}^{\infty}(Q_2-x_2)g(x_1,x_2)dx_1dx_2\right) \geq 0. \qquad \text{(C.6)}$$

$$\frac{\partial^2 C}{\partial \nu_1^2} = \frac{\partial^2 \Delta}{\partial \nu_1^2} = -\left(\frac{p_1}{r}\right)\left(\int_0^{Q_2}(Q_2-x_2)^2 g(Q_1+\nu_1(Q_2-x_2),x_2)dx_2\right) \leq 0. \qquad \text{(C.7)}$$

$$\frac{\partial C}{\partial \nu_2} = \frac{\partial \Delta}{\partial \nu_2} = \left(\frac{p_2}{r}\right)\left(\int_0^{Q_1}\int_{Q_2+\nu_2(Q_1-x_1)}^{\infty}(Q_1-x_1)g(x_1,x_2)dx_2dx_1\right) \geq 0. \qquad \text{(C.8)}$$

$$\frac{\partial^2 C}{\partial \nu_2^2} = \frac{\partial^2 \Delta}{\partial \nu_2^2} = -\left(\frac{p_2}{r}\right)\left(\int_0^{Q_1}(Q_1-x_1)^2 g(x_1,Q_2+\nu_2(Q_1-x_1))dx_1\right) \leq 0. \qquad \text{(C.9)}$$

The signs of these derivatives clearly confirm that the target's value to the bidder increases as either the target's or the bidder's facilities become more flexible. Put differently, a bidder with more flexible facilities should be willing to pay more for the same target.

D Proof of Proposition 3 and Its Specialization to Independent Demands

In order to see how stochastically larger X_2 affects C, we simplify and re-organize terms of equation 5 and obtain the following equivalent form:

$$C_0(Q_1,Q_2) = \begin{cases} p_2Q_2 - (p_2-p_1\nu_1)\int_0^{Q_2}F_2(x_2)dx_2 - (p_1\nu_1)\int_0^{Q_2}G_{X_1}(Q_1+\nu_1(Q_2-x_2)\mid_{x_2})F_2(x_2)dx_2 \\ +p_1\int_0^{Q_2}\left[\int_{Q_1}^{Q_1+\nu_1(Q_2-x_2)}\left(\frac{\partial G_{X_1}(y|_{x_2})}{\partial x_2}\right)dy\right]F_2(x_2)dx_2 \\ -p_2\int_0^{Q_1}\int_{Q_2}^{Q_2+\nu_2(Q_1-x_1)}f_1(x_1)G_{X_2}(x_2\mid x_1)dx_2dx_1 \\ +(p_2\nu_2)\int_0^{Q_1}F_1(x_1)dx_1. \end{cases} \qquad \text{(D.1)}$$

Note that the term $\left(\frac{\partial G_{X_i}(y|x_j)}{\partial x_j}\right)$, where $i \neq j$, represents the rate at which $\text{Prob}\{X_i \leq y \mid X_j = x_j\}$ changes as a function of x_j. It is negative if $X_{i|x_j}$ is stochastically increasing in x_j, for each x_j. This property is called positive dependence through stochastic ordering (Block, Savits, and Shaked, 1985). For several common distributions, for example, multivariate normal, positive dependence is implied by positive pairwise correlation.

Upon examining equation D.1, we find that the first and the last terms are neutral in X_2, the second, third, and the fifth terms are increasing in stochastically increasing $X_{2|x_1}$, for all x_1, and the fourth term could go either way. However, when X_1 is positively dependent on X_2 through stochastic ordering, i.e., $\frac{\partial G_{X_1}(y|_{x_2})}{\partial x_2} \leq 0$, this term is also increasing in stochastically increasing X_2. This confirms that C is increasing in stochastically increasing $X_{2|x_1}$, $\forall\, x_1$, when $PDS(X_1\,|_{x_2})$ holds.

When demands X_1 and X_2 are stochastically independent, we obtain the following simpler expression for C:

$$
\begin{aligned}
C_0(Q_1, Q_2) &= p_2Q_2 - (p_2 - p_1\nu_1)\int_0^{Q_2} F_2(x_2)dx_2 + (p_2\nu_2)\int_0^{Q_1}[1 - F_2(Q_2 + \nu_2(Q_1 - x_1))]F_1(x_1)dx_1 \\
&- (p_1\nu_1)\int_0^{Q_2} F_1(Q_1 + \nu_1(Q_2 - x_2))]F_2(x_2)dx_2. \qquad \text{(D.2)}
\end{aligned}
$$

It is now easy to see that as we replace F_2 with F_2', where $F_2'(y) \leq F_2(y)$, for all y, the terms in D.2 increase. Thus, the net effect of increasing X_2 is a larger $C_0(Q_1, Q_2)$, i.e., the maximum value of the target firm is increasing in increasing demand for its products.

E Proof of Proposition 4 and Its Specialization to Independent Demands

We provide detailed arguments for the case when X_1 is stochastically increasing. It has the same effect on value of C and Δ. The situation concerning the effect of X_2 on Δ can be treated using similar arguments. To begin, equation 5 can be simplified and re-organized to yield the following form:

$$
C_0(Q_1, Q_2) = \begin{cases}
p_2Q_2 - (p_2 - p_1\nu_1)\int_0^{Q_2} F_2(x_2)dx_2 - (p_2\nu_2)\int_0^{Q_1} G_{X_2}(Q_2 + \nu_2(Q_1 - x_1) \mid x_1)F_1(x_1)dx_1 \\
+p_2\int_0^{Q_1}\left[\int_{Q_2}^{Q_2+\nu_2(Q_1-x_1)}\left(\frac{\partial G_{X_2}(y|x_1)}{\partial x_1}\right)dy\right]F_1(x_1)dx_1 \\
-p_1\int_0^{Q_2}\int_{Q_1}^{Q_1+\nu_1(Q_2-x_2)} f_2(x_2)G_{X_1}(x_1\,|_{x_2})dx_1dx_1 \\
+(p_2\nu_2)\int_0^{Q_1} F_1(x_1)dx_1.
\end{cases} \qquad \text{(E.1)}
$$

The first two terms of equation E.1 are neutral in X_1, the third and the fifth terms are increasing in stochastically increasing $X_{1|_{x_2}}$, and the fourth term could go either way. Even if the fourth term is increasing in X_1, i.e., $\frac{\partial G_{X_2}(y|x_1)}{\partial x_1} \leq 0$, the overall direction of change in C is uncertain since the sixth term is decreasing in X_1. Clearly, when $\nu_2 = 0$ (bidder's capacity is inflexible), the sixth term vanishes. Thus, the target's value is increasing in $X_{1|_{x_2}}$, $\forall\, x_2$, if X_1 is positively dependent on X_2 through stochastic ordering and $\nu_2 = 0$. Additional results can be derived for the independent demand situation, as discussed below.

From equation D.2 we note that upon making X_1 larger in the stochastic sense, the first two terms of D.2 do not change, the third term decreases, whereas the fourth term increases. Thus, the net effect of increasing X_1 is not monotonic in all cases. However, the third term vanishes when either $\nu_2 = 0$ or $F_2(Q_2) \approx 1$, and the fourth term vanishes when either $\nu_1 = 0$ or $F_1(Q_1) \approx 1$. Thus, when either the existing capacity is inflexible or the target firm has sufficient capacity to meet its peak demand, increasing X_1 increases the maximum value of the target firm. On the other hand, when either the target firm has inflexible capacity or the firm's existing capacity is sufficient to meet its peak demand, then the value of the target firm is decreasing in X_1.

F The Bivariate Lognormal (BLN) Distribution

Consider the case in which the distribution of demands is bivariate lognormal. Let X_1 and X_2 be bivariate normal with means μ_i, variances σ_i^2, and correlation ρ. We define lognormal variates Z_i such that $X_i = \ln(Z_i)$, for all $i = 1, 2$. Then it can be shown that Z_1, Z_2 are BLN with the following distribution (see Mostafa and Mahmoud, 1964, for details):

$$g(z_1, z_2) = \left(\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}\right)\left(\frac{1}{z_1 z_2}\right) e^{-q/2}, \tag{F.1}$$

where $q = [1/(1-\rho^2)]\{[(\ln(z_1)-\mu_1)/\sigma_1]^2 + [(\ln(z_2)-\mu_2)/\sigma_2]^2 - 2\rho[(\ln(z_1)-\mu_1)/\sigma_1][(\ln(z_2)-\mu_2)/\sigma_2]\}$, and $0 < z_1, z_2 < \infty$.

The conditional density functions of Z_2 given Z_1, and Z_1 given Z_2 are, respectively,

$$\begin{aligned} g_{Z_1}(y \mid z_2) &= \left(\frac{1}{\sqrt{2\pi(1-\rho^2)}\sigma_1 z_1}\right) e^{-\frac{1}{2(1-\rho^2)}(A-\rho B)^2}, \\ g_{Z_2}(y \mid z_1) &= \left(\frac{1}{\sqrt{2\pi(1-\rho^2)}\sigma_2 z_2}\right) e^{-\frac{1}{2(1-\rho^2)}(B-\rho A)^2}, \end{aligned} \tag{F.2}$$

where the terms A and B are defined as follows:

$$A = \frac{\ln(Z_1)-\mu_1}{\sigma_1}, \tag{F.3}$$

$$B = \frac{\ln(Z_2)-\mu_2}{\sigma_2}. \tag{F.4}$$

The above relationships imply that the conditional distribution of $Z_1 \mid z_2$ and $Z_2 \mid z_1$ are each lognormally distributed with the underlying normal variate having means $\mu_1 + \rho(\sigma_1/\sigma_2)[(\ln(z_2)-\mu_2)/\sigma_2]$, and $\mu_2 + \rho(\sigma_2/\sigma_1)[(\ln(z_1)-\mu_1)/\sigma_1]$. Furthermore, the variances of underlying normal variates are constant and given respectively by $\sigma_1^2(1-\rho^2)$ and $\sigma_2^2(1-\rho^2)$. This makes it possible to compute the CDF of the conditional distributions from the standard normal CDF, where the latter is denoted by Φ, as follows:

$$G_{Z_1}(z_1 \mid z_2) = \Phi\left(\frac{\ln(z_1)-\mu_1-\rho(\sigma_1/\sigma_2)[(\ln(z_2)-\mu_2)/\sigma_2]}{\sigma_1\sqrt{1-\rho^2}}\right). \tag{F.5}$$

Upon examining $G_{Z_1}(z_1 \mid z_2)$, we observe that so long as $\rho > 0$, increasing z_2 makes $G_{Z_1}(z_1 \mid z_2)$ smaller. Similar relationship is also observed for $G_{Z_2}(z_2 \mid z_1)$. This confirms that positive correlation implies PDS even for the BLN distribution.

The conditional means $E[Z_1 \mid z_2]$ and $E[Z_2 \mid z_1]$ can be calculated as follows:

$$E[Z_1 \mid z_2] = e^{\{\mu_1+\rho(\sigma_1/\sigma_2)(\ln(z_2)-\mu_2)+\frac{1}{2}\sigma_1^2(1-\rho^2)\}}, \tag{F.6}$$

$$E[Z_2 \mid z_1] = e^{\{\mu_2+\rho(\sigma_2/\sigma_1)(\ln(z_1)-\mu_1)+\frac{1}{2}\sigma_2^2(1-\rho^2)\}}. \tag{F.7}$$

The marginal density functions of Z_i are lognormal with the following properties:

$$f_{Z_i}(z) = \frac{1}{\sqrt{2\pi}\sigma_i z} e^{-\frac{1}{2}\left(\frac{\ln(z)-\mu_i}{\sigma_i}\right)^2}. \tag{F.8}$$

$$E(Z_i) = e^{\mu_i + 0.5\sigma_i^2}. \tag{F.9}$$

Let ζ denote the correlation coefficient between Z_1 and Z_2, defined in the usual way as:

$$\zeta = \frac{cov(Z_1, Z_2)}{\sqrt{Var(Z_1)Var(Z_2)}}. \tag{F.10}$$

After substitution, the above equation can be simplified as follows:

$$\zeta = \frac{e^{\rho\sigma_1\sigma_2} - 1}{\sqrt{[e^{\sigma_1^2} - 1][e^{\sigma_2^2} - 1]}}. \tag{F.11}$$

It can be confirmed that $\zeta = 0$, when $\rho = 0$, and that $\zeta > 0$ when $\rho > 0$. However, for $\rho \neq 0$, $\mid \zeta \mid < \mid \rho \mid$.

Bidder or Target Characteristic	**Impact on the Value of Operational Synergy** (Δ)	
	Type of Impact	*Sufficient Conditions*
Larger ν_1	Increasing	—
Larger ν_2	Increasing	—
Larger Q_1	Decreasing	$\nu_2 = 0$, or $P(X_2 \leq Q_2) = 1$.
	Increasing	$P(X_1 \leq Q_1) = 1$.
	No effect	$P(X_i \leq Q_i) = 1$, $i = 1, 2$.
Larger Q_2	Decreasing	$\nu_1 = 0$, or $P(X_1 \leq Q_1) = 1$.
	Increasing	$P(X_2 \leq Q_2) = 1$.
	No effect	$P(X_i \leq Q_i) = 1$, $i = 1, 2$.
FSD-larger $X_{1\|x_2}$	Increasing	$PDS(X_2 \mid X_1)$, and $\nu_2 = 0$.
FSD-larger $X_{2\|x_1}$	Increasing	$PDS(X_1 \mid X_2)$ and $\nu_1 = 0$.
FSD-larger X_1	Increasing	X_1, X_2 are independent, & either $\nu_2 = 0$, or $P(X_2 \leq Q_2) = 1$.
	Decreasing	X_1, X_2 are independent, & either $\nu_1 = 0$, or $P(X_1 \leq Q_1) = 1$.
FSD-larger X_2	Increasing	X_1, X_2 are independent, & either $\nu_1 = 0$, or $P(X_1 \leq Q_1) = 1$.
	Decreasing	X_1, X_2 are independent, & either $\nu_2 = 0$, or $P(X_2 \leq Q_2) = 1$.
SSD-larger X_1	Decreasing	X_1, X_2 are independent, & $P(X_2 \leq Q_2) = 0$.
SSD-larger X_2	Decreasing	X_1, X_2 are independent, & $P(X_1 \leq Q_1) = 0$.

Table 1: The effect of bidder and target's production and demand characteristics on the value of operational synergy.

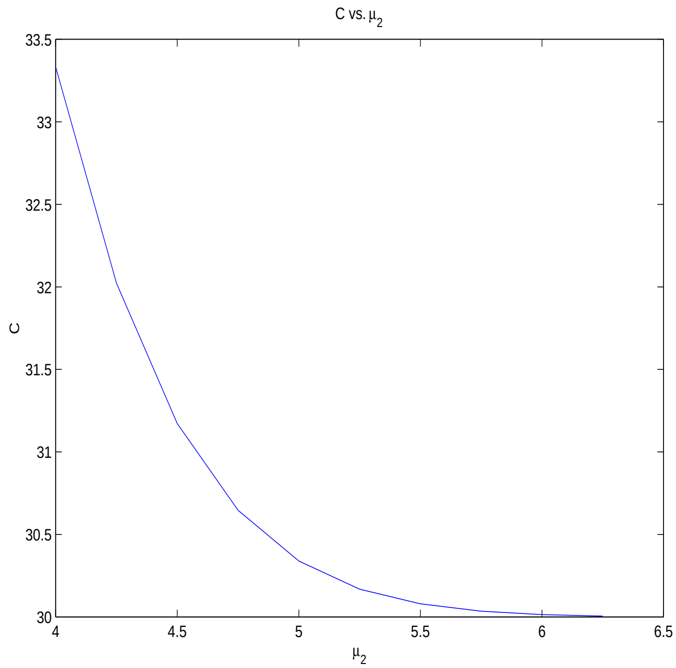

Figure 1: Plot of C versus μ_2 (demand for target's product is stochastically increasing).

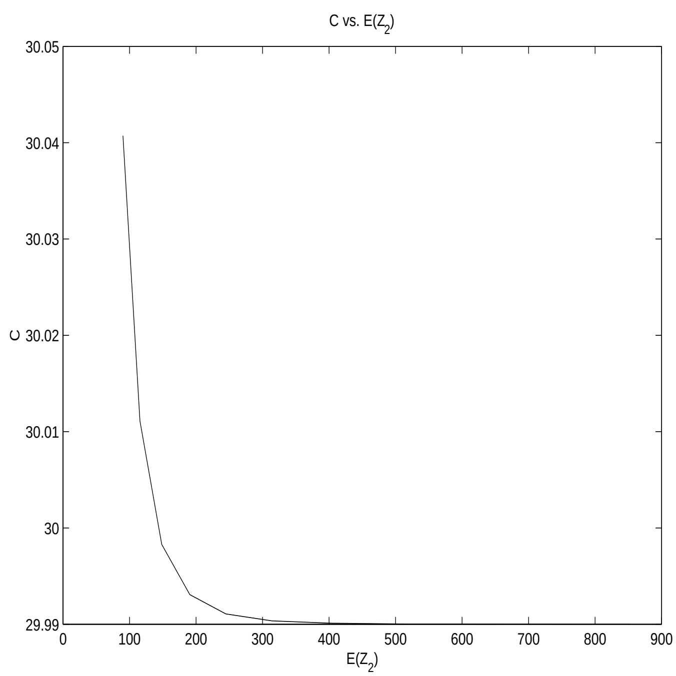

Figure 2: Plot of C versus $E(Z_2)$ (demand for target's product is stochastically increasing).

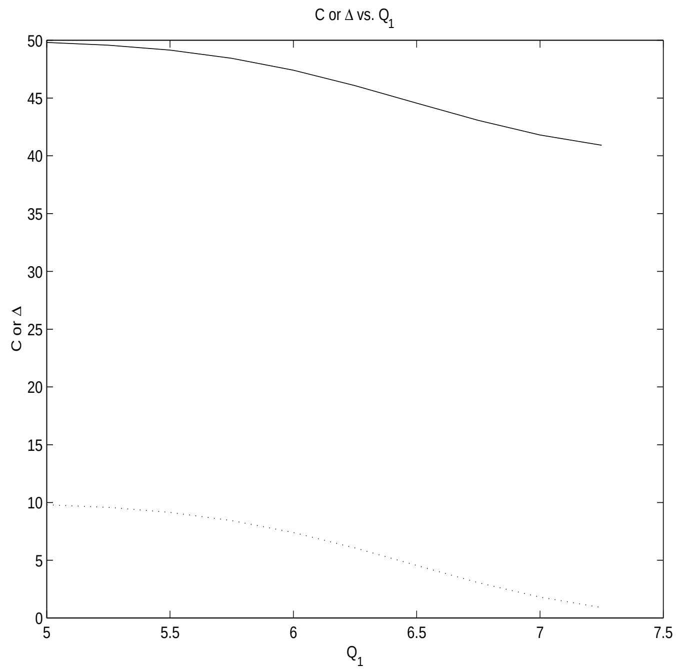

Figure 3: Plot of C and Δ versus Q_1 when target has ample capacity to meet its peak demand. The solid line shows C and the dotted line shows Δ.

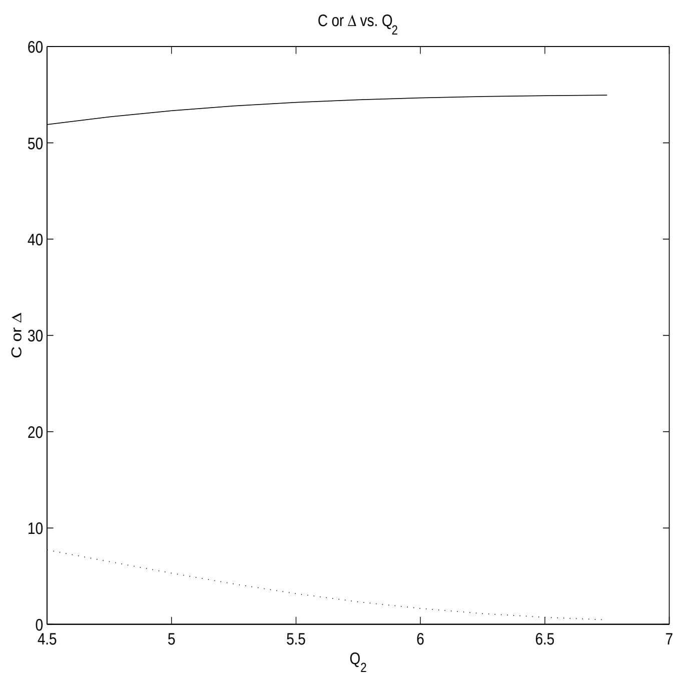

Figure 4: Plot of C and Δ versus Q_2 when the bidder has ample capacity to meet its peak demand. The solid line shows C and the dotted line shows Δ.

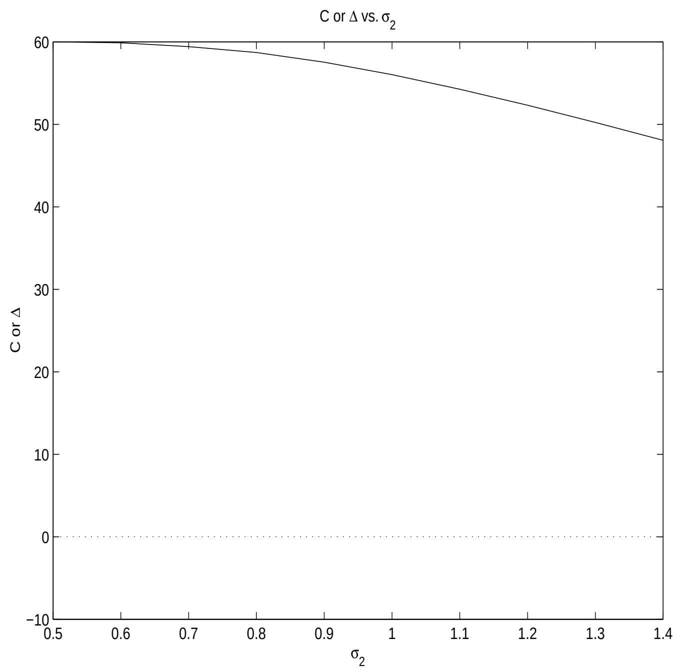

Figure 5: Plot of C and Δ versus σ_2 when the bidder has ample but inflexible capacity to meet its peak demand. Solid line shows C and dotted line shows Δ.