Digital Coding of Analog Sources

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Topics for Projects - Preliminary List

1. **Covering and Packing properties of Lattices [52]:** Summary of known facts at low dimensions (including the recent proof of "Kepler's conjecture"; see http://www.math.lsa.umich.edu/ hales/cou In addition, a simulation of a 3-dim lattice quantizer based on the FCC / BCC lattice, preferable with an efficiant implementation of the search for the closest lattice point.

2. **Size of granular region for fixed-rate high resolution scalar quantization:** Prove, by running Lloyd with increasing N, the recent result of Neuhoff.

3. **Irregular cells in ECSQ:** Find examples where the optimum Entropy Constraint Scalar Quantizer does not have convex cells (i.e., at least one cell is composed of a union of intervals).

4. **Lloyd Algorithm (scalar case) at various resolution levels [12, 11, 9]:** Write and run the algorithm for various source distributions $p(x)$, distortion measures $d(x, \hat{x})$, and distortion levels D. Find the point D at which quantizer converges to the high resolution approximation.

5. **Companding for non-uniform Vector Quantization [8, 55, 56, 45, 57]:** For a specific source (say, two dimensional Gaussian) find the optimum quantizer (via Lloyd Alg.) and the optimum companding (compressor-lattice-expander), and compare the achievable distortions.

6. **The same, for entropy-coded quantization**

7. **Entropy-coded TC / SBC [9, 33, 5]:** Investigate the rate reduction obtained by entropy coding a Transform-Code (or Sub band Code) quantizer, for Gaussian and non-Gaussian sources (check component-wise and vector-wise entropy coding).

8. **Entropy-coded DPCM [31, 35, 25]:** The same for DPCM. Consider also using a minimum-entropy (instead of minimum-MSE) predictor.

9. **Bit allocation in TC [33, 9, 59]:** Greedy algorithms for bit allocation in Transform Coding and in Discrete Multitone Modulation - a comparison.

10. **Lattice and Trellis Quantizers [52, 7, 9]:** Simulate and compare the performance of a 2-dimensional lattice quantizer with a 2-state trellis quantizer. The same for dimension / state = 3.

11. **Quantization under general distortion measures [43, 45]: 1)** For a uniform source, find the optimum VQ for the distortion measure $d(\underline{x}, \hat{\underline{x}}) = (\underline{x} - \hat{\underline{x}})W(\underline{x} - \hat{\underline{x}})^t$, where W is a 2×2 matrix, and for the distortion measure $d(x, \hat{x}) = \exp(-x^2/2\sigma_w^2)(x - \hat{x})^2$.

    2) Optimize the VQ of the 10 LPC parameters for a long speech "training sequence", relative to the Itakura-Saito distortion measure (= mean-squared prediction error with optimal gain); see [9, pp. 387-393].

12. **Karhunen-Loeve transform for picture data** For a sampled picture signal, compare the (empirical) Transform-Coding gains of the DCT versus that of the optimum (empirical) KL-transform.

13. **Adaptive DPCM applied to speech [31]:** Compare the prediction gain of a fixed predictor with that of a LMS-type adaptive predictor, in DPCM encoding of a sampled speech signal.

14. **Optimum joint source-channel quantizer:** For a Gaussian source and a Binary-Symmetric Channel, write a Lloyd-type algorithm for optimization of the partition and the reconstruction of a VQ.

15. **Staggered lattice quantizers for multiple descriptions:** Optimize the lattice and the shift, for two-dimensional quantization in a "two-description" setting.

16. **Stochastic quantizer optimization (to avoid local minima in Lloyd alg.) [9, pp. 369-372], [?]:** Compare design via simulated annealing and via deterministic annealing.

17. **Non-causal DPCM using a "turbo-decoder" mechanism for low-rate quantization:** For low rate DPCM, it is not necessary that a predictor followed by a scalar uniform quantizer makes optimum decision, even if reconstruction is still done by a conventional DPCM decoder. Show (i) the gain of a non-causal "predictor" at the encoder; and (ii) the performance of a "turbo"-like search. (References will be provided).

18. **Minimum entropy prediction for variable-rate DPCM:** [Elias59] , [Erez-Zamir99].

19. **The granular region of high-resolution fixed-rate quantizers [?]:** Study the optimum support size and step size of uniform quantizers versus optimum Bennet quantizers.

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