Multilevel Diversity Coding via Successive Refinement

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Abstract — **In a hierarchical multilevel diversity coding system, each decoder has access to a certain subset of the full source code, and the decoders are partitioned into fidelity levels in a hierarchical manner. We show that without loss in rate-distortion performance, this coding system can be separated into a successive refinement stage followed by a "lossless" multilevel diversity coding stage.**

I. Introduction

Transmission of compressed information via lossy packet networks provides an up-to-date motivation for the use of diversity techniques in source coding. Consider the following model for this problem. An information source block $X_1 \dots X_n$ is encoded into M subcodes, to which we refer as "packets", at rates $R_1, \dots, R_M$. There are a number of decoders, each can access a certain subset of the packets, called the *fan* of the decoder. In the *multiple description* problem, each of the decoders aims at optimizing its reconstruction relative to some distortion level. However, in our formulation below the decoders are partitioned into L *decoding levels*, such that the following two properties hold:

- **(Invariance Property)** all the decoders at level l must produce the same reconstruction, $\hat{X}_l$, and are subject to the same fidelity criterion

$$\frac{1}{n}\sum_{k=1}^{n} \mathrm{E}d_l(X_k, (\hat{X}_l)_k) \le D_l, \quad l = 1 \dots L; \qquad (1)$$

- **(Inclusion Property)** the fan of any decoder at level l contains the fan of some decoder at level $l-1$.

Note that these properties imply a hierarchy among the decoders, so that any decoder at level l has enough information to decode reconstructions $\hat{X}_1, \dots, \hat{X}_{l-1}$ in addition to its own reconstruction $\hat{X}_l$. Although the invariance property may seem somewhat restrictive and results in some cost in rate-distortion performance (as compared with the corresponding multiple description problem), it allows one to solve the problem completely in certain cases and to give the solution an intuitive and useful meaning.

II. Definitions

The MDC network $\mathcal{N}$ is specified by the coding rates $R_1 \dots R_M$, the fans of the decoders and their partition into decoding levels. Given the source X and the network $\mathcal{N}$, we say that the rate-distortion tuple $(R_1 \dots R_M, D_1 \dots D_L)$ is *MDC admissible* if there exists (for sufficiently large n) an MDC code which approaches these rates and distortions. The MDC rate-distortion region, $\mathcal{R}_{MDC}(X, \mathcal{N})$, is defined as the set of all such tuples..

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We now define the lossless diversity coding (MDC-IS) rate region of the network $\mathcal{N}$. Assume the encoder is fed with a source composed of L (independent) data streams:

$$X_1^n = ((Y_1)_1^n, \dots, (Y_L)_1^n),$$

and that the fidelity criterion d_l is the Hamming distance between $(\hat{X}_l)_1^n$ and $(Y_l)_1^n$. From Shannon's source coding theorem, the MDC admissibility of the $(M+L)$-tuple $(R_1 \dots R_M, 0 \dots 0)$, (i.e., with arbitrary small error rate) depends only on the entropies $h_l = H(Y_l)$, $l = 1 \dots L$ of the components of X. We say that the tuple $(R_1 \dots R_M, h_1 \dots h_L)$ is *MDC-IS admissible* if the corresponding tuple $(R_1 \dots R_M, 0 \dots 0)$ is MDC admissible. The MDC-IS rate region $\mathcal{R}_{MDC-IS}(\mathcal{N})$ is then defined as the set of all such tuples.

Recall the definition of L-level successive refinement [1] of the source X relative to the fidelity criteria (1). In terms of our formulation above, we say that the tuple $(h_1 \dots h_L, D_1 \dots D_L)$ is *SR admissible* if we can encode the source into L packets at rates $R_1 \dots R_L$, such that packets $j = 1 \dots l$ allow decoding of the source at distortion D_l, for $l = 1 \dots L$. The successive refinement rate-distortion region $\mathcal{R}_{SR}(X)$ is defined as the set of all such tuples. Theorem 1 in [1] gives a complete characterization of $\mathcal{R}_{SR}(X)$.

III. A Separation Theorem

Our main result shows that an MDC system can be separated into successive refinement of the source into L codes, followed by lossless diversity coding of the L codes into M packets.

Theorem 1 $(R_1 \dots R_M, D_1 \dots D_L) \in \mathcal{R}_{MDC}(X, \mathcal{N})$ *if and only if there exist nonnegative real numbers* $h_1 \dots h_L$ *such that* $(h_1 \dots h_L, D_1 \dots D_L) \in \mathcal{R}_{SR}(X)$ *and* $(R_1 \dots R_M, h_1 \dots h_L) \in \mathcal{R}_{MDC-IS}(\mathcal{N})$.

The direct part of the theorem implies a simple method to construct an MDC system. This method has been used implicitly in [3]. The idea is to use the numbers $h_1 \dots h_L$ which achieve the "if" part of the theorem as the rates of the successive refinement encoder, and then to MDC-IS-encode these rates losslessly into packets with rates $R_1 \dots R_M$ using the method of [3, 2].

References

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