Noise-Shaped Predictive Coding for Multiple Descriptions of a Colored Gaussian Source

Yuval Kochman, Jan Østergaard, and Ram Zamir

Abstract

It was recently shown that the symmetric multiple-description (MD) quadratic rate-distortion function for memoryless Gaussian sources and two descriptions can be achieved by dithered Delta-Sigma quantization combined with memoryless entropy coding. In this paper, we generalize this result to stationary (colored) Gaussian sources by combining noise shaping and source prediction. We first propose a new representation for the test channel that realizes the MD rate-distortion function of a Gaussian source, both in the white and in the colored source case. We then show that this test channel can be materialized by embedding two source prediction loops, one for each description, within a common noise shaping loop. While the noise shaping loop controls the tradeoff between the side and the central distortions, the role of prediction (like in differential pulse code modulation) is to extract the source innovations from the reconstruction at each of the side decoders, and thus reduce the coding rate. Finally, we show that this scheme achieves the MD rate-distortion function at all resolutions and all side-to-central distortion ratios, in the limit of high dimensional quantization.

1 Introduction

The traditional multiple description (MD) problem [1] describes a source sequence $X[n]$ which is encoded into two descriptions, $Y_1[n]$ and $Y_2[n]$, using rates R_1 and R_2 respectively. Given one of these descriptions, the decoder produces a reconstruction $\hat{X}_1[n]$ or $\hat{X}_2[n]$. If both descriptions are available, the reconstruction is $\hat{X}_C[n]$. The achieved distortion triplet is $D_1 \triangleq E\{\overline{d(X,\hat{X}_1)}\}$, $D_2 \triangleq E\{\overline{d(X,\hat{X}_2)}\}$, and $D_C \triangleq E\{\overline{d(X,\hat{X}_C)}\}$, where $d(\cdot,\cdot)$ is a distortion measure, and $\overline{(\cdot)}$ denotes time-averaging over the source sequence.

The MD quadratic rate-distortion function (RDF) for memoryless Gaussian sources was found by Ozarow [1] and the extension to stationary Gaussian sources was recently completed by Chen et al. [2].

In [3], it was shown that Ozarow's white Gaussian MD RDF can be achieved by dithered Delta-Sigma quantization (DSQ) and memoryless entropy coding. Furthermore, by exploiting the fact that Ozarow's test channel becomes asymptotically optimal for stationary sources in the high-rate regime [4], it was shown in [3] that, at high resolution, the stationary MD RDF is achievable by DSQ and *joint* entropy coding.

In [2] it is demonstrated how one can achieve any point on the boundary of the colored Gaussian achievable rates region by a frequency-domain scheme, where the source is divided into sub-bands, and in each sub-band the scheme of [5] is applied. In this paper, we apply a *time domain* approach: We show that these optimum points can be achieved at

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all resolutions, in the symmetric case, using noise-shaping predictive coding and *memoryless* entropy coding. We establish this result by forming a nested prediction / noise-shaping structure containing a dithered DSQ scheme similar to [3] in the outer loop and a predictive coder per each description in the inner loop, see for example Fig. 5. Each of the predictive coders has the structure of the DPCM scheme, shown to be optimal in the single-description (SD) setting in [6].

The idea of exploiting prediction in MD coding has previously been proposed by other authors, see for example the following related works [7–10]. All these works faced the basic problem: Since DPCM uses prediction from the reconstruction rather than from the source itself, and this prediction should be reproduced at the decoder, it is not clear which of the possible reconstructions should be used for prediction. This work solves this problem.

The role of the DSQ loop is to shape the quantization noise so that a desired tradeoff between the side distortions and the central distortion is achieved. It was shown in [3] that the central distortion is given by the power of the noise that falls within the in-band spectrum (i.e. the part of the frequency spectrum which overlaps the source spectrum) whereas the side distortion is given by the power of the complete noise spectrum, i.e. the in-band and the out-of-band noise spectrum. It was furthermore shown that any ratio of side-to-central distortion can be obtained by proper shaping of the quantization noise. We establish a similar result here.

To summarize, the predictive coders take care of the source memory and thereby minimize the coding rate and make sure that memoryless entropy coding is optimal. Moreover, the DSQ loop performs the noise shaping which is required in order to achieve any desired pair of distortions (D_S, D_C).

This paper is organized as follows. In Section 2 we describe the main problem which is considered in this work. Then, in Section 3, we propose a test channel which provides a new interpretation of the MD quadratic Gaussian RDF. With this test channel in mind, we present, in Section 4, an SD scheme which encodes a source subject to a distortion mask. Finally, in Section 5, we extend the SD scheme of Section 4 to the MD case.

2 Problem Formulation and Notation

In this work, we are interested in the symmetric case, where $R_1 = R_2 \triangleq R$ and $D_1 = D_2 \triangleq D_S$. We will consider a discrete-time stationary Gaussian source $X[n]$ with spectrum $S_X(e^{j2\pi f})$, $|f| \leq 1/2$. We assume that the spectrum obeys the Paley-Wiener conditions [12], such that it has a positive entropy-power $0 < P_e(X) < \infty$, where the entropy power of a spectrum $S(e^{j2\pi f})$ is defined as:[1]

$$P_e(S) \triangleq \exp \int_{-\frac{1}{2}}^{\frac{1}{2}} \log\Big(S(e^{j2\pi f})\Big) df \tag{1}$$

and where here and onwards all logarithms are taken to the natural base. Using this notation, a spectrum has a spectral decomposition:

$$S(e^{j2\pi f}) = P_e(S) \cdot A(z) A^*\left(\frac{1}{z^*}\right)\Bigg|_{z=e^{j2\pi f}} , \tag{2}$$

[1]For arbitrary distributed sources with finite differential entropy $h(X)$, $P_e(X) \triangleq \frac{1}{2\pi e} e^{2h(X)}$. For stationary Gaussian sources, $h(X) = \frac{1}{2}\log(2\pi e) + \frac{1}{2}\int \log(S_X(e^{j2\pi f}))df$ from which (1) follows.

where the causal and monic $A(z)$ is the *optimal predictor* associated with the spectrum S.

We consider the coding problem of this source under a mean squared error (MSE) distortion criterion.

We will be using entropy-constrained dithered (latttice) quantizers (ECDQs) for which it is known that the additive noise model is exact at all resolutions [13]. We will furthermore assume the existence of a large number K of identical and mutually independent sources (or e.g. a single source which is divided into K long blocks and jointly encoded as K parallel sources, see [6] for details). These sources are treated independently, except for the actual ECDQ which processes them jointly. Thus we will only present the scheme for one source, but the quantization noise has the properties of a high-dimensional ECDQ (cf. [6]). We provide an asymptotic analysis in the limit $K \to \infty$. In this asymptotic case, the quantization noise becomes approximately Gaussian distributed (in a divergence sense) [14]. Thus, for analysis purposes, we can replace the quantizer with a white additive noise model where the noise is approximately Gaussian distributed.

3 The Quadratic Gaussian Symmetric MD Rate Revisited

In this section we re-state known results about the quadratic Gaussian MD achievable rate in the symmetric case, in order to gain some insight and prepare the ground for what follows. In the high resolution limit, these results also hold for general sources with finite differential entropy rate [11].

For a white Gaussian source of variance σ_X^2, the minimum achievable symmetric side-descriptions rate was given by Ozarow [1]: [5]):

$$R_{white}(\sigma_X^2, D_C, D_S) \triangleq \frac{1}{4}\log\frac{\sigma_X^2(\sigma_X^2 - D_C)^2}{4D_C(D_S - D_C)(\sigma_X^2 - D_S)} \tag{3}$$

as long as $\frac{1}{D_C} \geq \frac{1}{D_{C,max}} = \frac{2}{D_S} - \frac{1}{\sigma_X^2}$. Under high-resolution conditions, i.e. $D_S \ll \sigma_X^2$, the above rate becomes:

$$R_{white,HR} = \frac{1}{2}\log\frac{\sigma_X^2}{2\sqrt{D_C(D_S - D_C)}} \tag{4}$$

as long as $D_C \leq D_{C,max,HR} \triangleq \frac{D_S}{2}$.

If the central decoder was to linearly combine two side descriptions of mutually independent distortions of variances D_S, it would achieve exactly the distortion $D_{C,max}$. This gives the motivation to the model of *negatively correlated* side distortions (see [4]). In the high resolution limit, the relation between the side and central distortions can be explained by the side distortions having a correlation matrix:

$$\Phi = D_S \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \tag{5}$$

where $\rho = -\frac{D_S - 2D_C}{D_S} \leq 0$. With this notation, (4) becomes:

$$R_{white,HR} = \frac{1}{2}\log\frac{\sigma_X^2}{\sqrt{|\Phi|}} = \frac{1}{2}\log\frac{\sigma_X^2}{D_S} + \frac{1}{2}\log\frac{1}{\sqrt{1-\rho^2}} \triangleq \frac{1}{2}\log\frac{\sigma_X^2}{D_S} + \frac{1}{2}\delta_{HR} \tag{6}$$

where δ_{HR} is the high-resolution excess rate [11]. Still in the high-resolution case, we take another step: Without loss of generality, we can represent the correlated noises as the sum

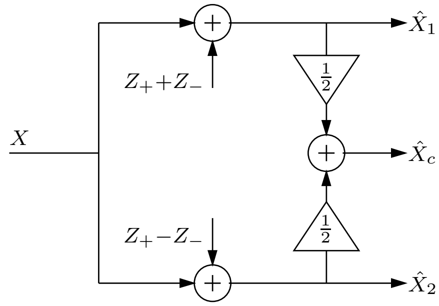

Figure 1: *A differential form of Ozarow's double-branch test channel for high resolution coding.*

of two mutually independent noises, one is added to both branches while the other is added to one branch and subtracted from the other, as depicted in Fig. 1. Note that the averaging eliminates Z_- from the central description. If we denote the variances of the noises Z_+ and Z_- as Θ_+ and Θ_-, respectively, then we can re-write (5) as:

$$\Phi = \begin{bmatrix} \Theta_+ + \Theta_- & \Theta_+ - \Theta_- \\ \Theta_+ - \Theta_- & \Theta_+ + \Theta_- \end{bmatrix}, \tag{7}$$

where the negative correlation $\rho < 0$ implies that $\Theta_- \geq \Theta_+$. In terms of these variances, we can define a spectrum:

$$\tilde{\Theta}(e^{j2\pi f}) \triangleq \begin{cases} 2\Theta_+, & |f| \leq \frac{1}{4} \\ 2\Theta_-, & \frac{1}{4} \leq |f| \leq \frac{1}{2}. \end{cases} \tag{8}$$

With the above definitions, we have that the entropy-power (1) of $\tilde{\Theta}(e^{j2\pi f})$ is given by:

$$P_e(\tilde{\Theta}) = \sqrt{|\Phi|} = 2\sqrt{\Theta_+\Theta_-}$$

and consequently the MD rate is:

$$R = \frac{1}{2}\log\frac{\sigma_X^2}{P_e(\tilde{\Theta})}. \tag{9}$$

The following proposition states this formally:

Proposition 1. *In the scheme of Fig. 1, let $\sigma_X^2 \geq \Theta_- \geq \Theta_+$. The distortions are given by:*

$$\begin{aligned} D_S &= \Theta_+ + \Theta_- \\ D_C &= \Theta_+. \end{aligned} \tag{10}$$

In the high resolution limit, for these distortions, the minimum rate (4) *is given by* (9).

Generalizing our view to all distortion levels, the equivalent channel is depicted in Fig. 2. A similar correlated-noises model to (5) can be obtained by expressing ρ in a rather complicated form. However, we can greatly simplify such an expression by proper use of pre- and post-factors as we show next. In a point-to-point scenario, it is convenient to make these factors equal [15], [13]. However, this is generally not possible in MD coding because the optimal post-factors (Wiener coefficients) are different for the side and central reconstructions. We choose the pre-factor to be equal to the *side* post-factor. While this choice seems arbitrary, it will prove useful when we turn to colored sources. Thus we have:

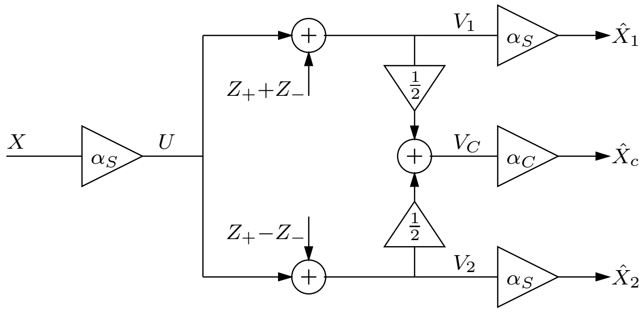

Figure 2: *Ozarow's test channel with pre and post factors.*

$$\begin{aligned} \alpha_S &\triangleq \sqrt{\frac{\sigma_X^2 - \Theta_+ - \Theta_-}{\sigma_X^2}} \\ \alpha_C &\triangleq \frac{\alpha_S \sigma_X^2}{\alpha_S^2 \sigma_X^2 + \Theta_+} = \sqrt{\frac{\sigma_X^2(\sigma_X^2 - \Theta_+ - \Theta_-)}{(\sigma_X^2 - \Theta_-)^2}}. \end{aligned} \tag{11}$$

Proposition 2. *In the scheme of Fig. 2, let $\sigma_X^2 \geq \Theta_- \geq \Theta_+$. The distortions are given by:*

$$\begin{aligned} D_S &= \Theta_+ + \Theta_- \\ D_C &= \frac{\sigma_X^2 \Theta_+}{\sigma_X^2 - \Theta_-}. \end{aligned} \tag{12}$$

For these distortions, the minimum achievable rate (3) *is given by* (9).

Note that at high resolution conditions $\sigma_x^2 \gg \Theta_-$, so (12) reduces to (10).

Proof: Between U and $\{V_1, V_2, V_C\}$ we have exactly the high-resolution scheme of Prop. 1, i.e. we have $V_1 = U + Z_1$, $V_2 = U + Z_2$, $V_C = U + Z_C$, where $\{Z_1, Z_2, Z_C\}$ are independent of U, and where $E\{Z_1^2\} = E\{Z_2^2\} = \Theta_+ + \Theta_-$ and $E\{Z_C^2\} = \Theta_+$. Since $\hat{X}_i = \alpha_S V_i$ and $\hat{X}_c = \alpha_C V_C$ it is, by use of (11), straightforward to show that $D_S = E\{(\hat{X}_i - X)^2\}$ and $D_C = E\{(\hat{X}_C - X)^2\}$ are given by (12). Now substitute these distortions in (3) to establish (9). ■

We now turn to general (colored) stationary Gaussian sources. In the high resolution limit, it was shown in [4] that the minimum rate is given by Ozarow's rate (3) with the source variance σ_X^2 replaced by its entropy-power $P_e(X)$ (1). Recalling (9) we define:

$$R_{colored} \triangleq \frac{1}{2} \log \frac{P_e(X)}{P_e(\tilde{\Theta})} \quad . \tag{13}$$

Proposition 3. *In the high resolution limit, for any $\Theta_- \geq \Theta_+$, the minimum achievable rate for the distortions* (10) *is given by* (13)*.*

For general resolution, the achievable colored Gaussian MD rate region was found by Chen et al. [2]. They prove, that the optimum rates for stationary Gaussian sources can be expressed as the sum of rates of parallel channels, each one representing a frequency band. Each of these channels must be tuned to a minimum Ozarow rate (3) for some distortions. The working point at each frequency is determined by a "water-filling" solution:

For all possible spectral distributions of the side and central distortions satisfying the total distortions, find the one which minimizes the side descriptions rate. No explicit solution to this optimization problem is presented in [2], and this remains an open problem. However, in terms of our representation for the white case, we can re-write the result of [2] (for the symmetric case) in a parametric form. For given source spectrum $S_X(e^{j2\pi f})$ and noise spectra $\Theta_+(e^{j2\pi f})$ and $\Theta_-(e^{j2\pi f})$, we generalize (8) to the form[2]:

$$\tilde{\Theta}(e^{j2\pi f}) = \begin{cases} 2\Theta_+\left(e^{j4\pi f}\right), & |f| \le \frac{1}{4} \\ 2\Theta_-\left(e^{j4\pi\left(f-\frac{1}{4}\right)}\right), & \frac{1}{4} < f \le \frac{1}{2} \\ 2\Theta_-\left(e^{j4\pi\left(f+\frac{1}{4}\right)}\right), & -\frac{1}{2} \le f < -\frac{1}{4} \end{cases} \tag{14}$$

and define the distortion spectra:

$$\begin{aligned} D_S(e^{j2\pi f}) &\triangleq \Theta_+(e^{j2\pi f}) + \Theta_-(e^{j2\pi f}) \\ D_C(e^{j2\pi f}) &\triangleq \frac{S_X(e^{j2\pi f})\Theta_+(e^{j2\pi f})}{S_X(e^{j2\pi f}) - \Theta_-(e^{j2\pi f})}, \end{aligned} \tag{15}$$

reflecting the use of pre- and post-filters. Then the result of [2] is equivalent in the symmetric case to the following Proposition and Corollary:

Proposition 4. *For any spectra*

$$S_X(e^{j2\pi f}) \ge \Theta_-(e^{j2\pi f}) \ge \Theta_+(e^{j2\pi f}) \ge 0 \ \ \forall f,$$

the minimum achievable side-description rate in symmetric MD coding of a Gaussian source with spectrum $S_X(e^{j2\pi f})$ with the side and central distortion spectra (15) *is given by* (13)*.*

Corollary 1. *The optimum symmetric MD side-description rate is given by the minimization of* (13) *over all $\Theta_+(e^{j2\pi f})$, $\Theta_-(e^{j2\pi f})$ such that the distortion spectra* (15) *satisfy:*

$$\begin{aligned} \int_{-\frac{1}{2}}^{\frac{1}{2}} D_S(e^{j2\pi f})df &\le D_S \\ \int_{-\frac{1}{2}}^{\frac{1}{2}} D_C(e^{j2\pi f})df &\le D_C. \end{aligned}$$

Note that in the high resolution limit, the spectrum $\Theta(e^{j2\pi f})$ becomes a two-step spectrum, as in [3].

4 Source Coding Subject to a Distortion Mask

We take a detour to a problem that is suggested by Proposition 4; coding of a source subject to a maximum distortion *mask* $D(e^{j2\pi f})$, rather than subject to a total distortion constraint. This is an SD problem, but the solution will be extended to the MD problem in the following

[2]Notice that the lowpass and highpass spectra of $\tilde{\Theta}(e^{j2\pi f})$ are formed by $\Theta_+(e^{j4\pi f})$ and $\Theta_-(e^{j4\pi f})$, which are compressed versions (by a factor of two) of the spectra $\Theta_-(e^{j2\pi f})$ and $\Theta_+(e^{j2\pi f})$, respectively.

section. Without loss of generality[3], we assume that $D(e^{j2\pi f}) \leq S_X(e^{j2\pi f})\ \ \forall f$. It is easy to verify, that the minimum rate for this problem is given by (recall (13)):

$$R\Big(S_X(e^{j2\pi f}), D(e^{j2\pi f})\Big) = \frac{1}{2}\log\frac{P_e(X)}{P_e\Big(D(e^{j2\pi f})\Big)}. \tag{16}$$

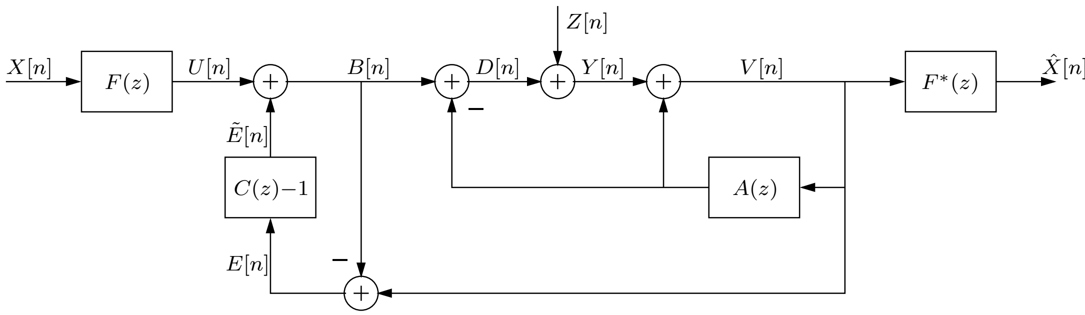

Figure 3: *A DSQ/DPCM equivalent channel for SD coding subject to a distortion mask.*

Fig. 3 presents a *time domain* scheme which achieves this rate. Motivated by the ratio of entropy powers (16), we strive to achieve the optimal rate by the combination of *source prediction* in order to present the quantizer with a prediction error of power $P_e(X)$, and *noise shaping* in order to shape the white quantization noise of power $P_e(D)$ into the spectrum $D(e^{j2\pi f})$[4]. These two tasks, we perform by a DPCM loop [6] and a noise-prediction loop [3], respectively. In this scheme, $Z[n]$ is AWGN of variance $P_e(D)$, $A(z)$ and $C(z)$ are the optimal predictors (2) of the source spectrum $S_X(e^{j2\pi f})$ and the equivalent distortion spectrum $D(e^{j2\pi f})$, respectively. Note that $E[n]$, the input to the noise-shaping filter, is equal to $Z[n]$. The pre-filter $F(e^{j2\pi f})$ satisfies:

$$|F(e^{j2\pi f})|^2 = \frac{S_X(e^{j2\pi f}) - D(e^{j2\pi f})}{S_X(e^{j2\pi f})}. \tag{17}$$

Theorem 1. *The channel of Fig. 3 with the choices above, satisfies:*

$$S_{\hat{X}-X}(e^{j2\pi f}) = S_{V-U}(e^{j2\pi f}) = D(e^{j2\pi f}) \tag{18}$$

with the scalar *mutual information* $I(D[n]; Y[n]) = R\Big(S_X(e^{j2\pi f}), D(e^{j2\pi f})\Big)$ *of* (16).

Proof: Since $E[n] = Z[n]$, we have that $V[n] = U[n] + Z[n] * c[n]$ so $V[n]$ and $U[n]$ are connected by an additive noise channel with noise spectrum $D(e^{j2\pi f})$. From here, using the pre/post filter given by (17), the distortions follow immediately. Since $S_V(e^{j2\pi f}) = S_X(e^{j2\pi f})$, it also means that the mutual information rate $\bar{I}(U[n]; V[n])$ equals the desired rate (16). Since $V[n] = A[n] + Z[n]$ the mutual information rate $\bar{I}(A[n]; V[n])$ is the same. Applying [6, Thm.1], the scalar mutual information follows. ■

[3]Otherwise, there is just wasted allowed distortion which does not serve to reduce the rate.

[4]An alternative time-domain approach, is to accommodate for the distortion mask by changing the pre and post-filters. However, we choose the noise-shaping approach for the sake of extending this scheme to the MD setting.

We remark that, in the special case of a white distortion mask $D(e^{j2\pi f})$, the constraint becomes (by the water-filling principle) equivalent to a regular quadratic distortion constraint. Indeed, the channel collapses in this case to the pre/post filtered DPCM channel of [6]. Much of the analysis there remains valid for this problem as well. In particular, we can construct an optimal coding scheme using this channel, substituting the AWGN for an ECDQ, and the scalar mutual information $I(D[n]; Y[n])$ is also equal to the directed mutual information $I(D[n] \rightarrow Y[n])$.

5 Optimal Time-Domain Colored MD Coding

The similarity between the rates (13) and (16) is evident. We also note, that Thm. 1 deals with achieving the minimum rate subject to a distortion mask constraint, while Proposition 4 tells us that we must minimize the rate subject to *two* distortion mask constraints.

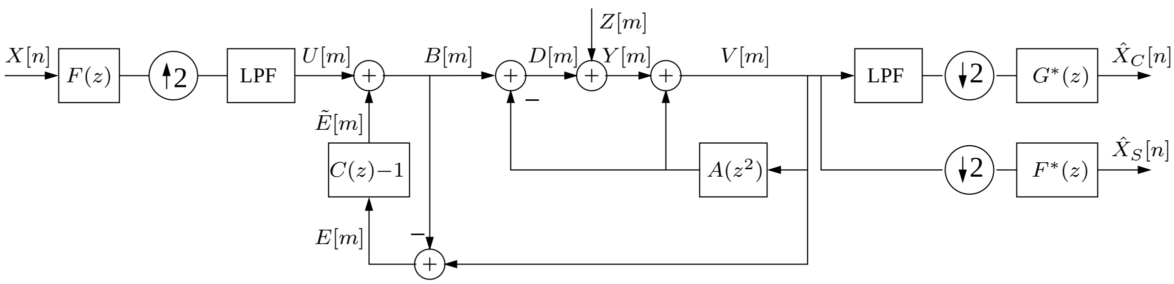

Figure 4: *A DSQ/DPCM equivalent channel for MD coding of a colored source.*

Fig. 4 shows the adaptation of the distortion-mask equivalent channel to the MD problem.[5] Following [3], we combine upsampling by a factor of two with the noise-prediction loop, forming a DSQ loop. $C(z)$ and $A(z)$ are the optimal predictors (2) of the spectra $S_X(e^{j2\pi f})$ and $\tilde{\Theta}(e^{j2\pi f})$, as before. Note that we apply an upsampled version of the source predictor, namely $A(z^2)$. Since the two side descriptions consist of the even and odd instances of $V[m]$, this is equivalent to applying the predictor $A(z)$ to each description in the original source rate. The DSQ loop, on the other hand, works in the upsampled rate. For a white source, $A(z) = 0$ and the channel reduces to the DSQ MD scheme of [3], while for optimal side distortion, $C(z) = 0$, and the channel reduces to an upsampled version of the DPCM equivalent channel of [6].

The filters $F(e^{j2\pi f})$ and $G(e^{j2\pi f})$ play the roles of pre/post filters and satisfy:

$$
\begin{aligned}
|F(e^{j2\pi f})|^2 &= \frac{S_X(e^{j2\pi f}) - \Theta_+(e^{j2\pi f}) - \Theta_-(e^{j2\pi f})}{S_X(e^{j2\pi f})} \\
G(e^{j2\pi f}) &= \frac{S_X(e^{j2\pi f})}{S_X(e^{j2\pi f}) - \Theta_-(e^{j2\pi f})} F(e^{j2\pi f}).
\end{aligned} \tag{19}
$$

Theorem 2. *The channel of Fig. 4 with the choices above, satisfies:*

$$
\begin{aligned}
S_{\hat{X}_C - X}(e^{j2\pi f}) &= D_C(e^{j2\pi f}) \\
S_{\hat{X}_S - X}(e^{j2\pi f}) &= D_S(e^{j2\pi f})
\end{aligned} \tag{20}
$$

[5] We use the index n for sequences which are "running" at the source rate, and the index m when referring to the upsampled rate.

where the distortion spectra were defined in (15)*, while the* scalar *mutual information* $I(D[n];Y[n])$ *equals twice the rate* $R_{colored}$ *of* (13).

The proof basically follows from Thm. 1 by taking appropriate care of rate changes. However, due to lack of space, we omit the details here.

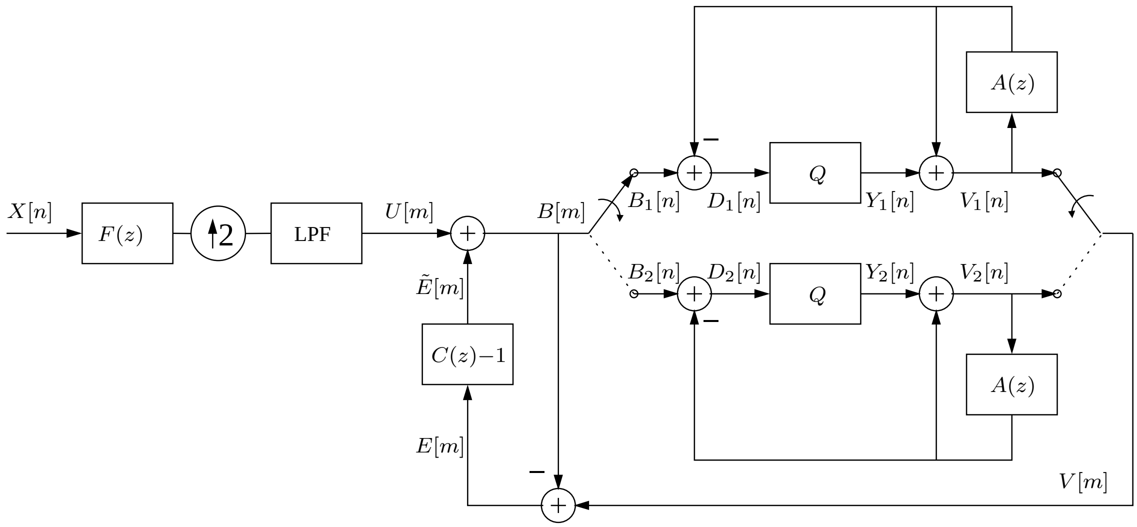

Figure 5: *Nested DSQ/DPCM MD encoder.*

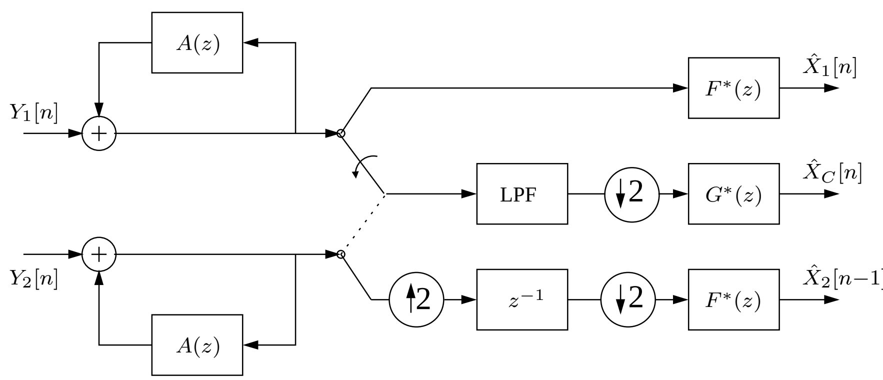

Figure 6: *DSQ/DPCM MD decoder.*

The encoder and decoder which materialize this equivalent channel are presented in Fig. 5 and Fig. 6, respectively. All of the switches in the encoder and the decoder are synchronized.[6] The up sampling operation followed by lowpass filtering introduces a half-sample delay on the odd samples. This delay is corrected at the decoder by the delay operator z^{-1} combined with the pair of up and downsamplers, see Fig. 6. If each quantizer block Q is taken to be a high-dimensional ECDQ with the required rate, and the two quantizer dither sequences are mutually independent, then these quantizers are equivalent to the additive noise $Z[m]$ of the equivalent channel. Consequently, the two descriptions $Y_1[n]$ and $Y_2[n]$

[6]It is to be understood that the switches change their positions with the upsampled rate (m). Thus, in the encoder shown in Fig. 5, the even samples $B_1[n]$ of $B[m]$ will go on the upper branch and the odd samples $B_2[n]$ will go on the lower branch.

are equivalent to the odd and even samples, respectively, of $Y[m]$ in the equivalent channel, and finally the whole scheme from the source to the central and side reconstructions is equivalent to the channel from $X[n]$ to $\hat{X}_C[n]$ and $\hat{X}_S[n]$, respectively.

Since we see that this scheme achieves the optimal rate for any choice of spectra, it will become globally optimal when its parameters are chosen according to the minimizing spectra of Proposition 4. Thus, the encoder/decoder pair of Figs. 5 and 6 is able to achieve the complete quadratic MD RDF for stationary Gaussian sources at all resolutions and for any desired side-to-central distortion ratio.

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